A Two Population Model for the Stock Market Problem

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Abstract. The development of the last year disaster in the Stock Markets all over the world gave rise to reconsidering the previous models used. It is clear that, even in an organized international or national context, large fluctuations and sudden losses may occur. This paper explores a two populations' model. The populations are conflicting into the same environment (a Stock Market) by following the main rules present, that is mutual interaction between adopters, potential adopters, word-of-mouth communication and of course by taking into consideration the innovation diffusion process. The proposed model has special futures expressed by third order terms providing characteristic stationary points.
Keywords: Chaotic modeling, The stock-market problem, Stock-Market, Innovation diffusion modeling, Lotka-Volterra, Simulation, Chaotic simulation.

1 Introduction

Several attempts to model the stock-market problem can be found in the literature. Between these models the Lotka-Volterra modeling approach is of considerable interest. The model can be used if we assume two interacting populations and can be generalized to more than two. The case of two interacting populations is explored by using two coupled differential equations and the results found give rise to a limit cycle and the corresponding oscillating graphs over time, Skiadas, 2009 [7]. However, the Lotka-Volterra system of differential equations, a non-linear system of the second order, fails to explain the sudden growth and decline resulting into the stock-market environment and even more the stability to high or low values (capital gains or losses). In view of the last year losses in the stock markets globally it would be very important to reconsider the classical Lotka-Volterra theory by introducing into the corresponding equations terms having to do with the diffusion aspect of the communication process. The proposed model has special futures expressed by third order terms providing characteristic stationary points. It should be noted that a seminal paper was published by Harold Hotteling [1] during March 1929

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few months before the big crash in the New York stock exchange. Hotelling, in this paper explored the "stability in competition" problem. The analysis proposed was based in a simplification of the problem. Only two markets where competing; however, the results where extremely important and clarified the situation. In this paper we accept the same methodology supposing that there is a stock market with two interacting populations and we explore the arising case. That is new is the introduction of the theories of the last decades on the adoption-diffusion of innovations in developing the equations of the proposed model.

2 The Model and Simulations

To model the specific situation we take into account that two populations x and y are present into the stock market and interact each other. Even more to simplify the case we suppose that the variables x_t and y_t stand for the number of players or for the number of transactions or for the values of the stocks belonging to each part of the players at a specific time period t. The aim is to explore the behavior of the two populations during time and especially in the limits. A model including the main characteristics of two interrelating or even conflicting populations into a stock market can be expressed by the following set of differential equations:

$$\dot{x} = a_1 y + a_2 xy + a_3 x(l-y) + a_4 yx(k-x)$$
$$\dot{y} = b_1 x + b_2 xy + b_3 y(k-x) + b_4 xy(l-y),$$

where a_1, a_2, a_3, a_4 and b_1, b_2, b_3, b_4 are parameters expressing the mutual interaction of the populations x and y. The parameters k and l express the upper limit of the populations x and y respectively. The first term to the right stands for the flows from the one part to the other whereas the second term expresses the mutual interaction between the active parts of the populations x and y. If we retain the first two terms to the right the proposed model is of the Lotka-Volterra type. The non active parts of the populations are $(k-x)$ for x and $(l-y)$ for the population y. The interaction of these parts with the active populations y and x is expressed with the third terms to the right of the above differential equations (see C. H. Skiadas [2],[3],[4], Skiadas and Giovanis [5] and Skiadas *et al* [6]). The fourth terms to the right include the terms $x(k-x)$ and $y(l-y)$ multiplied by y and x respectively. These terms account to the rates of adoption-diffusion that is extremely important in order to express the word-of-mouth communication between adopters (the active part of the players) and potential adopters (the non-active part of the players).

Looking back to the set of the two differential equations above we see that we have two third degree equations for x and y. In the following we check the influence of the third order term to the stock-market model. To this end the above model is simplified to the following form:

$$\dot{x} = -ay + cyx^2$$

$$\dot{y} = bx - cxy^2,$$

Where a, b and c are parameters of interaction. Especially the parameter c is selected to be the same in both equations expressing the coupling of both populations x and y.

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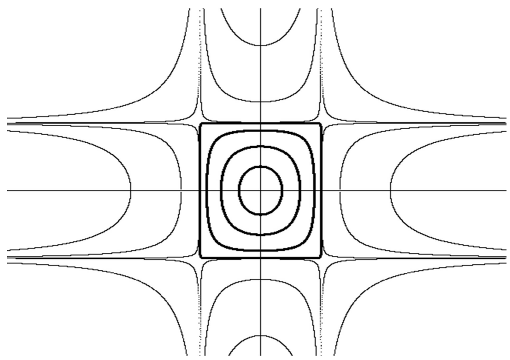

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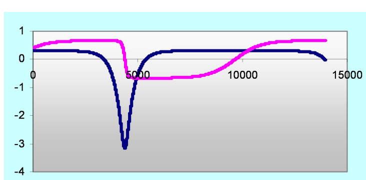

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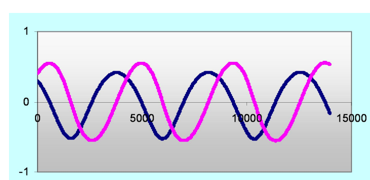

Fig. 3. The caption for figure 3

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3 Conclusions

A model expressing two conflicting populations in the stock-market is developed. A general model is formulated and a simpler one is explored and simulated. The results support the well known process of fluctuations, oscillations and further sudden growth and decrease of gains-losses of the two conflicting populations. It was derived that it could arise that both would be stabilized to gains or losses or to stabilize the one in gains and the other to losses.

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