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Adaptive Control for Synchronization of Li Chaotic Systems with Unknown Parameters

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Abstract. In this paper, the problems on chaos control and synchronization of Li chaotic systems with unknown parameters are considered. Firstly, the Li chaotic systems are introduced. Secondly, the adaptive control system with only two state feedbacks for synchronization of the Li chaotic systems is presented and the parameter identification method is also given. Sufficient conditions for the stability of the synchronized errors are provided. Finally, numerical studies are performed to verify the effectiveness of presented schemes.
Keywords: chaos, synchronization, unknown parameters, adaptive controller.

1 Instroduction

Since the synchronization of the coupled chaotic dynamical system with different initial conditions was observed by Pecora and Carroll [1], the idea of synchronization of chaotic systems has gained a lot of attention from various disciplines due to its potential application in many fields such as chemical reactions[2], biological systems[3], and secure communications[4], etc. In the past two decades, a variety of types of synchronization approaches in dynamical systems have been proposed such as adaptive control [5], observer based control [6], variable structure control [7], back stepping control [8], impulsive control [9], nonlinear control [10], and so on.

In practical engineering situations, it is often the case that the parameters of chaotic systems are unknown. Therefore, there is an immense need for algorithms that can effectively synchronize chaotic systems with unknown parameters for theoretical research and practical application.

This paper presents the synchronization between Li systems with unknown parameters accompanied by the presented control system contains only two state feedbacks. Based on the Lyapunov stability theory, we prove that the suggested approach can realize chaos synchronization globally and asymptotically and recognize the unknown parameters as well. Numerical simulations demonstrate the effectiveness of the proposed synchronization methods.

The rest of this paper is organized as follows. The problem formulation and systems description are performed in section 2 and section 3, respectively.

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Section 4 presents the adaptive control method for the Li systems with unknown parameters. Numerical simulations are performed in section 5 to verify the effectiveness of the presented schemes, and concluding remarks are made in the final section.

2 Synchronization

Consider a class of chaotic system which can be described as

$$\dot{x} = f(x) \tag{1}$$

where $x = x_1, x_2, ..., x_n$ are the state vectors. Take system (1) as the drive system, and the response system is defined as

$$\dot{y} = g(y) + u(t, x, y) \tag{2}$$

where $y = y_1, y_2, ..., y_n$ represent the the state vectors, and $f, g : R^n \to R^n$ are two continuous nonlinear vector functions, $u(t, x, y)$ is an $n-$dimensional control signals.

Let $e(t) = y(t) - x(t)$ is the synchronization error. The control goal is to design the controller $u(t, x, y)$ for the response system (2), such that the error system $e(t)$ can be asymptotically stable at the zero equilibrium. In this sense, that is $\lim\limits_{t\to\infty} e(t) = 0$, which implies that the error dynamic system $e(t)$ between the drive system and the response system is globally asymptotically stable.

3 Systems description

Recently, Li constructed a three-dimensional chaotic system [11], which is described by

$$\begin{cases} \dot{x_1} = a(x_2 - x_1) \\ \dot{x_2} = x_1 x_3 - x_2 \\ \dot{x_3} = b - x_1 x_2 - c x_3 \end{cases} \tag{3}$$

where x_1, x_2, x_3 are state variables, and a, b, c are system parameters. When the system parameters are $a = 5, b = 16, c = 1$, the system (3) demonstrates a chaotic attractor. The three-dimensional view of the chaotic strange attractor and some dynamical behavior in different planes for system (3) are shown in Fig. 1.

4 Synchronization of Li system with unknown parameters

Suppose the master system is defined in (3) which drives the slave system given in the following form

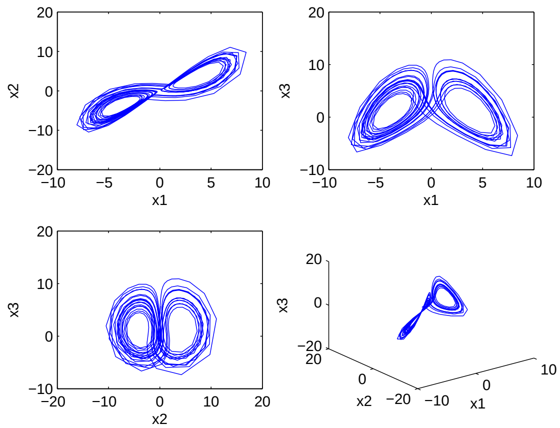

Fig. 1. Typical dynamical behaviors of five-dimensional hyperchaotic Lorenz system.

$$\begin{cases} \dot{y_1} = \widetilde{a}(y_2 - y_1) + u_1 \\ \dot{y_2} = y_1 y_3 - y_2 + u_2 \\ \dot{y_3} = \widetilde{b} - y_1 y_2 - \widetilde{c} y_3 + u_3 \end{cases} \tag{4}$$

where $y_i (i = 1, 2, 3)$ are state variables, $u_i (i = 1, 2, 3)$ are external control inputs, and $\widetilde{a}, \widetilde{b}, \widetilde{c}$ are unknown parameters to be identified.

The error vector can be defined as

$$\begin{cases} e_1 = y_1 - x_1 + u_1 \\ e_2 = y_2 - x_2 + u_2 \\ e_3 = y_3 - x_3 + u_3 \end{cases} \tag{5}$$

So the detail error dynamics is as follows

$$\begin{cases} \dot{e_1}(t) = \widetilde{a}(e_2 - e_1) + \overline{a} x_2 - \overline{a} x_1 + u_1 \\ \dot{e_2}(t) = -e_2 + e_1 e_3 + x_1 e_3 + x_3 e_1 + u_2 \\ \dot{e_3}(t) = \overline{b} - e_1 e_2 - x_1 e_2 - x_2 e_1 - \widetilde{c} e_3 - \overline{c} x_3 + u_3 \end{cases} \tag{6}$$

where $\overline{a} = \widetilde{a} - a$, $\overline{b} = \widetilde{b} - b$ and $\overline{c} = \widetilde{c} - c$.

Theorem. The drive system (3) and the response system (4) can be asymptotically synchronized for any different initial condition with following adaptive controller

$$\begin{cases} u_1 = -\widetilde{a}(e_2 - e_1) - e_1 \\ u_2 = 0 \\ u_3 = e_1 e_2 + x_1 e_2 + x_2 e_1 + (\widetilde{c} - 1) e_3 \end{cases} \tag{7}$$

and the following parameter laws of $\widetilde{a}$,$\widetilde{b}$ and $\widetilde{c}$

$$\begin{cases} \dot{\widetilde{a}} = -x_2 e_1 + x_1 e_1 \\ \dot{\widetilde{b}} = -e_3 \\ \dot{\widetilde{c}} = x_3 e_3 \end{cases} \tag{8}$$

Proof. Firstly, choose Lyapunov function as follows

$$V_1 = \frac{1}{2}(e_1^2 + (\widetilde{a} - a)^2) \tag{9}$$

Its time derivative along the trajectories of (6) is

$$\begin{aligned} \dot{V_1} &= (e_1 \dot{e_1} + \overline{a}\dot{\widetilde{a}}) \\ &= -e_1^2 + e_1(a(e_2 - e_1) + \overline{a}(e_2 - e_1) + e_1 + \overline{a}x_2 - \overline{a}x_1 + u_1) + \overline{a}\dot{\widetilde{a}} \end{aligned} \tag{10}$$

Substituting (7) and (8) into (10), we have

$$\dot{V_1} = -e_1^2 \leq -2V_1(t) \tag{11}$$

which leads to $V_1(t) \leq V_1(0)\exp(-t)$, thus we have $\lim\limits_{t\to\infty} e_1(t) = 0$.

On the other hand, we take another Lyapunov function as

$$V_2 = \frac{1}{2}(e_3^2 + (\widetilde{b} - b)^2 + (\widetilde{c} - c)^2) \tag{12}$$

Its time derivative along the trajectories of (6) is

$$\begin{aligned} \dot{V_2} &= (e_3 \dot{e_3} + \overline{b}\dot{\widetilde{b}} + \overline{c}\dot{\widetilde{c}}) \\ &= -e_3^2 + e_3(\overline{b} - e_1 e_2 - x_1 e_2 - x_2 e_1 - \overline{c}e_3 - \overline{c}x_3 + (1-c)e_3 + u_3) + \overline{b}\dot{\widetilde{b}} + \overline{c}\dot{\widetilde{c}} \end{aligned} \tag{13}$$

Substituting (7) and (8) into (13), we haven

$$\dot{V_2} = -e_3^2 \leq -2V_2(t) \tag{14}$$

which leads to $V_2(t) \leq V_2(0)\exp(-t)$, thus we have $\lim\limits_{t\to\infty} e_3(t) = 0$.

Now, by solving the second equation of (6) we get the following result

$$e_2 = \exp(-t)(e_2(0)) + \int_0^t \exp(t)(e_1 e_3 + x_1 e_3 + x_3 e_1)dt \tag{15}$$

Because $\lim\limits_{t\to\infty}(e_1 e_3 + x_1 e_3 + x_3 e_1) = 0$, thus by basic calculation one can yield $\lim\limits_{t\to\infty} e_2(t) = 0$. Therefore, the error $e_i(t) = 0 (i = 1, 2, 3)$ will converge to zero.

Consider the first and the third equation of (6) we get the following result

$$\begin{cases} \lim\limits_{t\to\infty} \overline{a}(x_2 - x_1) = 0 \\ \lim\limits_{t\to\infty} \overline{b} = 0 \\ \lim\limits_{t\to\infty} \overline{c}x_3 = 0 \end{cases} \tag{16}$$

then we can have

$$\begin{cases} \lim\limits_{t\to\infty} \widetilde{a} = a \\ \lim\limits_{t\to\infty} \widetilde{b} = b \\ \lim\limits_{t\to\infty} \widetilde{c} = c \end{cases} \tag{17}$$

This completes the proof.

5 Simulation examples

In this section, some numerical simulations about the synchronization between the drive system (3) and the response system (4) are given to verify the effectiveness of the proposed method. In the numerical simulations, the fourth-order Runge-Kutta method is used to solve the system. The system parameters are selected as $a = 5, b = 16, c = 1$ and $a_1 = 4, b_1 = -3, c_1 = 9$, such that the drive system and the response system are chaotic with no control applied. We employed the initial conditions be $x(0) = (2, -4, 6)$ for the drive system and $y(0) = (-1, 1, 2)$ for the response system, respectively.

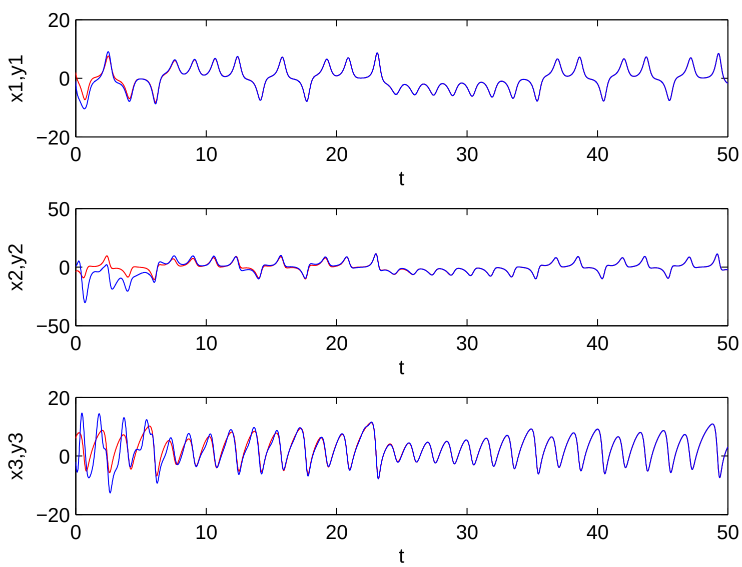

Fig. 2. State trajectories of synchronization between the drive system and the response system.

Numerical results are displayed in Figures 2-4. Fig.2 shows shows the time evolution curves of the drive system and the response system with controllers (7) and parameter laws (8). Fig.3 shows the time evolution of the synchronization errors, which displays that the errors tend to zero as $t \to \infty$. In addition,

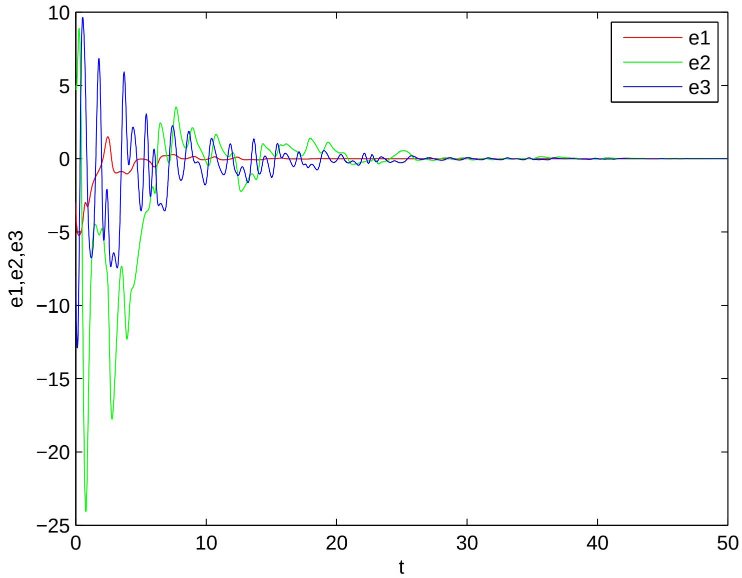

Fig. 3. Time response of the synchronization errors.

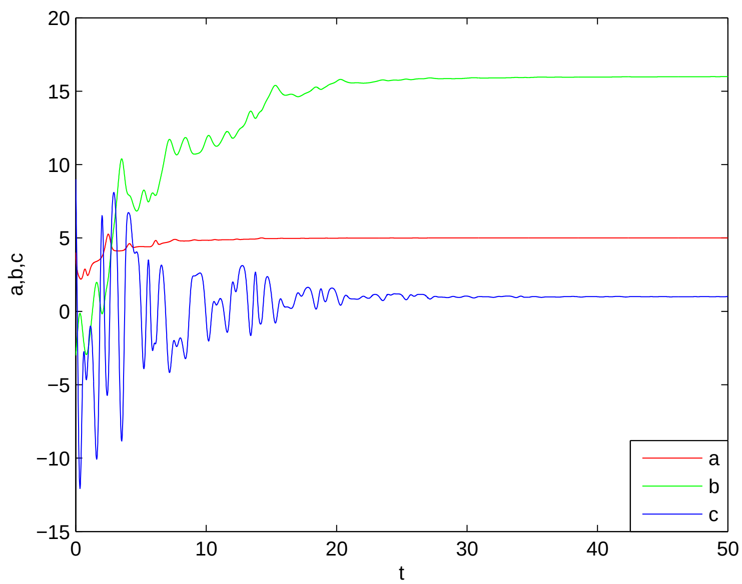

Fig. 4. Graph of the estimate parameters results.

the estimations of the parameters are shown in Fig.4. These results show that synchronization between Li chaotic systems with unknown parameters has been achieved with our designed adaptive controller and parameter laws.

6 Conclusions

In this paper, we have studied the robust adaptive synchronization between two Li chaotic systems with unknown parameters based on adaptive control and stability theory. The effectiveness of the proposed approach has been verified by the numerical simulations. The proposed adaptive control method can not only achieve synchronization but also identify the system parameters at the same time. The presented control method can also be applied in many other chaotic systems.

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