The profile of temperature in the dissipative over-dense plasma layer

L. Rajaei1⋆, S. Miraboutalebi2

1 Physics Department, Qom University, Qom, Iran
(E-mail: rajaeel@yahoo.com)
2 Physics Department, Islamic Azad University, North Tehran Branch, Tehran, 1651153311, Iran
(E-mail: smirabotalebi@gmail.com)

Abstract. An investigation is undertaken to introduce an effective mechanism of plasma heating which significantly enhances and facilitates the heating of a dense plasma layer. This mechanism directly related to the phenomena of anomalous transparency of a dense plasma layer through the resonant excitation of the coupled surface waves. It is shown that the collisional effects reduce the rate of the energy transmission through the plasma layer under resonant conditions. This dissipative effects cause heating of the plasma layer to a considerable amount of temperature. The temperature distribution in the plasma layer during the transmission of the electromagnetic waves is studied.

Keywords: Microwave, Dissipation, Incident wave, Over-dense plasma, Surface plasma, Transparency.

1 Introduction

The investigations of circumstances and important factors in the plasma heating is the subject of relevance for many fields of plasma physics ranging from laboratory experiments to astrophysics [1]. The heat flow and temperature gradient in plasma are fundamental process that take place during plasma phenomena and are of great importance effects. These processes in some situations, have inevitable destructive effects. One of the challenges of embedding plasma in electric field is the appearance of high temperature electrons which acts like an internal transport barrier. The three well known heating mechanisms of plasma are ohmic heating, neutral beam injection and high frequency electromagnetic waves. In the high frequency electromagnetic waves mechanism of plasma heating, the energy of waves is converted into thermal electron motion through electron-ion collisions [2,3]. Also the interaction of the electromagnetic waves with plasma may lead to the resonant excitation of the electrostatic waves which subsequently decays generating energetic electrons [4].

⋆ Corresponding author: Tel +98-25-32852067. E-mail address: rajaeel@yahoo.com

1 Received: 7 August 2013 / Accepted: 10 April 2014
© 2014 CMSIM
ISSN 2241-0503
In our previous works we studied the total transparency conditions of an overdense plasma layer due to the resonant excitation of the coupled surface modes, [5]-[9]. In the present paper we demonstrate another mechanism of plasma heating that significantly enhances and facilitates the heating of the dense plasma. We will show that for a collisional plasma under resonant conditions, the dissipative effects cause heating of the dense plasma to a considerable amount of temperature. Hence the process of resonant excitation of the coupled plasmons can be considered as an effective mechanism of plasma heating. In fact this process has a dual acting effect, it cause the transparency of the plasma slab and at the same time it gives rise to plasma heating.

This paper is organized as follows: In section two the geometrical construction of the problem and the transmission of the electromagnetic waves through the considered structure. In section three the solutions of the heat equations at steady states condition is given and the temperature is predicted. Finally, section four presents the results.

2 the model

![Diagram showing the spatial distribution of the effective electric permittivity.](image)

Fig. 1. Schematic diagram showing the spatial distribution of the effective electric permittivity.

Let us consider a geometrical structure fulfills the conditions under which the high transparency of a normally reflected overdense plasma can be observed , [7]. The system is modeled as an overdense plasma layer with length $b$ placed between two equal ordinary dielectrics or equivalently cold plasma layer (see Fig. 1). In order to examine the resonant conditions of the excitation of the surface modes let us consider a p-polarize wave specified by the magnetic field component $B = (0, 0, B_z)$ and the electric field in the plane of incidence $(x, y)$,
namely $E = (E_x, E_y, 0)$. Considering the geometrical structure of Fig. (1), the spatial part of the electric and the magnetic fields must be proportional to $\exp(iky)$ and the waves amplitudes become functions only of the variable $x$. In this case the magnetic field and temperature at the steady state, obtain the following forms:

$$\kappa \frac{\partial^2 T}{\partial x^2} + Q = 0. \quad (1)$$

$$\epsilon \frac{\partial}{\partial x} \left( \frac{1}{\epsilon} \frac{\partial B_z}{\partial x} \right) + (\epsilon - k_y^2)B_z = 0. \quad (2)$$

where $\kappa$ refers to the thermal conductivity. Also,

$$Q = \frac{\epsilon_0}{2} \epsilon'' \omega E_0^2 |E|, \quad (3)$$

where $\epsilon''$ is the imaginary part of the permittivity. Here $\epsilon = (1 - \frac{\omega^2}{\omega_p^2})$, $s = (1 + i\nu/\omega)$, also $k_0 = \varpi$ and $\omega_p = \frac{4\pi n_0 e^2}{m}$. In these equations, all field quantities have become dimensionless and redefined as follows:

$$(\tilde{r} = k_0 r, \tilde{t} = \omega t), \tilde{E} = \frac{E}{E_0}, \tilde{\nu} = \frac{\nu}{\omega}, \tilde{T} = \frac{T}{T_0}. \quad (4)$$

where the tilde quantities are dimensionless, but for simplicity, we ignore the tilde sign of our field quantities in equations. Likewise, the temperature should satisfy the surface heat balances. where $k_y = \sin \theta$. Analytical solutions of Eqs.(2) gives the magnetic field in the both dielectrics and the over dense plasma mediums, and are obtained in the following forms:

$$B_z = (A_1 e^{\alpha x} + A_2 e^{-\alpha x}), \quad (5)$$

where $\alpha = \sqrt{k_y^2 - \epsilon}$. The electromagnetic fields in the vacuum regions, $x < -a$ and $x > b$, have the following forms:

$$B_z = E_0 e^{i \cos \theta x} + R e^{-i \cos \theta x} \quad x < -a, \quad (6)$$

$$B_z = T e^{i \cos \theta x} \quad x > b, \quad (7)$$

where $E_0$, $R$ and $T$ respectively assign the field components for the incident, reflected and transmitted waves.

These field solutions should satisfy the conditions of continuity of $dB_z/dx$ and $B_z$, on all the boundaries and as a result, one achieve eight equations for the unknown coefficients, specifically the coefficient $R$ and $T$, are obtained. For resonant values of the incident angle $\theta$, we should expect anomalous transmission of the electromagnetic waves.

Fig. (2) shows the transmission curve $T_r$ and $R$ as a function of the incidence angle $\theta$ for two different values of the collisional frequencies, namely for $\nu = 0.1$ and $\nu = 0.2$. Two important results are obtained from these plots. As the first
result, there is an acute increase in the transmission properties of the system, when the angle of incidence reaches to one of its resonant values. According to the figure.(2), the maximum of the transmission coefficient $T$ occurs about $\theta = 0.18 \text{rad}$ for two different value of collisional frequencies.

Fig. 2. The transmission coefficients vs incident angel for two different value of collisional frequency

As the second result, comparing the two maximums indicates that, with the increase of the collisional frequency $\nu$, the maximum rate of the electromagnetic waves decreases. In order to investigate the effect of this dissipated energy on the increase of the plasma temperature, the electric field amplitude in the over-dense plasma at the maximum points is needed. To provide this we note that for the case $\nu = 0.1$, the deficiency at the maximum point is about $1 - (|R|^2 + |T_r|^2) = 0.3071$, while for the case $\nu = 0.2$ the deficiency is about $1 - (R^2 + T_r^2) = 0.4373$.

3 The heat equation

In order to investigate the temperature variations or the increase of the temperature degree of the over-dense plasma due to passing of the electromagnetic waves one should consider the solutions of the heat equation (1). The factor $Q$ that is given in Eq.(3) can be considered as a thermal source for the heat equation. The imaginary part of the permittivity explicitly appears in the heat source and hence is vitally important to produce heat. Also the heat source $Q$ depends on the amplitude of the passing electric field in the over-dense plasma layer. The electric field amplitudes, obtained in previous section, in the over-dense plasma layer under the resonant transmission of waves. Substituting them in Eq. (3) for each value of the collisional frequency $\nu$, the heating source $Q$ would be obtained. Subsequently one can study the temperature variations and the key factors of these variations by solving the corresponding heat equation via Eq.(1).

Here we follow the above procedure and analyze the temperature variations from Eq.1 in steady state both numerically and analytically.

we require only the boundary condition which includes convection relation on both boundaries of the plasma.

$$T(x) = 1, x = 0, \quad \frac{\partial T}{\partial x} = 0, x = b.$$
It should be noticed that the sample or the plasma will be kept in room temperature, therefore, \( T_0 \) is the room temperature also here \( T \) is dimensionless and equals to \( T/T_0 \). Heat equation will be solved through analytic as well as the numerical method and, subsequently, compare the data obtained. The analytical answers of this equation are as follows:

\[
T(x) = -\frac{1}{\kappa} \int \int Q(x)dx\,dx + C_1x + C_2,
\]

where \( C_1 \) and \( C_2 \) are calculated by boundary conditions. Also Eq. 8 can be written in the following form

\[
T(x) = -F_1(x) + F_2(0)x + F_1(0),
\]

where \( F_1(x) = \int \int Q(x)dx, F_2(x) = \int Q(x)dx \).

The above relation demonstrates that the curve of temperature is an ascending function of space. To investigate this more closely, the analytical solutions of the temperature functions obtained from Eq. 1 are plotted in Fig. (3a). This figure contains two different curves corresponding to two different values of the collisional frequencies \( \nu \). As it is shown in the figure, the plasma temperature increases as one move away from the plasma fronting edge. It should be noticed that this temperature rising takes place due to the collisional effects which appears in the form of the thermal source term \( Q \).

It is also plotted the temperature curve by using a numerical method and the results are given in Fig. (3b). To solve the heat equation (1) numerically, the finite difference method has been employed. These numerical results are in a good agreement with the analytical results and show an ascending function of space for the temperature function.

Fig. 3. The profile of temperature for analytic and numeric solutions.

4 Conclusions

It has been shown here that the coupled resonant excitation of the surface modes could create a condition for suitably heating of a dense collisional plasma layer. To provide the resonant conditions, the plasma layer was considered between two dielectric layers. The slab then was supposed to be subjected to the electromagnetic waves. After solving the equation of the wave for a cold
plasma, the electric field in all mediums was obtained. Eventually, the energy transmission on the rear side of the dielectric layer was examined. Here, the important issue is the effect of the collision on the amount of energy transmission which, as researchers have shown, diminishes as the collisional effects increase. Equally the important issue is the way in which the dissipative energy in the plasma appears. The results indicate that the dissipated energy in the plasma led to the heating of the particles and the increasing of the plasma temperature.

References