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Attractors and Deformation Field in the Coupled Fractal Multilayer Nanosystem

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Abstract. The behavior of the deformation field of coupled multilayer nanosystems: fractal nanotrap – fractal bulk structure is investigated. It is shown that when crossing the surface of the coupled structure critical planes formed a line of singular points (attractors) of the deformation field. Analysis of the isolines of singular points core shows that the behavior of coupled systems with the same or different centers of gravity varies considerably. It is shown that the deformation field of coupled structures is determined by mutual influence of stochastic processes on each other. Such effects as the moving of fractal structures, the alteration of a thin quasiperiodic structure inside the trap, the appearance of an extra thin structure outside the trap are possible when changing the parameters.
Keywords: Fractal Bulk Structure, Fractal Nanotraps, Coupled Systems, Attractors, Deformation Field, Multilayer Nanosystem.

1 Introduction

The Nobel Prize 2014 on physiology and medicine was awarded to J. O'Keefe, M.-B. Moser and E. Moser for the discovery of cells system in the brain that determine the position in space (Fyhn *et al.*[1], Hafting *et al.*[2], Sargolini *et al.*[3]). J. O'Keefe has opened active neurons or "place cells". M.-B. Moser and E. Moser have opened coordinate neurons or "grid-cell". The brain of living organisms is a specific neural network. The active cells respond to an intersection of nodes of imaginary hexagonal lattice, that is the passage of certain distance in a certain direction. In describing neuron networks (Heyman[4]) model multilayer nanosystems and bulk fractal structures appearing there can find its application.
In describing the fractal bulk structures (V. Abramov[5,6], O. Abramova, S. Abramov [7-9]) in the multilayer nanosystem bulk lattice nodes that are attractors can be as active elements (C.H. Skiadas and C. Skiadas[10]). These attractors form a surface of active elements of the deformation field (displacement field). To determine the position of singular points (attractors) of a displacement field in a single active layer it is necessary to cross this surface by the plane. As a result it is possible to find isolines of singular points in a separate layer.
In the works (O. Abramova, S. Abramov[8, 9]) the possibility of creating fractal nanotraps based on bulk structures in a multilayer nanosystem is demonstrated. Fractal bulk structures can be used as traps for trapping or capturing other

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fractal structures (particles or groups of particles) in order to investigate their physical properties. Experimentally, such traps can be created, for example, using two lasers (Tosi *et al.*[11]) or occur at self-organizing of neuron networks (Fyhn *et al.*[1], Hafting *et al.*[2], Sargolini *et al.*[3], Heyman[4]). At the same time many of the physical properties of the resulting coupled fractal systems (trap – bulk structure) differ from those of the individual fractal trap, bulk structure.

The aim of this work is a description of attractors and the behavior features of the deformation field of coupled systems: fractal nanotrap – fractal bulk structure.

2 Attractors in Fractal Bulk Structures

We consider the coupled system: nanotrap – fractal bulk structure (NT-FBS), which is in the bulk discrete lattice $N_1 \times N_2 \times N_3$, whose nodes are given integers n, m, j. The evolution of states of this system depends essentially on the choice of the iterative process for solving basic nonlinear equations (O. Abramova, S. Abramov[7-9]). The nonlinear equation for the dimensionless displacement function u of lattice node of this coupled system is given in the form

$$u = R_1(1-\alpha)(1-2sn^2(u-u_0,k))/Q_1 + R_2(1-\alpha)(1-2sn^2(u-u_0,k))/Q_2\,; \quad (1)$$

$$Q_i = p_{0i} - b_{1i}(n-n_{0i})^2/n_{ci}^2 - b_{2i}(m-m_{0i})^2/m_{ci}^2 - b_{3i}(j-j_{0i})^2/j_{ci}^2\,; \quad (2)$$

$$p_{0i} = p'_{0i} + p'_{1i}n + p'_{2i}m + p'_{3i}j\,. \quad (3)$$

In the expression (1) for the displacement u the first term characterises nanotrap (NT) and the second term is the fractal bulk structure (FBS); α is the fractal dimension of the deformation field u along the Oz-axis ($\alpha \in [0,1]$); u_0 is the constant (critical) displacement; k is the modulus of the elliptic sine; parameters R_i, p'_{0i}, p'_{1i}, p'_{2i}, p'_{3i}, b_{1i}, b_{2i}, b_{3i}, n_{0i}, n_{ci}, m_{0i}, m_{ci}, j_{0i}, j_{ci} characterise NT and different FBS.

In general case these parameters may depend on the layer index j and the dimensionless time t. Functions Q_1, Q_2 take into account the interaction of the nodes as in the main plane of the discrete rectangular lattice as well as interplane interaction.

Singular points (attractors) of the deformation field of the multilayer nanosystem are located on the surface, the core of which is determined by the condition

$$Q_1 \cdot Q_2 = 0\,. \quad (4)$$

If the surface (4) is crossed the plane $j = j_k$, we obtain the equation of the isolines. Consider the following fractal bulk structures (FBS): the fractal quantum dot (FQD), the fractal ellipsoid (FE), the fractal elliptical paraboloid (FEP) and the fractal elliptical cylinder (FEC). The bulk discrete lattice is defined by the parameters: $N_1 = 30$, $N_2 = 40$, $N_3 = 200$ ($n = \overline{1, N_1}$; $m = \overline{1, N_2}$; $j = \overline{-N_3, N_3}$). On the basis of these structures coupled systems NT-FBS are formed, in which the centers of gravity NT and FBS coincide (fig. 1, 2), where parameters are $n_{0\mathrm{i}} = 14.3267$; $n_{\mathrm{ci}} = 9.479$; $m_{0\mathrm{i}} = 19.1471$; $m_{\mathrm{ci}} = 14.7295$; $j_{0\mathrm{i}} = 31.5279$; $j_{\mathrm{ci}} = 11.8247$ (i=1,2).

The surface of the core NT-FBS with NT as the FQD (such imaginary FE) with $p'_{01} = -1.0123$ and FBS as the FE with $p'_{02} = 1.0123$ is given on fig. 1 a, where parameters are $b_{1\mathrm{i}} = b_{2\mathrm{i}} = b_{3\mathrm{i}} = 1$, $p'_{1\mathrm{i}} = p'_{2\mathrm{i}} = p'_{3\mathrm{i}} = 0$. Isolines of singular points core (4) are shown on fig. 1 b-d. For the lower ($j_k = 20$) and an upper ($j_k = 43$) critical layers isolines are close to the hexagonal type structure, and for the average ($j_k = 31$) layer isolines are close to the structure of the ellipse. The surface of the core NT-FBS with NT as the FQD with $p'_{31} = -0.03375$ and FBS as the FE with $p'_{32} = 0.03375$ is given on fig. 1 e, where other parameters are the same as for fig. 1 a. In this case, the core of the coupled system NT-FBS shifted in the region $N_1 \times N_2$, the shape of the core surface is changed in comparison with fig. 1 a, appearing holes. This is connected to the fact that the parameters $p'_{3\mathrm{i}} \neq 0$. Isolines of singular points core are given on fig. 1 f-h. The presence of discontinuities on the isolines with $j_k = 31$, $j_k = 43$, confirms the presence of holes on the surface of the core.

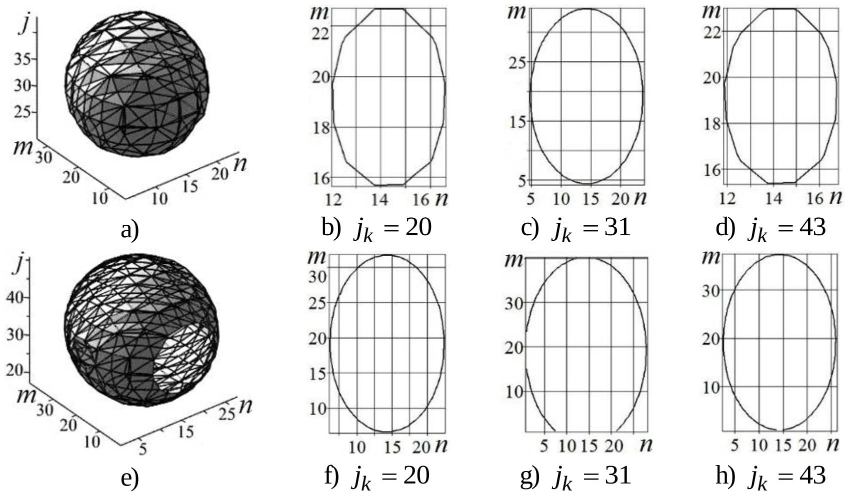


Fig. 1. The surfaces (a, e) and isolines (b-d, f-h) of singular points cores for coupled systems NT-FBS with coinciding centers of gravity.

The surface of the core NT-FBS with NT as the FQD (type of the imaginary FEP) with $p'_{01} = -1.0123$, $p'_{31} = -0.03375$ and FBS as the FEP with

$p'_{02}=1.0123$, $p'_{32}=0.03375$ is given on fig. 2 a, where parameters are $b_{1i}=b_{1i}=1$, $b_{3i}=0$, $p'_{1i}=p'_{2i}=0$ ($i=1,2$). Isolines of singular points cores are given on fig. 2 b-h. When changing $j\in(-60;-20)$ the effect of narrowing forming a region of "neck" type is observed on the surface of the core. In this region there is a separate layer with $j_k=-30$, near which the cylindrical surface with forming parallel to the axis Oj is formed. The presence of angular points on broken isolines reflect the fractal structure of the core. The transition from layer to layer in multilayer nanosystem is accompanied by changes in the surface structure of the singular points core, that is reflected on the behavior of isolines when changing j_k (fig. 2 b-h).

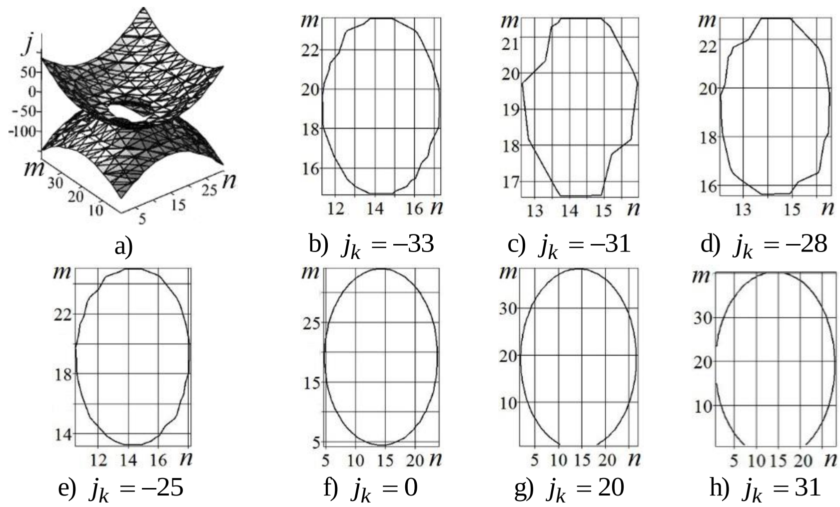
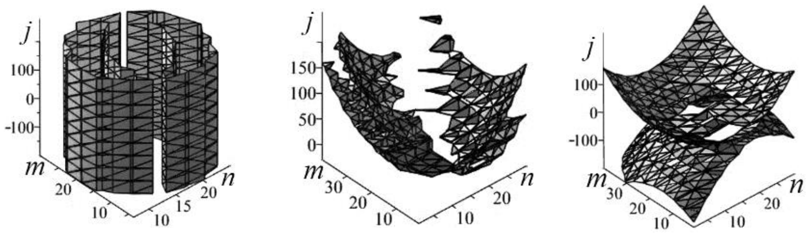


Fig. 2. The surface (a) and isolines (b-h) of singular points cores for coupled systems FQD-FEP with coinciding centers of gravity.

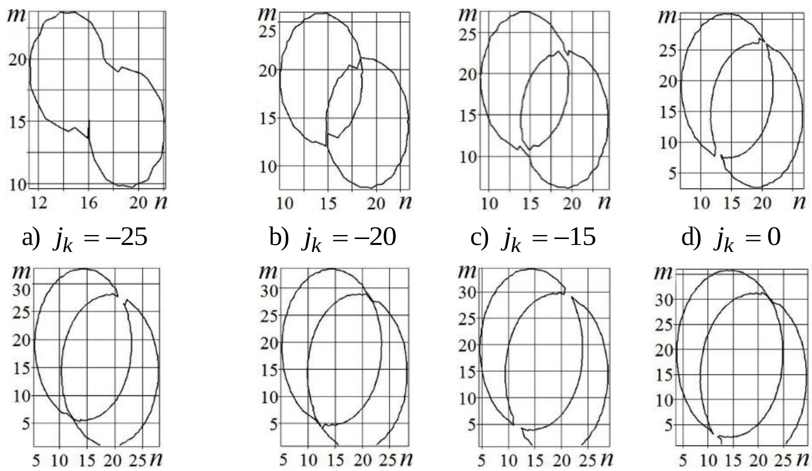
Further we will consider coupled systems NT-FBS, in which centers of gravity (parameters n_{0i}, m_{0i}, j_{0i}) for the NT and FBS are different. For these structures (fig. 3) parameters $n_{ci}=7.4793$, $m_{ci}=11.7295$, $j_{0i}=31.5279$, $j_{ci}=11.8247$ ($i=1,2$) selected the same. The surface of the core NT-FBS with NT as the FEC with $p'_{01}=1.0123$, $n_{01}=14.3267$, $m_{01}=19.1471$ and FBS as the FEC with $p'_{02}=1.0123$, $n_{02}=19.1471$, $m_{02}=14.3267$ is given on fig. 3 a, where parameters are $b_{1i}=b_{1i}=1$, $b_{3i}=0$, $p'_{1i}=p'_{2i}=p'_{3i}=0$ ($i=1,2$). Isolines of the core of singular points (for the surface from fig. 3 a) for all values are of the same type (fig. 4 d). In the region of the intersection of cylindrical surfaces two “gaps” are formed, which position allows to determine the distance between them and the direction in space. When changing the parameters p'_{31}, p'_{32} the cylindrical surface from fig. 3 a pass into the parabolic (fig. 3 b, c).



a) b) c)

Fig. 3. The surfaces of singular points cores for coupled systems with different centers of gravity.

The surface of the core NT-FBS with parameters $p'_{3i} \neq 0$ compared with fig. 3 a for NT as the FEP with $p'_{31} = 0.03375$ and FBS as the FEP with $p'_{32} = 0.03375$ is given on fig. 3 b. All other parameters are such as for fig. 3 a. Isolines of singular points cores (for the surface from fig. 3 b) are given on fig. 4. When $j_k = -25$ (fig. 4 a) an integration of the individual cores from FEP with different centers of gravity into an uniform dual-core system is observed. A further change j leads to a complex transformation of the isolines: the intersection effects (fig. 4 b, f.), the appearance of new internal inclusions (fig. 4 c), the formation of "gaps" (fig. 4 d, e, g, h.).



a) $j_k = -25$ b) $j_k = -20$ c) $j_k = -15$ d) $j_k = 0$

e) $j_k = 10$ f) $j_k = 15$ g) $j_k = 20$ h) $j_k = 30$

Fig. 4. Isolines of singular points cores for coupled systems from fig. 3 b.

The surface of the core NT-FBS with NT as the FEP with $p'_{01} = 1.0123$,

$p'_{31} = 0.03375$, $n_{01} = 14.3267$, $m_{01} = 19.1471$ and FBS as the FEP with $p'_{02} = 1.0123$, $p'_{32} = -0.03375$, $n_{02} = 19.1471$, $m_{02} = 14.3267$ is given on fig. 3 c, where parameters are $b_{1i} = b_{1i} = 1$, $b_{3i} = 0$, $p'_{1i} = p'_{2i} = 0$ (i=1,2). Isolines of singular points cores (for the surface from fig. 3 c) are given on fig. 5.

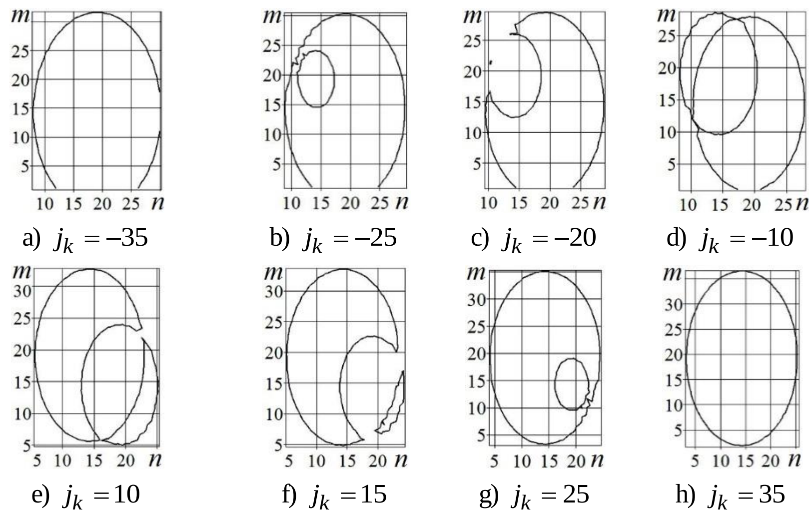


Fig. 5. Isolines of singular points cores for coupled systems from fig. 3 c.

Changing the sign of parameter $p'_{32} \neq 0$ FBS from fig. 3 b leads to significant structural rearrangement of the surface core with the formation an internal “bridge” (fig. 3 c) between the boundaries of the surface in the region of the hole from fig. 2 a. The behavior of isolines when changing j confirms the appearance of internal inclusions, their moving from one boundary (fig. 5 b) to the other (fig. 5 g), which is associated with the presence of the “bridge” (fig. 3 c). The effect of moving the internal inclusions can be used to determine the distance between the boundaries and the direction of movement in separate planes nOm of multilayered nanosystems.

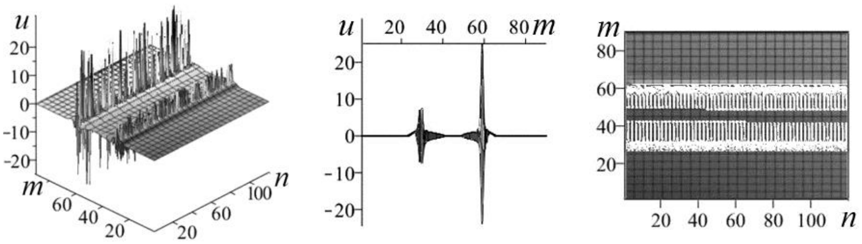
3 The Behavior of the Deformation Field of Fractal Structures in Fractal Nanotrap

As nanotrap (NT) the splitting dislocation (SD) is selected, which is a fractal structure with parameters $p'_{01} = 1.0123$; $b_{11} = 0$; $b_{21} = 1$; $b_{31} = 0$; $p'_{11} = p'_{21} = p'_{31} = 0$, $n_{01} = 89.1471$; $n_{c1} = 19.4793$; $m_{01} = 44.3267$; $m_{c1} = 14.7295$ (fig. 6). The solution of the nonlinear equation (1) for $R_1 = 1$,

$R_2 = 0$ is carried out by the iteration method on the variable n with the following fixed values: $N_1 = 120$; $N_2 = 90$; $\alpha = 0.5$; $u_0 = 29.537$; $k = 0.5$.
For each of the two branches of the splitting dislocation the stochastic behavior of the displacements u along the axes of the dislocation (fig. 6 a) is characteristic. The coordinates of the dislocation cores on an axis Om are defined by the formulas

$$m = m_1 = m_{01} - m_0; \quad m = m_2 = m_{01} + m_0; \quad m_0 = (m_{c1}^2 \cdot p'_{01})^{1/2}. \qquad (5)$$

The calculated values $m_1 = 29.507$, $m_2 = 59.147$ by formulas (5) are consistent with the position of the peaks of fig. 6 b. Inside the region of the splitting dislocation for $m \in (m_1, m_2)$ zones with quasi-periodic thin (along the axis On) structure and region with almost constant negative displacement $u \approx -0.001$ (fig. 6 c) are observed. The almost regularly change positive displacement is observed outside of region $m \in (m_1, m_2)$ of the dislocation with the value $u \approx 0.003$ by the boundary values of $m = 1$ and $m = 90$.



a) b) c)

Fig. 6. Nanotrap: (a) – the dependence u on the lattice indices n, m;
(b) – projection u onto a plane mOu; (c) – cross-section $u \in [-0.001;1]$ (top view).

Further we will consider coupled systems NT-FBS, for which $R_1 = 1$ and $R_2 = 1$, nanotrap (NT) is selected as the splitting dislocation (SD) with parameters (fig. 6).
System I: FBS is selected as the fractal elliptical cylinder (FEC).
The structure of FEC described by parameters $p'_{02} = 1.0123$; $b_{12} = 1$; $b_{22} = 1$; $b_{32} = 0$; $p'_{12} = p'_{22} = p'_{32} = 0$; $n_{02} = 39.1471$; $n_{c2} = 19.4793$; $m_{02} = 44.3267$; $m_{c2} = 14.7295$.
System II: FBS is selected as the fractal quantum dot (FQD).
The structure of the FQD is described by the same parameters as the FEC, but for $p'_{02} = -3.457 \cdot 10^{-11}$.
Fractal structures FEC and the FQD are located inside the trap SD. The explicit

behavior of the displacement function for the coupled **systems I, II** is shown on fig. 7. As the parameter n_{02} varies the moving of fractal structures FEC (fig. 7 a, b, c) and FQD (fig. 7 d, e, f) inside the trap along the axis On are observed. For multilayer nanosystem these movings can be taken into account by changing the parameter $n_{02}(j)$ from layer to layer by the formula

$$n_{02}(j) = n'_{02} + a_{02}(j - j_{02}) / j_{c2}, \qquad (6)$$

where parameters are $j_{02} = 31.5279$; $j_{c2} = 11.8247$; $n'_{02} = 165.5064$; $a_{02} = 591.0165$. According to (6) a value $n_{02} = 39.1471$ corresponds to layer with $j = 29$, $n_{02} = 89.1471$ corresponds to layer with $j = 30$, and $n_{02} = 119.1471$ corresponds to a position of the fractal structure between the layers with $j = 30$ and $j = 31$. Essential changes in the structure both inside the trap, and the trap itself are observed.

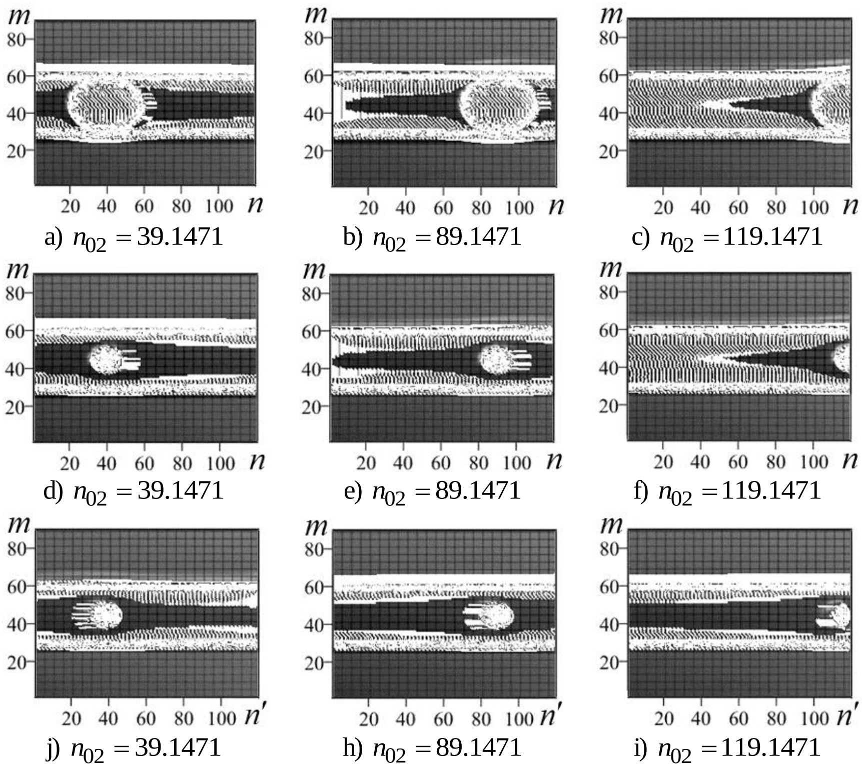


Fig. 7. Cross-section $u \in [-0.001;1]$ (top view). Forward (a - f) and backward (j - i) iterative processes for coupled systems: (a-c) - SD-FEC; (d-i) - SD-FQD.

If the functions $Q_1 = Q_1(n', m)$, $Q_2 = Q_2(n', m)$ from (2) depend on n', where

$$n' = N_1 - n + 1,$$

than the iterations process on the variable n' is performed in backward order.
The behavior of the deformation field FQD in the trap (fig. 7 j) essentially changes in comparison with the (fig. 7 d).
“Tails” at the quantum dot pass from a position to the right relative to the center of the quantum dot (fig. 7 d) in position on the left (fig. 7 j).
Also, there is a change of the structure of SD.
Changing a parameter n_{02} leads to an effect of moving FQD in the trap (fig. 7 j, h, i). Such a change of parameter, unlike from (6) can be associated with the time dependency of the type $n_{02}(t) = n'_{02} + g_{02}(t - t_{02}) / t_{c2}$ in a separate layer.

4 The Deformation Field of Two Quantum Dots in Fractal Trap

We consider the coupled **system III** (NT-FBS): nanotrap (SD) and FBS in the form of two fractal quantum dots FQD1 and FQD2.
The nonlinear equation for the dimensionless displacement function u of lattice node this coupled system based on (1) is given in the form

$$u = R_1(1-\alpha)(1-2sn^2(u-u_0,k))/Q_1 + R_2(1-\alpha)(1-2sn^2(u-u_0,k))/Q_2 + \\ + R_3(1-\alpha)(1-2sn^2(u-u_0,k))/Q_3; \quad (7)$$

$$Q_3 = p_{03} - b_{13}(n' - n_{03})^2 / n_{c3}^2 - b_{23}(m - m_{03})^2 / m_{c3}^2 - b_{33}(j - j_{03})^2 / j_{c3}^2; \quad (8)$$

$$p_{03} = p'_{03} + p'_{13}n' + p'_{23}m + p'_{33}j, \qquad n' = N_1 - n + 1. \quad (9)$$

In the expression (7) for the displacement u the first term characterises nanotrap in the form of SD with parameters as for (fig. 6), second and third terms characterise FBS in the form of two fractal quantum dots FQD1 and FQD2. Here, the parameters are $R_1 = R_2 = R_3 = 1$.
Fractal quantum dots FQD1 (i=2) and FQD2 (i=3) are described by the same parameters: $p'_{0i} = -3.457 \cdot 10^{-11}$; $b_{1i} = b_{2i} = 1$; $b_{3i} = 0$; $p'_{1i} = p'_{2i} = p'_{3i} = 0$; $n_{ci} = 19.4793$; $m_{0i} = 44.3267$; $m_{ci} = 14.7295$.
However, the parameter n_{02} is variable for FQD1, but for FQD2 the parameter $n_{03} = 60.6529$ is constant.
Fractal quantum dot FQD2 is a fixed, but FQD1 moves inside nanotrap by changing parameter n_{02} (fig. 8).

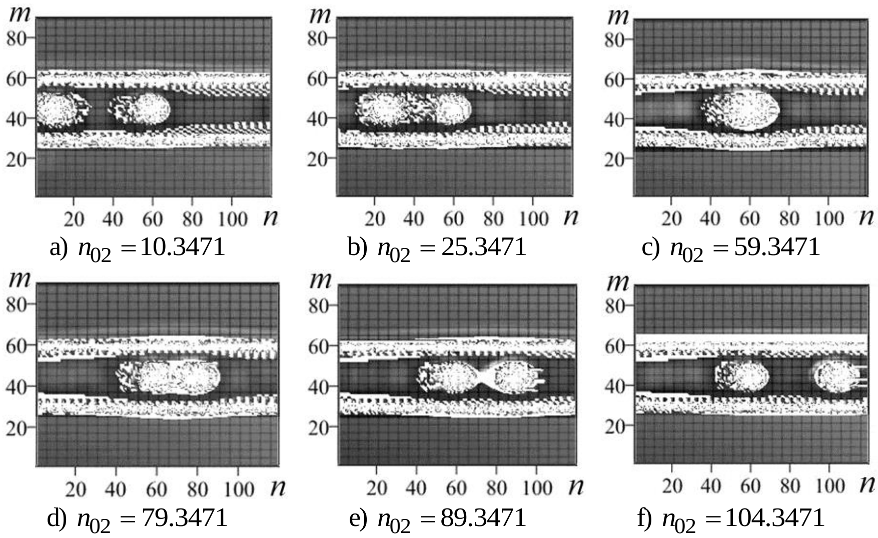


Fig. 8. Two quantum dots. Movement of the first quantum dot.
Cross-section $u \in [-0.001;1]$ (top view).

Analysis of these results shows that the forward and backward iterative processes for the two quantum dots in nanotrap influence on each other.
When FQD1 is located at a considerable distance from the FQD2 (fig. 8 a, f) mutual influence is practically absent.
When approaching the quantum dots their “tails” are overlaping, and there is a pronounced stochastic behavior between them (fig. 8 b, d).
When the parameter n_{02} is located in the vicinity of n_{03}, quantum dots cores are overlapping and the amplitudes of displacement u increases sharply (fig. 8 c).
When n_{02} becomes larger than n_{03} the region of overlapping quantum dots cores decreases and the narrowing is formed between them (fig. 8 e).
When changing a parameter n_{02} in a coupled system with two quantum dots the difference of structure itself SD compared to fig. 7 d-i is also observed, where a variant of the coupled system with a single quantum dot is considered.

Conclusions

It is shown that in a multilayer system coupled system (NT-FBS) can be formed. This is confirmed by an investigation of attractors, which are singular points of the deformation field of the coupled systems.
The singular points core is located on the surfaces.
Analysis of isolines of the singular points core shows that the behavior of coupled systems with the same or different centers of gravity varies significantly.

Isolines structure changes from smooth to hexagonal; the formation of holes, gaps, bridges between the inner boundaries of surfaces are observed.
These structural features of coupled systems allow to determine the characteristic distances and directions in the multilayer nanosystem (possibility of navigating).
The behavior of the deformation field for the coupled **systems I - III** significantly differs from the behavior of the deformation field as a separate trap (NT), both and fractal structures (FEC, FQD, FQD1, FQD2).
It is explained by the influence of stochastic processes on each other as a result of solving nonlinear equations by the iteration method for coupled systems.
Effects of moving fractal structures inside the trap when changing a parameter of the fractal structure n_{02} from layer to layer or in a separate layer over time are observed.
This significantly changes the thin quasi-periodic structure inside the trap, additionally there is the thin structure outside the trap.
The forward and backward iterative processes for the two quantum dots in nanotrap also influence on each other.
When approaching the quantum dots their "tails" are overlapping, and there is a pronounced stochastic behavior between them.
When quantum dots cores are overlapping, the amplitudes of displacement u increases sharply.

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