

Turbulence of Optical Dissipative Solitons

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Abstract. The optical representation of turbulence and chaos phenomena open the door to the statistical and thermodynamic theory of the dissipative coherent and partially coherent structures, in a whole, that can be attributed to “one of the central problems of theoretical physics.” Our extensive numerical simulations of the generalized cubic-quintic nonlinear Ginzburg-Landau equation, which models, in particular, dynamics of mode-locked fiber lasers, demonstrate a close analogy between the properties of dissipative solitons and the general properties of turbulent and chaotic systems. In particular, we show a scenario of transition to turbulence related to “*spectral condensation – temporal thermalization*” duality and disintegration of dissipative soliton into a non-coherent (or partially coherent) multisoliton complex. Thus, the dissipative soliton can be interpreted as a complex of nonlinearly coupled coherent “internal modes” that allows developing the kinetic and thermodynamic theory of the non-equilibrium dissipative phenomena. Also, we demonstrate an improvement of dissipative soliton integrity and, as a result, soliton disintegration suppression due to non-instantaneous nonlinearity caused by the stimulated Raman scattering. This effect leads to an appearance of a new coherent structure, namely, a dissipative Raman soliton.

Keywords: Optical turbulence, Dissipative solitons, Chaos in nonlinear optical systems, Generalized cubic-quintic nonlinear Ginzburg-Landau equation.

1 Introduction

In the last decades, rapid progress in modern nonlinear science was marked by the development of the concept of a dissipative soliton (DS). DS is a strongly localized, coherent, partially coherent or even incoherent structure emergent in a nonlinear dissipative system far from the thermodynamic equilibrium. This concept is highly useful in very different fields of science ranging from field theory and cosmology, optics, and condensed matter physics to biology and medicine [1]. The existence of DS under non-equilibrium conditions requires a well-organized energy exchange with an environment so that this energy flow forms a non-trivial internal structure of DS, which provides the energy redistribution inside it and can distort the soliton coherence. Such a DS with nontrivial internal structure can develop in lasers [2,3], and the DS dynamics can be chaotic and turbulent [4]. The range of turbulence, noise, and rogue wave phenomena emulated by the optical DS is so broad that it turns them into a universal playground for studies in the fields of nonlinear dynamical systems

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and nonequilibrium thermodynamics, as well as provides us with the highly-effective and controllable tools for “metaphoric” computing and analog modelling, big data and rare events analysis which are suitable for different branches of knowledge [5].

In this work, we conjecture an analogy between the internal structure of a DS and the turbulence phenomena. Such close relation leads to chaotization of DS dynamics with the energy growth. Simultaneously, the nonlocality (“inertia”) in a dissipative system caused by the stimulated Raman scattering (SRS) can suppress chaos and stabilize DS.

2 DS and turbulence

As was shown in [7], the most natural description of DS with strongly inhomogeneous phase (i.e., large “chirp”) can be realized in the spectral domain. Using the adiabatic approximation (for details see [7,8]) shows that the DS spectrum can be expressed in the following form:

$$p(\omega) = \frac{T}{\omega^2 + \Omega_L^2} \theta(\Delta^2 - \omega^2), \quad (1)$$

where $p(\omega)$ is a DS spectral power, ω is a frequency, Δ is a cut-off frequency ($\theta(x)$ is the Heaviside function). Eq. (1) represents a truncated Lorentz profile with the characteristic width Ω_L and can be interpreted by analogy with thermodynamics as the Rayleigh-Jeans distribution [9,10], so that Ω_L^2 plays a role of negative “chemical potential.” The parameter T is analogous to “temperature” and is closely connected with the system nonlinearity:

$$T = \frac{6\pi\gamma}{\kappa\zeta}. \quad (2)$$

The cut-off frequency Δ is defined by the dissipation (see below).

The master model for Eqs. (1,2) is based on the famous complex nonlinear cubic-quintic Ginzburg-Landau equation describing a broad variety of nonequilibrium phenomena, in particular, a propagation of dissipative nonlinear waves and beams [1,2]:

$$\frac{\partial a}{\partial z} = i \left(\beta \frac{\partial^2 a}{\partial t^2} - \gamma |a|^2 a \right) + \sigma a + \alpha \frac{\partial^2 a}{\partial t^2} + (\kappa - \zeta |a|^2) |a|^2 a. \quad (3)$$

Here z and t are the propagation and time coordinates, respectively; $a(z, t)$ is an optical field, β is a dispersion (“kinetic energy”) coefficient (we assume

$\beta > 0$ for the normal dispersion), and γ is a phase nonlinearity (“self-interaction”) coefficient. Limiting the right-hand side of the equation to the first part gives precisely the nonlinear Schrödinger equation, and its applications include such phenomena as turbulence, Bose-Einstein condensate, etc.

The next part of Eq. (3) describes the dissipative factors. The σ -term defines an average energy in/out-flow (“gain”) and depends on the field energy $E \propto \int |a|^2 dt$, in general case. The α -term defines the spectral dissipation (“kinetic cooling” in Bose-Einstein condensates), κ is a coefficient of self-amplitude modulation, and ζ -coefficient describes a saturation of self-amplitude modulation, which is necessary for DS stabilization.

The running wave steady-state solution of Eq. (3)

$$a(z,t) = |a(t)| \exp(i\phi(t) - iqz) \quad (4)$$

with the instant local phase $\phi(t)$ results in the Langmuir dispersion relation (Fig. 1) [10] for the DS wave number q [7]:

$$q = \gamma P_0 = \beta \Delta^2 \quad (5)$$

which connects the DS peak power $P_0 \equiv |a(t=0)|^2$ and the cut-off frequency $\pm \Delta$. Such cut-off corresponds to the edge of a spectral dissipation window (“transparency window”):

$$\Delta^2 \approx 1/\alpha \quad (6)$$

and these spectral losses have to be compensated by a nonlinear gain $\approx \kappa P_0$ at the DS peak (the relative carrier frequency is set to $\omega=0$ at $t=0$) with the subsequent redistribution of the energy (Fig. 2):

$$\kappa P_0 \approx \alpha \Delta^2. \quad (7)$$

Eqs. (5,6,7) give the threshold condition for the DS existence:

$$\frac{\alpha \gamma}{\kappa \beta} \approx 1 \quad (8)$$

(more precise consideration gives 2/3 in the high-energy limit $E \rightarrow \infty$ and 2 in the low-energy limit $E \rightarrow 0$). This relation shows that both spectral filtering and nonlinear dissipation are highly important for DS stability.

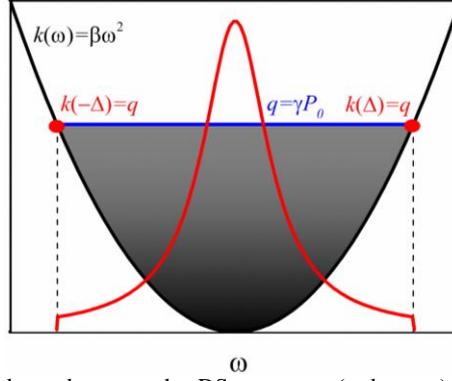


Fig. 1. Correspondence between the DS spectrum (red curve) and the turbulence spectral distribution in Fourier space ω (black curve shows the Langmuir dispersion relation): cut-off frequency Δ is defined by resonance condition between linear waves with a wave number k and DS with a wave number q (P_0 is a DS peak power). Spectral condensation at $\omega=0$ is illustrated by shading [10].

These relations and the DS spectral properties presented above reveal a very close analogy with the main features of strong Langmuir turbulence [10]. Such analogy is deepened by analysis of energy flows inside the DS (Fig. 2).

Let's neglect the nonlinear gain saturation (i.e. $\zeta=0$). Then, the exact solution of Eq. (3) can be expressed as

$$a(t) = a_0 \operatorname{sech}\left(\frac{t}{\tau}\right)^{1+i\psi}, \quad (9)$$

where a_0 is amplitude, τ is a DS width, and ψ is a dimensionless *chirp* as a measure of phase inhomogeneity [11]. The intra-DS energy in/out-flow is [2]:

$$\varepsilon \equiv \frac{i}{2} \frac{\partial}{\partial t} \left(a \frac{\partial a^*}{\partial t} - a^* \frac{\partial a}{\partial t} \right) = 2\sigma |a|^2 + 2\kappa |a|^4 - 2\alpha \left| \frac{\partial a}{\partial t} \right|^2 + \alpha \frac{\partial^2 |a|^2}{\partial t^2}, \quad (10)$$

and it is shown in Fig. 2.

Fig. 2 reproduces a classical turbulence energy cascade: an energy nucleation (“spectral condensation”) at $\omega=0$ with a declining high-frequency transfer which falloffs at the cut-off frequency $\pm\Delta$ (i.e., at the DS edges in the time domain).

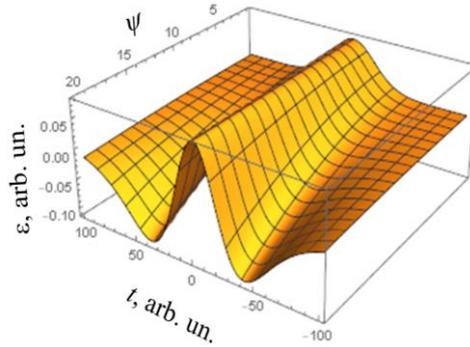


Fig. 2. Profile of energy generation in dependence on the chirp parameter for a DS of Eqs. (9,10) with $\kappa=0.05\gamma$, $\beta=10\alpha$, $\sigma=-0.01$.

As one can see from Fig. 2, such behavior is closely connected with the chirp increase, i.e., enhancement of the phase inhomogeneity. The last can destroy the internal DS coherency. Such partially coherent DS can be treated as consisting of “internal modes” [12] or as a “multisoliton complex” [13]. These properties of strongly chirped DS would allow applying the methods of wave kinetic theory [14] that waits for its pursuance.

3 DS fragmentation in high-energy and high-dispersion limits

Here we present the results of numerical simulation of Eq. (3) in the normal dispersion ($\beta > 0$) regime bearing in mind a Yb-all-fiber laser [3]. Since the turbulent regimes are highly non-stationary, these simulations offer difficulty and are time-consuming. We used the reduced split-step Fourier method [15]:

$$a(z+h) = \exp\left(\frac{h}{2}\Lambda\right) \exp(h\Pi(x))x, \quad x = \exp\left(\frac{h}{2}\Lambda\right)a(z), \quad (11)$$

where Λ and Π are the linear and nonlinear parts of Eq. (3), respectively, and h is a propagation step. Using the Agrawal’s method [16], the full split-step method [15] and the different numerical schemes for the Π -calculation allowed accelerating the simulations to some extent, but such acceleration was not significant in our case.

It is convenient to normalize the propagation coordinate z to a laser length L . The “propagation unit” becomes a laser cavity round-trip number. We used $h=L/500$ to avoid the numerical instabilities which can imitate physical turbulence phenomena. Other normalizations are: field power is normalized to the γ -coefficient corresponding to the fused silica, time is normalized to the time mesh step $\Delta t = 1$ fs ($2^{19} \div 2^{20}$ mesh points were used in calculations), energy was normalized to $\gamma/\Delta t$. Then, the reasonable value of the dimensionless

spectral dissipation parameter for a Yb-fiber laser is $\alpha = 366 \text{ fs}^2$ ($\approx 30 \text{ nm}$ spectral filter width), $\gamma = 6 \text{ W}^{-1}\text{km}^{-1}$ [25], $\kappa = 0.1\gamma$, and $\zeta = 0.05\gamma$ [17]. The propagation time equals to 5000 laser cavity round-trips.

Eq. (3) must be supplemented by an additional mechanism of “gain saturation,” i.e., the dependence of the σ -term on the pulse energy E . We used the simplest law [18]:

$$\sigma = \delta \left(1 - \frac{E}{E_s} \right), \quad (12)$$

where δ is a “stiffness” coefficient defining a saturation efficiency, and E_s is an energy of continuous-wave non-coherent radiation corresponding to $\sigma = 0$ (such a solution of Eq. (3) is unstable if DS exists).

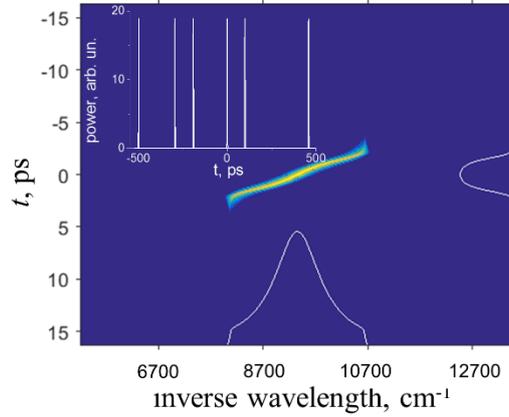


Fig. 3. The Wigner function and its marginals: the DS spectrum (bottom) and temporal profile (right) for a single DS from a multi-soliton complex. The latter is shown in the inset. The dimensionless $E_s = 3 \times 10^5$ (that corresponds to 50 nJ for given parameters and normalization), $\beta = 0.01 \text{ ps}^2$, and $\delta = 0.05$. Dimensional wavelength range corresponds to an all-normal-dispersion Yb-fiber laser [19].

We used the fixed value of $E_s = 3 \times 10^5$ and varied β in our calculations. When β is below some critical value ($\beta < 0.012 \text{ ps}^2$ in our case), the well-known multi-DS regime exists (e.g., see [20]). Fig. 3 (inset) shows an example of such regime. The Wigner distribution [21]:

$$\begin{aligned}
 W(t, \omega) &= \int a\left(t + \frac{t'}{2}\right) a^*\left(t - \frac{t'}{2}\right) \exp(-i\omega t') dt' = \\
 &= \int p^*\left(\omega + \frac{\omega'}{2}\right) p\left(\omega - \frac{\omega'}{2}\right) \exp(-i\omega' t) d\omega',
 \end{aligned} \tag{13}$$

in Fig. 3 with its marginals corresponding to the spectral (bottom) and time (right) profiles $a p \varepsilon$ shown for the central DS in the multi-pulse complex. Since Ω_L is sufficiently large for decreasing β [7], the spectral energy more uniformly distributed within the range defined by a cut-off frequency $\pm\Delta$ which increases with the β decrease (see the Langmuir dispersion relation in Fig. 1). Both factors enhance the spectral dissipation described by the α -parameter in Eq. (3). DS splitting reduces the DS peak power and, thereby, narrows spectrum (again, see the Langmuir dispersion relation in Fig. 1) that is beneficial energetically due to a decrease of spectral dissipation.

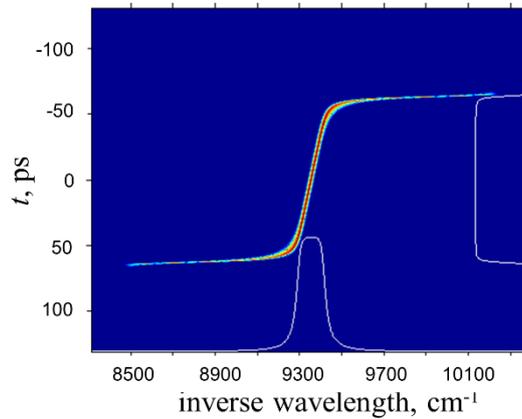


Fig. 4. The Wigner function, its projections, i.e., DS spectrum (bottom) and temporal profile (left) for a sole DS for dimensionless $E_s = 3 \times 10^5$, $\beta = 0.02 \text{ ps}^2$.

The phenomenon of multiple DSs creation with the dispersion decrease can be understood from the thermodynamic point of view, as well. If Ω_L^2 is treated as a “chemical potential” (Eq. (1)), taking into account the dependence of the “long-range” correlation time λ on the chemical potential [9]:

$$\lambda \propto \frac{\beta}{\Omega_L^2} \tag{14}$$

demonstrates degradation of correlation with the dispersion decrease and the Ω_L growth that prevents the DSs merging within a multisoliton complex. That means excitation of non-coherent radiation, when σ becomes positive (i.e.,

when DS energy becomes lower than the “critical energy” E_s in Eq. (12)), with subsequent creation of new DSs which cannot merge into a single coherent structure (single DS) due to a limited correlation time λ . In practice, DSs can disappear completely so that only non-coherent radiation remains. Dependencies of critical energy E_s on parameters for different types of laser systems form the so-called “master diagrams” [3,4,18] and finding such dependencies is a critical issue for the DS theory.

When $E > E_s$ with the growth of β , a stable DS develops (Figs. 4,5). The Wigner function demonstrates that the high-energy DS is strongly chirped and high spectral density is localized at the spectrum centrum (i.e., a “spectral condensation” takes place [10]). Simultaneously, the temporal profile has a flat-top shape with sharply truncated edges. One can see from Fig. 5, that the building-up stage is accompanied by power bursts that can be an obstacle for self-emergence of DS in a high-energy laser [22].

The key feature of the high-energy regime is that the DS peak power is fixed due to saturation of self-amplitude modulation: $P_0 \sim 1/\zeta$. Then, Fig. 1 allows concluding that the cut-off frequency decreases with the dispersion growth: $\Delta \approx \sqrt{\gamma/\beta\zeta}$ and, thereby, the spectral condensation enhances: $\Omega_L \rightarrow 0$. The last enlarges the long-range correlation time in agreement with Eq. (14), so that DS broadens and its chirp increases.

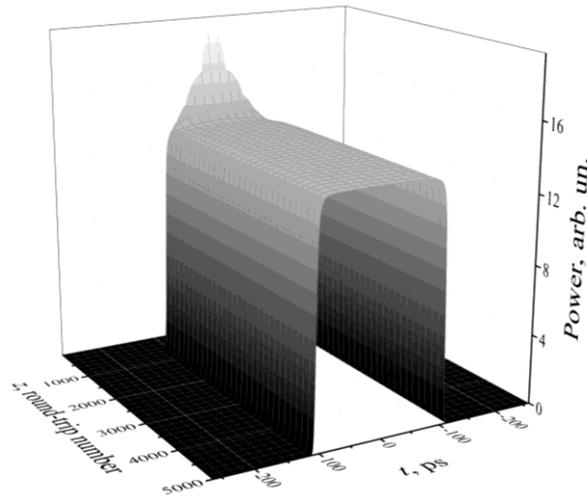


Fig. 5. Evolution of DS power profile for dimensionless $E_s = 3 \times 10^5$ and $\beta = 0.05 \text{ ps}^2$.

It is reasonable to assume that a large chirp (i.e., large phase inhomogeneity) weakens the intra-DS phase coherence, which is defined by the ratio of a “short-range” correlation time $\propto 1/\Delta$ to a “long-range” correlation time λ . This value decreases with dispersion, and such phenomenon was observed in a middle-

nonlinear regime when the β -growth induced P_0 -fluctuations and irregular modulation of spectrum shape [12]. It was interpreted as excitation of the DS “internal modes.” In this case growth of phase difference between different parts of DS leads to their phase decoupling under the action of small perturbations, and the DS loses its coherence. In a high-nonlinear regime considered here, it leads to DS splitting through a turbulent transitional phase (Fig. 6). Such phase decoupling of internal components of DS enhances an analogy between DS and turbulence that would allow developing a kinetic theory of DS. Some first steps on this wave were made [14], but a complete theory is not developed to date.

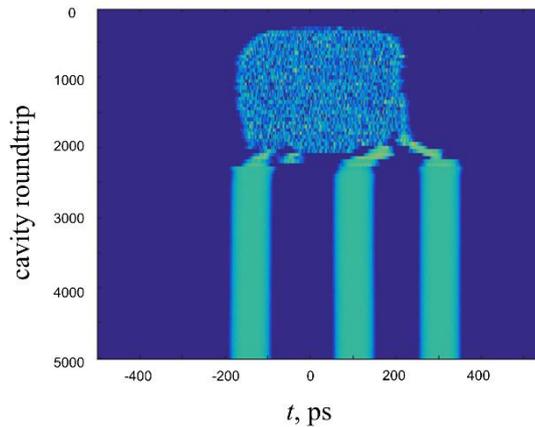


Fig. 6. Contour plot of evolving DS power for $E_s = 3 \times 10^5$ and $\beta = 0.2 \text{ ps}^2$.

We observed numerically that the coherence degradation is related closely to the Δ -decrease, and the turbulence appears for $\Delta^{-1} \sim 1 \text{ ps}$ so that the short-range correlation time is about of 1 ps within an incoherent DS with the spectrum width $\propto 1/\Omega_L$ (Fig. 7). The quantitative theory of this phenomenon is under development at present.

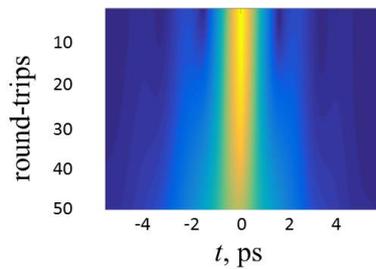


Fig. 7. Contour plot of autocorrelation function in a turbulence regime of Fig. 6.

4 Effect of stimulated Raman scattering and Raman dissipative soliton

The stimulated Raman scattering (SRS) is an example of a nonlocal nonlinear response which affects the DS dynamics of fiber laser and can limit its energy scalability and stability [23,24]. With SRS, the nonlinear phase term in Eq. (3) has to be replaced by [16]

$$\begin{aligned} & -i(1-f_R)\gamma|a(z,t)|^2 a(z,t) - if_R\gamma a(z,t) \int_{-\infty}^t dt' h(t-t')|a(z,t')|^2 \approx \\ & \approx -i(1-f_R)\gamma|a(z,t)|^2 - iT_R\gamma a(z,t) \frac{\partial |a(z,t)|^2}{\partial t}, \end{aligned} \quad (15)$$

where f_R is a fraction of SRS in total phase nonlinearity and the SRS response function for glasses is usually approximated by [16]

$$h(t) = \frac{T_1^2 + T_2^2}{T_1 T_2} \exp\left(\frac{t}{T_2}\right) \sin\left(\frac{t}{T_1}\right) \quad (16)$$

(T_1, T_2 are the response times). The approximation showed in the second line of Eq. (15) can be used for comparatively long ($\gg T_1, T_2$) but spectrally broad pulses and such approximation was used in the present work

(here, $T_R = f_R \int_0^\infty h(t) t dt$).

The numerical simulations, taking into account the SRS demonstrate that the minimal β providing the DS stabilization against multi-pulsing (i.e., satisfying $E > E_s$) is lower than that without SRS (Fig. 8).

It is obvious that SRS lowers DS energy and reduces its peak power. That narrows the DS spectrum and suppresses spectral dissipation. As was stated above, the last factor is the main source of multi-pulsing for small β . Therefore, SRS stabilizes DS in the vicinity of minimal dispersion.

The most exciting effect of SRS for large β is suppression of both turbulence and DS splitting. We assume that this phenomenon is closely connected with the emergence of new type of DS - Raman dissipative soliton - which is downshifted in frequency [24] (Fig. 9). The latter effect can play a role of passive negative feedback when the increase of power is suppressed by spectral dissipation due to the SRS-induced self-frequency shift. This topic requires further numerical and analytical investigations.

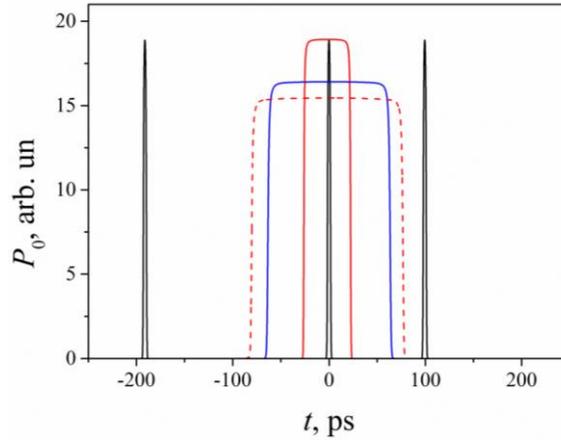


Fig. 8. DS profiles without ($f_R = 0$; black and blue solid lines) and with SRS ($f_R = 0.22$, $T_R = 1.8$ fs; solid and dashed red curves) for $E_s = 3 \times 10^5$, $\beta = 0.01$ (solid black and red lines) and 0.02 ps^2 (solid blue and dashed red lines).

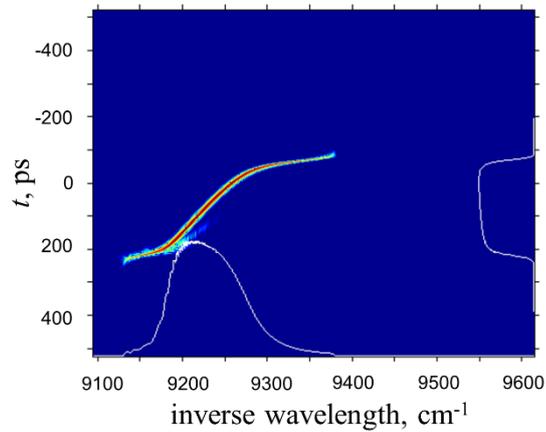


Fig. 9. Wigner function and its projections (spectrum, bottom; time profile, right) for the Raman dissipative soliton. Parameters correspond to those of Fig. 8, but $\beta = 0.7 \text{ ps}^2$.

Conclusions

DS dynamics was considered numerically on an example of the complex nonlinear cubic-quintic Ginzburg-Landau equation describing a variety of dissipative phenomena far from thermodynamic equilibrium. As a physical testbed, we chose a Yb-fiber laser operating in the all-normal-dispersion regime. It was conjectured that a structure of DS resembles that of turbulence in the

spectral domain. In particular, the spectrum is described by the Rayleigh-Jeans distribution with the characteristic “chemical potential.” There is the cut-off frequency, which is defined by the Langmuir dispersion relation and the spectral dissipation. Spectral density concentrates at $\omega=0$ with a subsequent declining frequency transfer to the cut-off frequency. The long-range correlation time defined by “chemical potential” degrades with the dispersion decrease that results in multi-pulsing. On the other hand, dispersion growth reduces the cut-off frequency that leads to spectral condensation, DS extra-broadening, and its substantial chirping. As a result, the small perturbation can cause phase decoupling between components of DS, so that it loses coherency. The latter leads to turbulence and DS decomposition. Stimulated Raman scattering, which is considered traditionally as a destabilizing factor, enhances stability against multi-pulsing due to the reduction of DS peak power and spectral width. For large dispersion, the SRS suppresses turbulence and DS decomposition and forms a new type of DS – Raman dissipative soliton – with down-frequency-shifted spectrum.

Acknowledgements

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