Chaotic dynamics of ice crystals scattering sun light

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Abstract. We are reporting the existence of the nonlinear dynamics in some curious phenomena observed in the context of atmospheric optics: jumping sundogs, halo formation and the miracle of the Sun. A possible connection between these atmospheric patterns of the atmospheric optics and light scattering in particles with the same size of the ice crystals could be based in the concept of diffracted rays.
Keywords: Chaotic Modelling, Jumping Sundogs, Ferrofluids, Parlaseric circle.

1 Introduction

There are some interesting phenomena involving atmospheric optics, for example, the jumping sundog is an amazing band of light continuously oscillating over the sky [1], sweeping downward and jumping up again, as it is shown in Fig. 1. Besides this phenomenon, Wirowski [2, 3] studied the bizarre effect known as “the miracle of the Sun”, in which many people observed oscillations in the size of the Sun, as it is shown in Fig. 2, and suggested the dynamical system describing this effect. These phenomena lasted a few minutes and some of them were recorded by some amateur observers. Another report of a luminous column of white light appeared next to the rocket Atlas V and followed the rocket up into the sky [4], when the rocket penetrated the clouds, creating a new kind of halo. These reports raise the question: how these optical phenomena are created?

The most orthodox explanation for these phenomena involves the light scattering by the ice crystals present in the clouds, and this hypothesis holds that atmospheric electric fields exert torques on these ice crystals, leading to an oscillation in their orientation. In this way, an oscillating electric field in the atmosphere can temporally reorient the position of charged ice crystals reflecting sunlight, which scatter sun light differently [5], pulsing at frequency of 1 or 2 Hz. These frequencies are compatible with the oscillations from the amplitude of motions obtained from the videos available at social media shown in Figs. 1 and 2.

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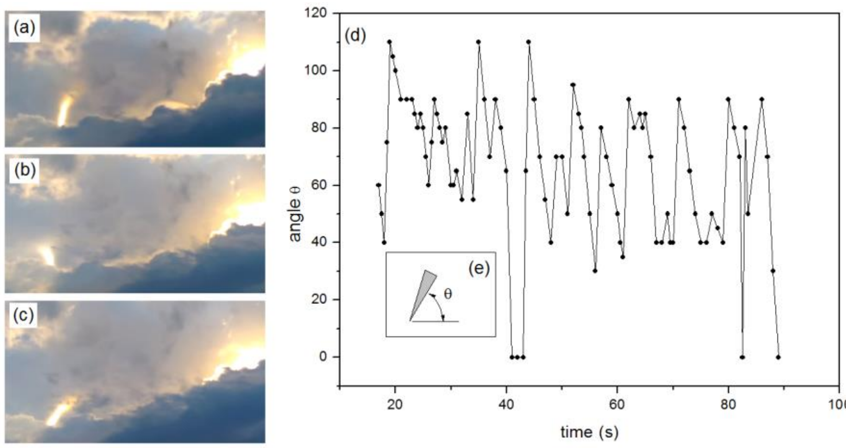

Fig. 1. The jumping sundog phenomenon observed by the YouTube user QuadeM13 (https://youtu.be/CPk0mKVnnCs). We can see in the pictures from (a) to (c) a sequence of the oscillations of a streak of light over the clouds at left, with the Sun at right side. In (d) we have obtained the erratic oscillations of this band of light based on the diagram shown in (e).

The explanation of the white column creation and new kind of halo for the case of the Atlas rocket is another example of collective behavior of ice crystals. According to the explanation of some researchers [5], the ice crystals hexagon plates were not randomly scrambled, but they were organized by shock waves as a congregation of microscopic spinning tops, with crystals tilting between 8 and 12 degrees, then they gyrate, so that the main crystal axis described a conical motion. These three phenomena, the motion of the jumping sundogs, the apparent oscillations of the Sun size, or the explanation of the sundog destruction by the Atlas V rocket, enable us to have some insights of the dynamics present in such systems. Taken together, the observation of light effects previously described suggested that the ice crystals could be represented by some sort of nonlinear dynamics. For example, the erratic motions of the jumping sundog shown in the plot of Fig. 1(d), or the different periodic oscillations of the plots in period 1 of Fig. 2(d) and the period 2 of Fig. 2(e) of the apparent Sun size are a simple way to demonstrate that we are dealing with nonlinear systems. In the context of the dynamical systems, we are observing period doubling behavior in the reconstruct attractor of Fig. 2(f) from the time series of Fig. 2(e), which present the typical double loop pattern of the diagram of Fig. 2(g).

The aim of our study is to discuss some models representing the existence of chaotic dynamics of the light scattering in a system constituted by ice crystals subjected to an external oscillating electric field. We explore the optical effects observed and related them to the optics observed by nanoparticles controlled by an external field.

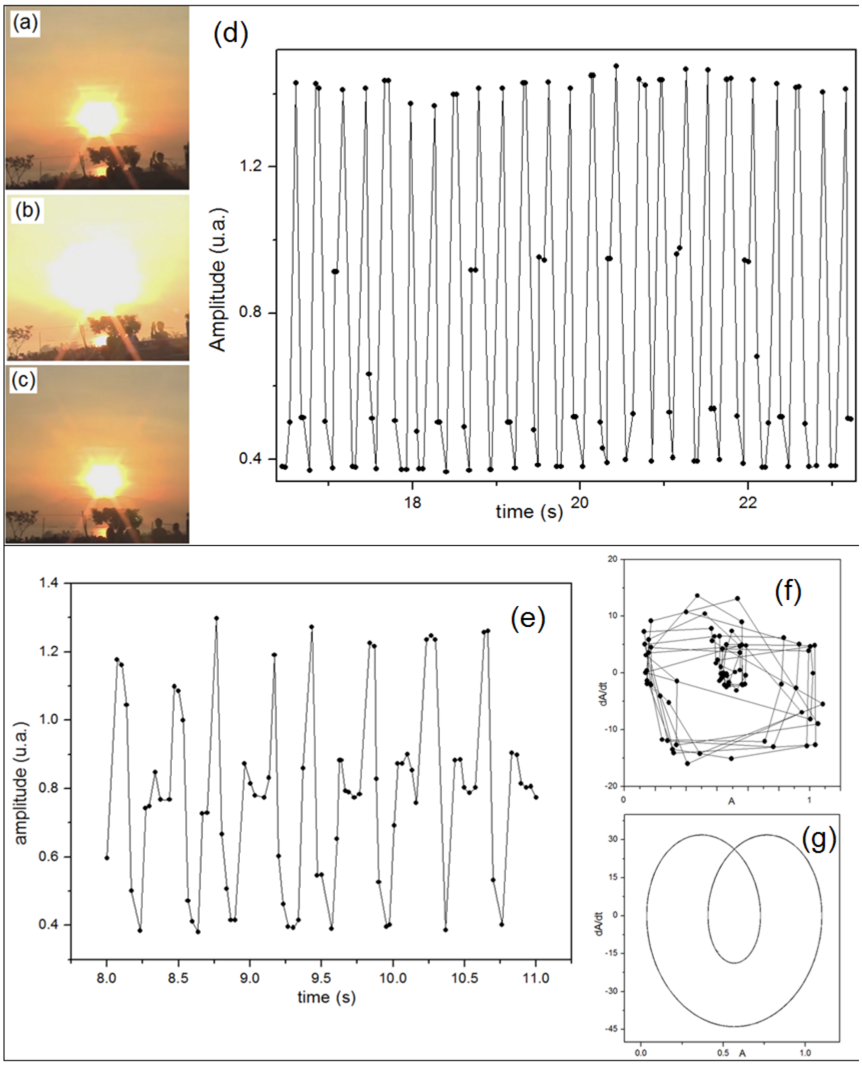

Fig. 2 The miracle of the Sun phenomenon in (a)-(c) represents the Sun size oscillations, and in (d) the time series of the light amplitude of the Sun changing very fast. Time series of the "miracle of the Sun phenomenon" in a period 2 behavior in (e). In (f) the reconstructed attractor from the time series in (e), and in (g) the smoothed reconstructed attractor of the previous plot.

2 The cloud as a complex fluid

Even though clouds are ubiquitous, they form a complex system that changes constantly, in which ice nuclei may catalyze the formation of a few frozen droplets. The ice crystals grow at expense of liquid droplets, and there is the formation of particles with different shapes with sizes around hundreds of micrometers. The sundogs result from different types of refraction and reflection through the basal faces of hexagonal plate or column crystals.

According to literature [5, 6], there are three main processes of the ice formation and precipitation. First, the homogeneous freezing, in which a supercooled liquid drop freezes without the assistance of ice nuclei. Second, the heterogeneous freezing, in which a supercooled drop freezes with the assistance of a solid particle, such as the aerosol particle, which is able to act as ice nuclei. Finally, the heterogeneous deposition, in which water vapor is deposited onto ice nuclei, taking a crystalline shape without first being in the liquid phase. For example, in Fig. 3(a) the aerosol particles, known as ice nuclei, catalyze the formation of a few frozen cloud droplets (typical several micrometers in diameter) from supercooled droplets of liquid water. These ice crystals grow at the expense of the remaining majority of liquid droplets, through transfer of water vapor (blue arrows). At right part of Fig. 3(a), the resulting large ice particles (often several tens to more than 100 micrometers in diameter) have higher fall velocities than the small liquid droplets, and may initiate precipitation. Atkinson *et al*. report that feldspar particles are the most effective mineral ice nuclei [6]. The halo formation occurs in clouds known as cirrostratus, as the Sun shines through the layer of ice particles [7], and this type of clouds consist almost entirely of ice particles. One feature that is not often explained is, however, that to produce very bright parhelia or halos shown in Fig. 4, the face of the ice crystals should be nearly perfect like a manufactured macroscopic prism. The ice crystals have a basic hexagonally symmetrical structure, but may grow into a number of different shapes, and it is important to note that natural ice crystals formed in the clouds are not perfect, and columnar ice crystals are often found with hollow interiors, or they have hollow faces that have grown in a stepped fashion, depending on the condition under they have grown. The irregular faces of these crystals will scatter light more irregularly, rather than producing the clear refractions required to generate parhelia. In this way, in addition to the geometric optics, we can observe some effects of the wave optics.

Because of this, a cloud can be thought as a complex fluid, in which the gas phase contains structures, such as ice crystals (Fig. 3(b)). The cloud in this condition distinguishes from simple fluids, presenting properties creating new forms of cooperative behavior and self-organization in the presence of external field. This self-organization is responsible for observation of jumping sundogs and miracle of the Sun observed in Figs. 1-2.

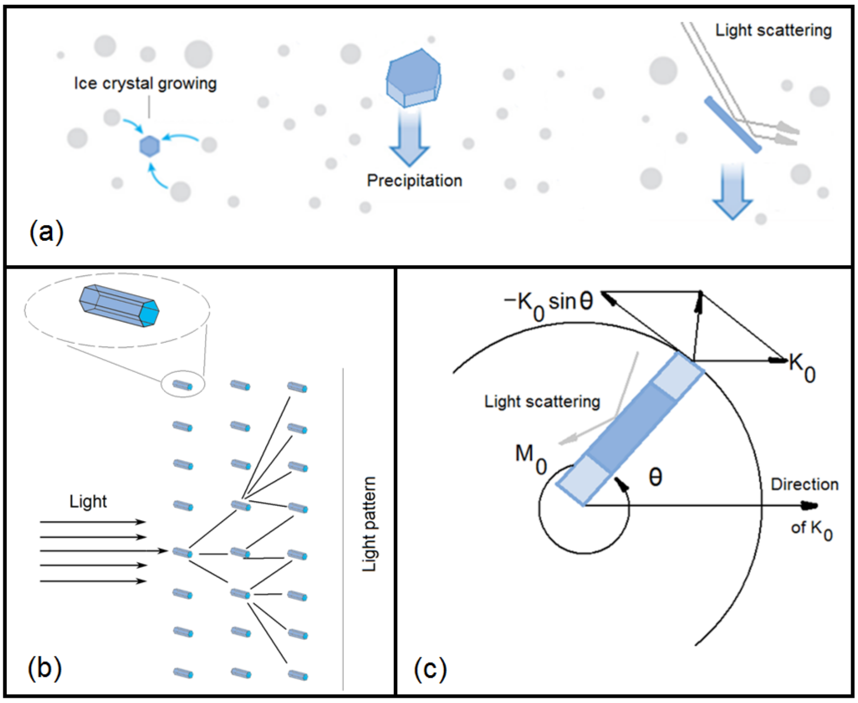

Fig. 3. The process of ice crystal formation in (a). In (b) there is a diagram of the multiple light scattering in the ice crystals. In (c) the ice crystal dynamics.

Fig. 4. Typical picture of the parhelia with Sun, sundogs and Sun pillars.

3 The dynamical system in sky with crystals

Equations that describe the motion of ice crystals in three-dimensional spaces are based on Newton's second law where the forces include friction due to viscosity, inertia and external forces exerted on the ice crystals. More specifically, the atmospheric electric fields exert torques on the ice crystals, leading to an oscillation in their orientation. In this way, an oscillating electric field in the atmosphere can temporally reorient the position of charged ice crystals reflecting sunlight, which scatter light differently. Foster and Hallett [5] reported that electric field of 4.6 KV/m pulsing at frequency of 1 or 2 Hz exhibit visual light intensity changes, while frequency of 3 to 5 Hz did not display visual effects, giving an indication that the time to realign plate crystals in these circumstances is of the order of a half second. The field exerting a torque on the crystal can be expressed as

$$\tau = 4\pi\varepsilon_0 E \sin(\theta)\cos(\theta)\,. \qquad \text{Eq.1}$$

The ice crystals are more or less two-dimensional or flat in shape, and the equation of the rotational motion of an individual crystal was obtained by Wirowski [2] taking into account the continuous distribution of the electric charge of the ice crystals, their rotation by large angles, the external forcing and the air resistance. This equation represents the non-linear vibrations of the cloud of the electrically charged ice crystals in the continuous form in two dimensions was described by the following equation:

$$\begin{aligned}
&\mu^2 \frac{\partial^2\theta(x,y,t)}{\partial t^2} + \psi^2 \frac{\partial\theta(x,y,t)}{\partial t} + \\
&\alpha^2\left(\frac{\partial^2\theta(x,y,t)}{\partial x^2} + \frac{\partial^2\theta(x,y,t)}{\partial y^2}\right) + \qquad \text{Eq. 2}\\
&-\beta^2\theta(x,y,t) + \gamma\big(\theta(x,y,t)\big)^3 = -M^E(x,y,t)
\end{aligned}$$

where M is the modulus of the total resultant moment acting on the crystal.

These equations can be converted into a simpler form of a dynamical system known as circle map, represented in the diagram of Fig. 3(c), given by

$$\begin{aligned}
\theta_{n+1} &= \theta_n + \Omega - \frac{K}{2\pi}\sin(2\pi\theta_n) + br_n \pmod 1 \\
r_{n+1} &= br_n - \frac{K}{2\pi}\sin(2\pi\theta_n)
\end{aligned} \qquad \text{Eq. 3}$$

where, θ_n denotes the angle of the rotator after to the n-th impulse, r_n is proportional to the angular velocity, Ω is the frequency ratio between the electric field and ice crystal oscillation, b is the dumping constant related to the contraction of the phase space, and K is the coupling strength.

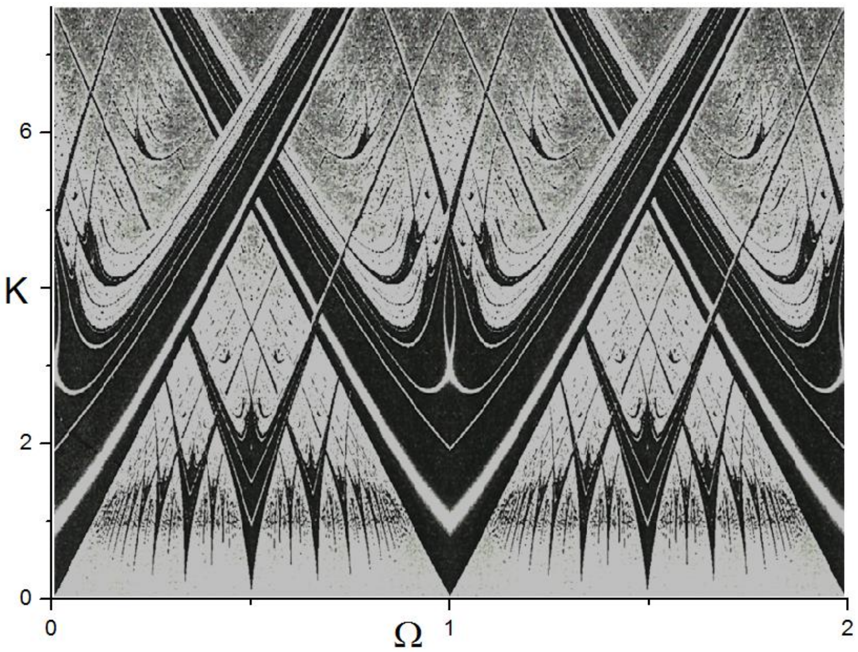

Fig. 5. The Lyapunov exponent σ for the circle map of Eq. 3 with parameter $b = 0$.

The connection between the time series of two-phase flows and light scattering can be found in the literature in dripping faucet experiment [9] and bubble formation systems [10]. This approach enables us to compare directly the time series obtained from the videos of the atmospheric phenomena and the angle of rotation of the circle map, based on the Takens theorem, in which a dynamical system can be reconstructed from a sequence of observations of the state of the dynamical system of Fig. 2(f), and smoothed in Fig. 2(g). This type of data analysis preserves the main properties of the dynamical system.

The dynamics of this system can be explored if we calculate the Lyapunov exponent σ for this map, and plot it as a function of Ω and K, as it is shown in Fig. 5, in the same way as a traveler explores new places using a city map. For example, in Fig. 6, we have the periodic behavior similar to those observed in the light scattering of ice crystals for a period 2, for values of Ω equals to 0.5 and $K = 1.1$. The chaotic behavior is observed in Fig. 6(c) ($K = 6.01$, $\Omega = 0.6$, and $b = 0.2$). Another chaotic behavior is observed in the time series of the Fig. 8. The reconstructed attractor of this time series, the attractor with the aspect of a “rhino horn” is shown in Fig. 7.

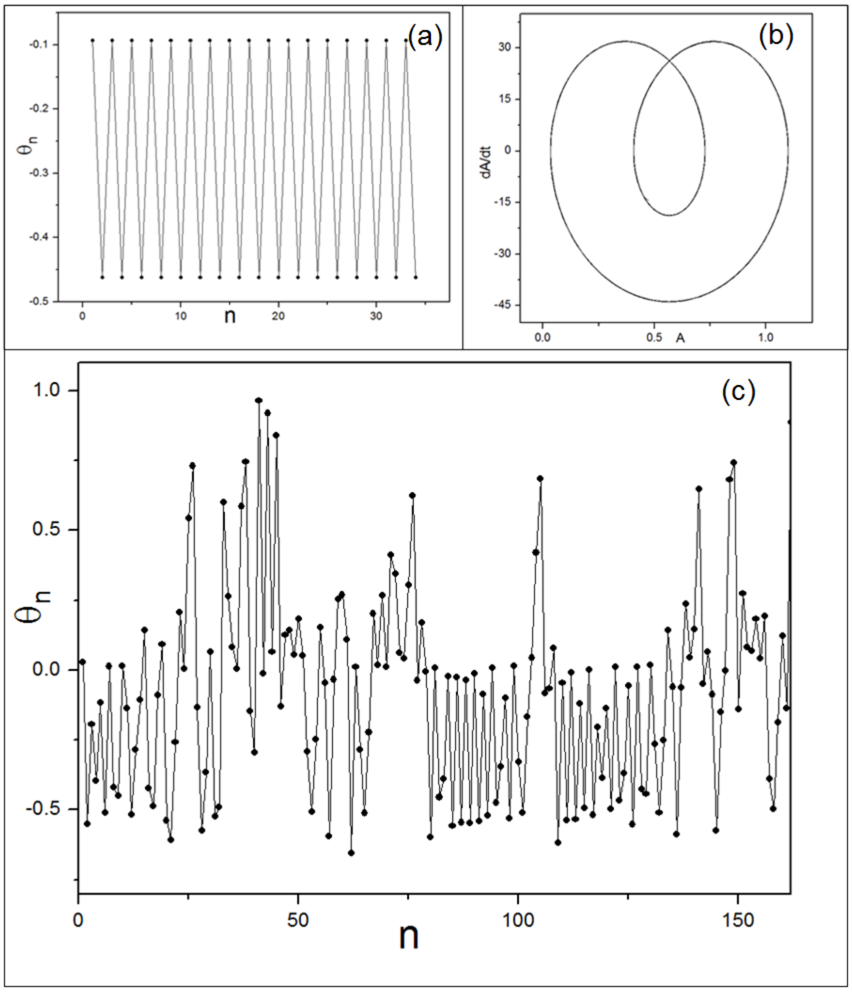

Fig. 6. Graph of the circle map function for the periodic behavior (period 2), which represents the flow of the period 2 form the reconstructed attractor of (b). In (c), the time series of the "rhino horn" attractor, representing the chaotic motion of the ice crystals.

The chaotic behavior of the time series of Fig. 6(c) is represented with the reconstructed attractor of Fig. 7, showing that the ice crystals motion concentrate around the origin of the plot, with orbits jumping in the structure of the "rhino horn" attractor, resembling the dynamics observed in Fig. 1 for the case of the jumping sundogs observed in the thunderstorm cloud.

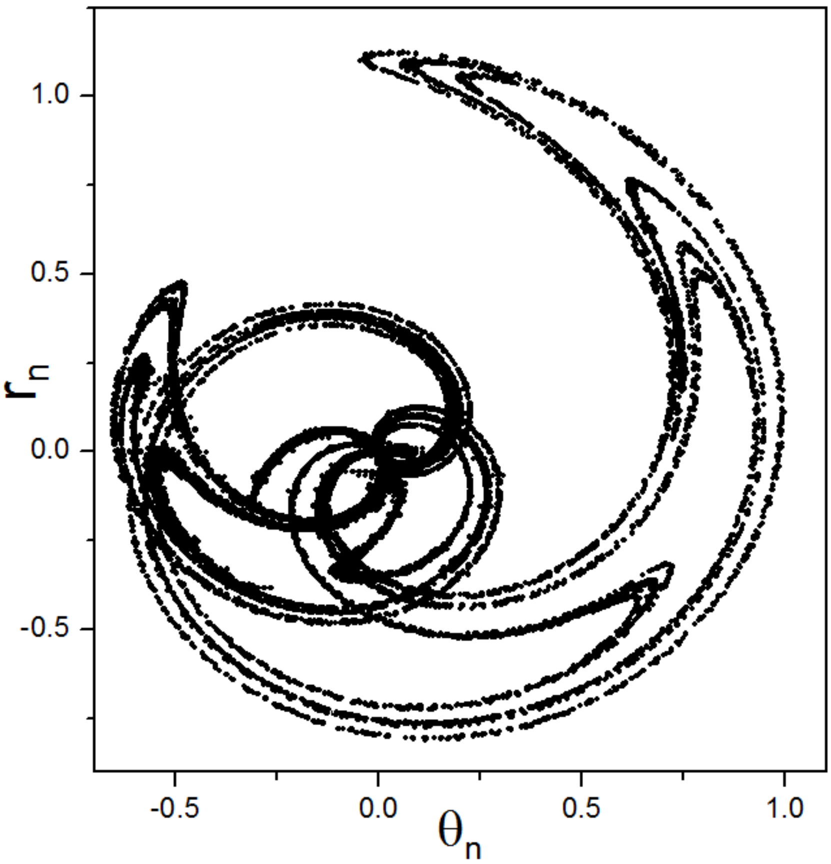

Fig. 7. The attractor “rhino horn”

4 Some experiments with laser dogs

A possible connection between some light patterns of the atmospheric optics and light scattering in particles with the same size of the ice crystals could be based in the concept of diffracted rays, introduced by the Geometrical Theory of Diffraction [11]. The behavior of the light scattering in some elements of foams is like the electromagnetic wave scattering involving high-frequency fields in sharp obstacles. For example, when a laser beam is focused at the center of a curved triangular prism with cross section around 100 μm, we have observed the formation of two laser dogs and one laser spot, along with the halo formation. Increasing the value of the angle, we can see bifurcations in some of these light spots, as it is shown in Fig. 8.

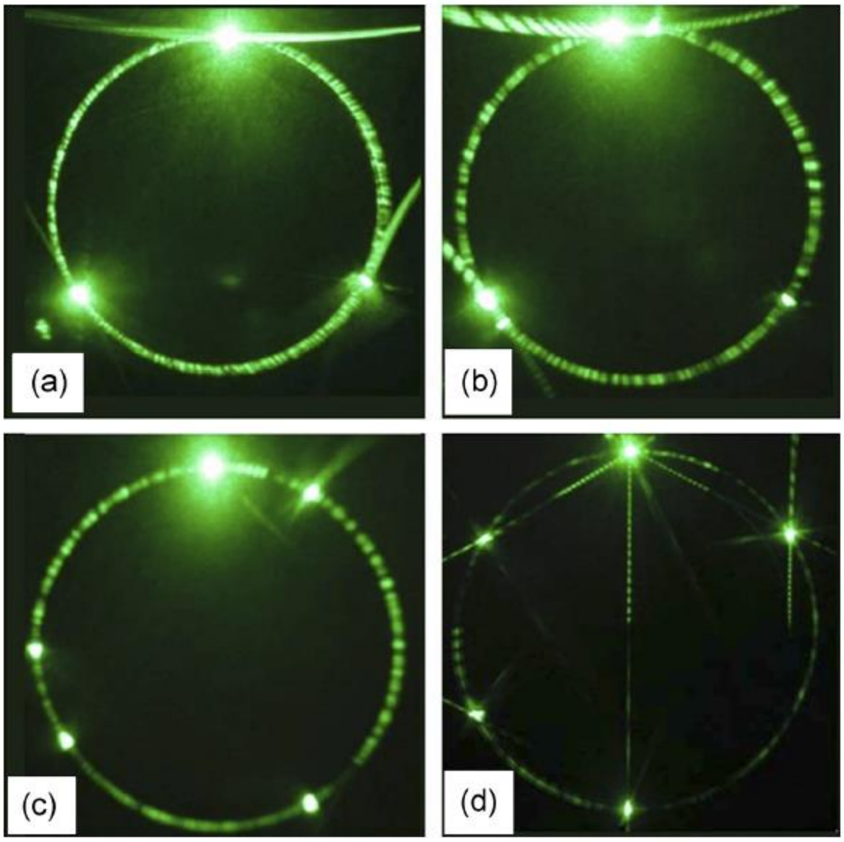

Fig. 8. The parlaseric circle. In (a) one laser spot and two laser dogs. Changing the angle of incidence in (b), creates the bifurcation of the laser spot and one of laser dogs. In (c), we can observe four laser dogs, and the laser spot at the top of the circle. In (d) the pattern with the parlaseric circle, laser spot, laser dogs, and some laser pillars.

The same effect can be observed when light hits some needlelike formation in ferrofluids [12]. In this setup, we have one Hele-Shaw cell acting as a beam splitter, in which part of the green laser is passing trough the Hele-Shaw cell forming the laser spot at the right side of Fig. 9(a), and part of the laser beam is reflected forming the laser dog at left side. The Hele-Shaw cell is subjected to an external magnetic field *B*. In this analogy, the ice crystals are represented by the needlelike structures of ferrofluid, and the external magnetic field plays the same role of the electric field in the atmosphere. See more details about this experiment in our paper about jumping laser dogs [1].

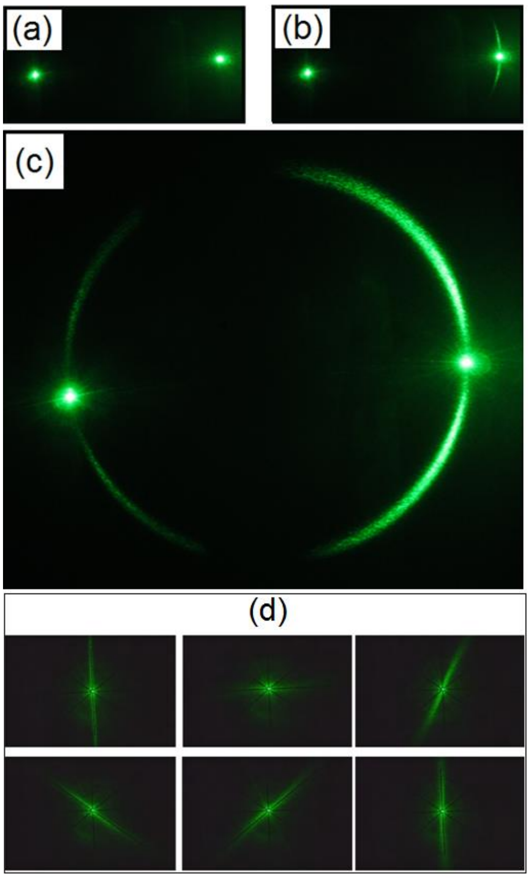

Fig. 9. In (a) laser dog and laser spot. (b) applying a magnetic field of B = 200 gauss, and in (c) applying 600 gauss. In (d) the motion of the jumping laser dog.

In Fig. 9(b), when a magnetic field B is applied around 200 G, we can observe the formation of two curved diffraction streaks, because there is the formation of small needles inside the ferrofluid. Increasing the magnetic field to B = 600 G in Fig. 9(c), we can observe the formation of the parlaseric circle. According to Dai and Pu [12] there is a difference between this type of diffraction and the regular one, due to the absence of characteristic well-defined fringes observed in diffraction patterns. The light diffracted by the ferrofluid is an integral sum of diffraction events from individual needles, and the observed pattern can be understood as a combination of diffraction and interference. The jumping laser dog motion for different orientations of the external magnetic field is shown in the sequence of Fig. 9(d). In Fig. 10(a)-(b) there is the jump of a sundog recorded in Russia in July of 2016 and presented by the YouTube user "Earth Watchers" [13]. The same effect can be observed when light hits some needlelike formation in ferrofluids [12]. In Fig. 10(c), when a magnetic field B

is applied around 400 G, we can observe the formation of a curved diffraction streak, because there is the formation of small rods inside the ferrofluid. Increasing the magnetic field to B = 800 G in Fig. 10(d), we can observe the formation of a large streak then the previous case. More examples of these phenomena and related experiments can be found in Refs. [14-16].

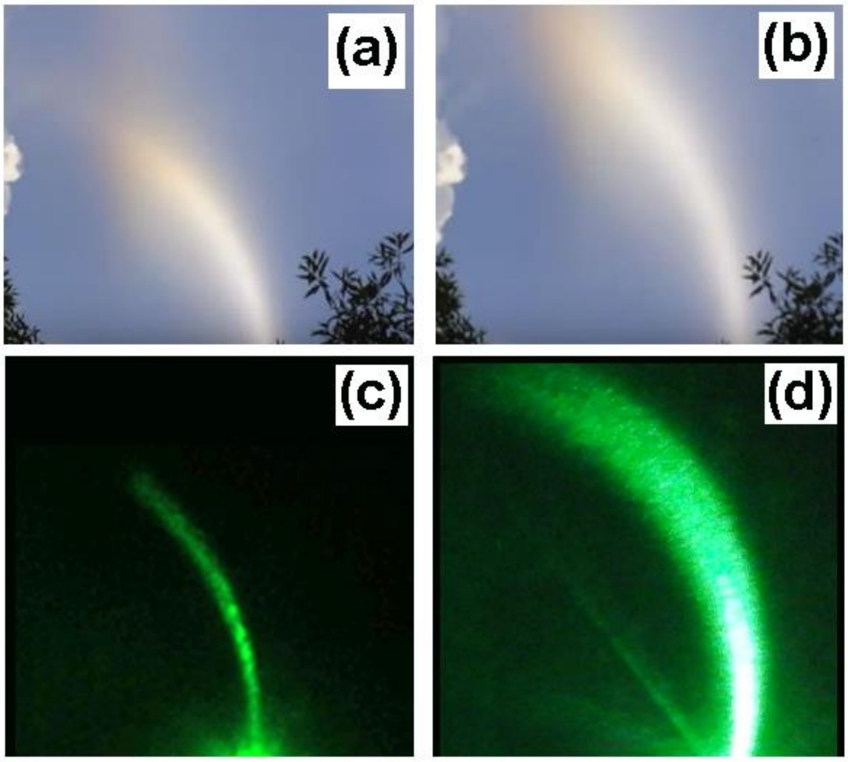

Fig. 10. In (a) and (b) show the jumping sundog. In (c) applying a magnetic field of B = 400 G, and in (d) applying 800 G, we have a similar motion of the jumping laser dog.

Conclusions

We have described some curious phenomena involving sunlight scattering and ice crystals in the clouds, the jumping sundogs, the miracle of the Sun and halo creation with a rocket, from the point of view of chaotic systems. The study of light scattering in ice crystal clouds has been evolving in line with the progress being made in other areas of science and technology, triggered by a continuous improving of the image recording systems by the general public and the respective sharing of these observations in social networks. Considering the possibility of these reported phenomena as legit, we are suggesting that atmosphere acts as a complex fluid under certain conditions, and using an experiment, we have simulated some aspects of halo formation and the jumping sundogs. A possible connection between some light patterns of the atmospheric optics and light scattering in particles with the same size of the ice crystals could be based in the concept of diffracted rays. In this way, the light scattered by ice crystals is an integral sum of events from individual ice crystals, and the observed pattern can be understood as a combination of multiple scattering.

Acknowledgments

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