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Chaos in Bubbles, Drops, and Foams

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Abstract. The dynamics of bubble, drop, and foam formation can present chaotic behavior. Under certain conditions these systems present routes to chaos such as period doubling, period adding, or intermittency. In addition to these aspects, some interesting emergent types of fragile objects can be observed, such as antibubbles.
Keywords: Bubble, Drop, Foam, Antibubble, Integrate-and-Fire Dynamics, Routes to Chaos, Circle Map.

1 Introduction

One of the best examples of a dynamical system exhibiting chaotic behavior is the bubble formation from a nozzle. This system presents a myriad of routes to chaos, such as period doubling route, intermittency route and period-adding route, just to cite some of them. We have developed this study from the paradigmatic case of a chaotic system, the dripping faucet experiment, suggested by Rössler [1]. In addition to this aspect, drops and bubbles can create foam, the fluid dynamics of foams is nonlinear, and we have observed the butterfly effect in foam systems. According to the Nobel Prize winner Pierre-Gilles de Gennes [2], bubbles, drops and foams can be considered as fragile objects, due to the ephemeral feature of these systems. In this paper we present the main features of the chaotic behavior of these fragile objects, indicating some general concepts that sum up the main aspects of chaotic systems in two phase flow and correlated systems.

2 Period Doubling and Intermittency

Bubbles, drops and foams consist mainly of a two phase flow system, consisting of liquid and a gas [1-4]. For the case of bubble and drop formation, we have to consider also the existence of a solid phase, such as a nozzle. In Fig. 1(a)-(e) we present a complete route to chaos

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in a bubbling system [5], with the related bifurcation diagram in Fig. 1(f).

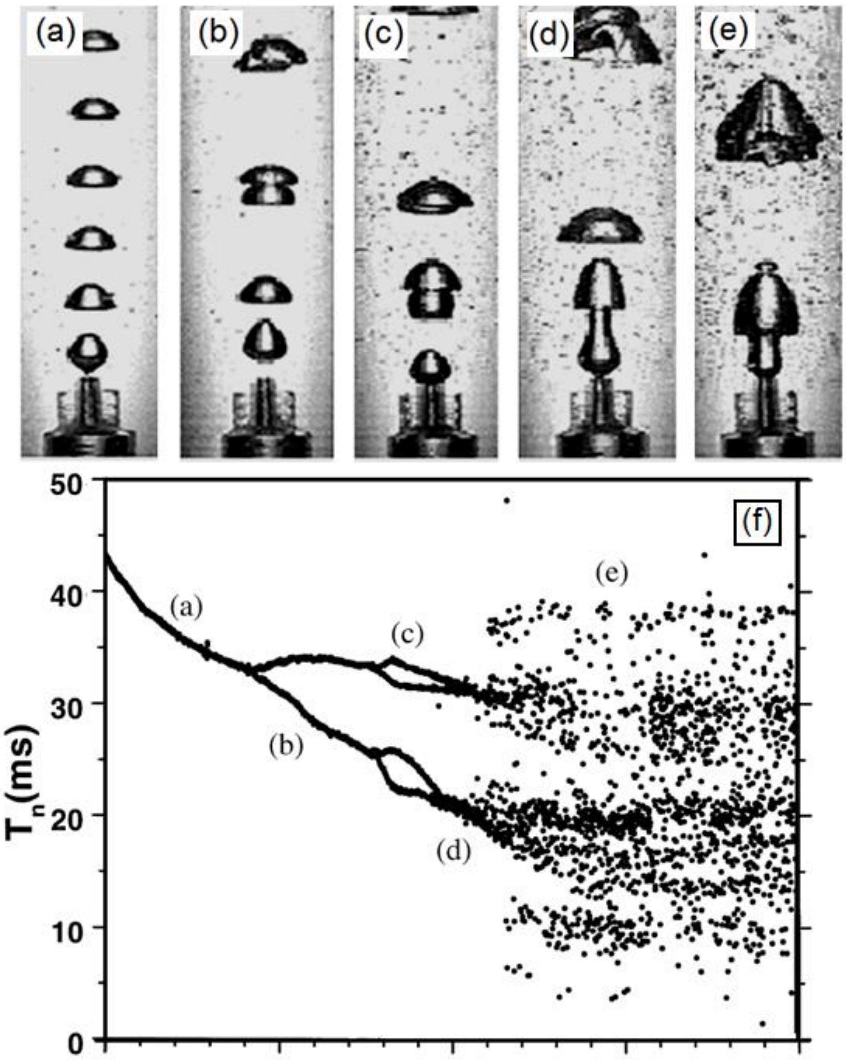

Figure 1. Chaotic behavior of a bubbling system involving period doubling and intermittency at same time, increasing the air flow rate of bubble formation.

The interface present in bubbles, drops and foams is one of the main features of the existence of nonlinearity in these systems, and this feature can lead to chaotic behavior.

Another example of a system presenting chaotic behavior is the dripping faucet experiment [1][4]. In Fig. 2(a) we present a bifurcation diagram for this system presenting period doubling, jumps and chaotic behavior. In Fig. 2(b) we are presenting a chaotic reconstructed attractor from the dripping faucet system.

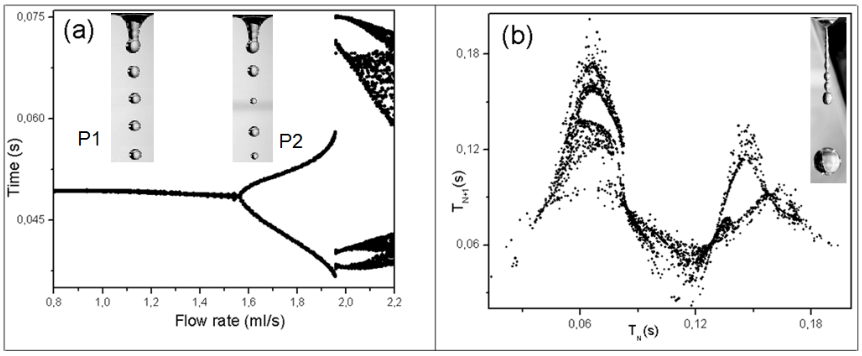

Figure 2. In (a), bifurcation diagram of the dripping faucet system showing periodic behavior, period doubling, jump, and chaos. One example of a chaotic behavior in the dripping faucet system is shown with the chaotic attractor in (b).

A model for the dripping faucet experiment is giving by:

$$\begin{aligned} dx/dt &= v, \\ d(Mv)/dt &= Mg - kx - bv, \\ dM/dt &= Q, \end{aligned} \tag{1}$$

where x is the coordinate of the mass center of the drop, Q is the liquid flow rate, k and b is surface tension and viscosity respectively, and M is the mass of liquid attached to the faucet. A drop with mass ΔM detaches from the nozzle with velocity v_c, with the factor α, when the attached drop reaches the point x_c. See more information about this system in Refs. [1][4]

3 Period-Adding bifurcations and Circle Map dynamics

The second important feature for the observation of chaotic behavior in systems involving fragile objects is the injection of mass and energy. This injection is related to the redistribution of matter with time, and the flow of the liquid/gas phases give the energy necessary to create states of out of equilibrium. In presence of these features, we can observe circle map and period-adding dynamics [5-13].

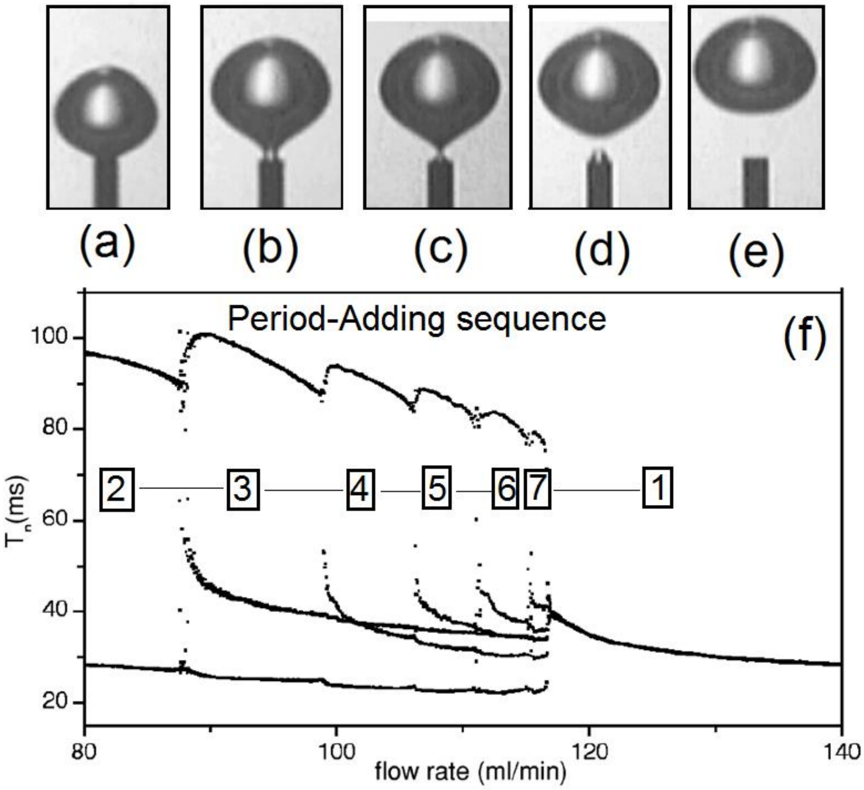

Figure 3. From (a) to (e) the profiles of a bubble growing with an air flow rate of 134 ml/min. This type of bubble growing is associated with period adding bifurcations (f).

The flow of gas is represented in Fig. 3 in the bubble formation from a nozzle. The bubble grows in Fig. 3(a) and in Fig. 3(b) the bubble lifts off and the neck appears collapsing latter in Fig. 3(c). In Fig. 3(d) a small bubble appears, and it retracts in (e). This process is related to the period-adding bifurcation presented in Fig. (f), which is a sequence of periodic bifurcations of k to $k+1$. A model for this type of dynamical system is present in Fig. 4, which is a map with periodic reinjection. Besides bubble formation, period adding behavior is observed in the interspike interval series generated by a neural pacemaker inserted in rats, or in the Fitzhugh-Naguno model [9] for mimicking the firing neurons. In this way, we have observed that both bubble creation model of Fig. 3, and drop formation of Eq. (1) are somehow explained by integrate and fire dynamics [10-11].

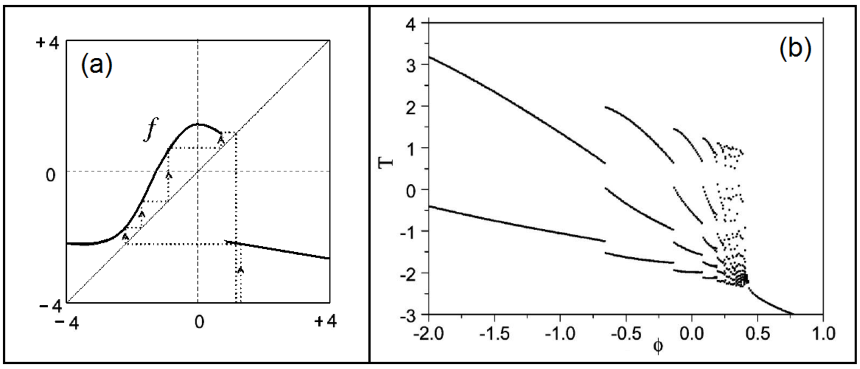

Figure 4. Example of function and iterated map in (a) for the period adding behavior with chaotic windows. Bifurcation diagram showing period adding behavior in (b).

In order to study this sensitiveness to initial conditions during bubble or drop formation, we have studied the perturbation of the formation of bubbles using sound waves, as it is shown in the diagram of Fig. 5(a), with the reconstructed attractors from the experiment in Fig. 5(b), showing period 1, bifurcations, and chaotic behavior.

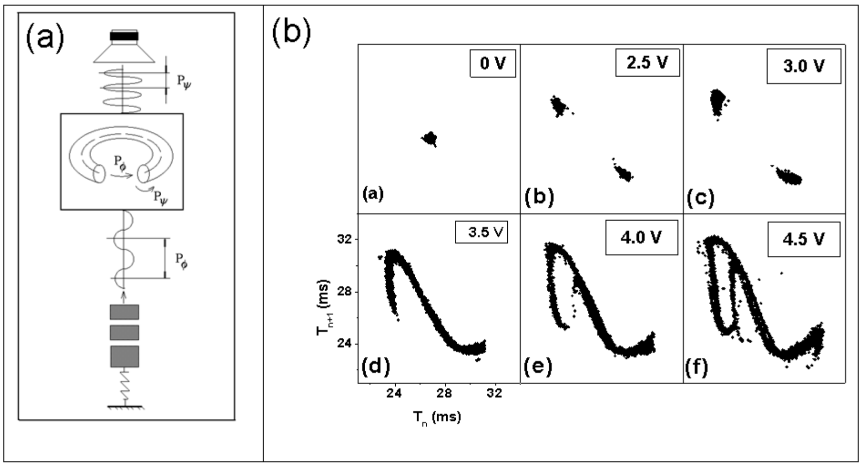

Figure 5. In (a) diagram of the bubble formation experiment subjected to a sound wave. In (b) a sequence of different types of attractors obtained from the experiment for different values of sound wave amplitude.

It is interesting to note the similarity between the experiment and the model based on the circle map for the case of bubble formation disturbed by sound waves [5-10], as it is shown in Fig. 6.

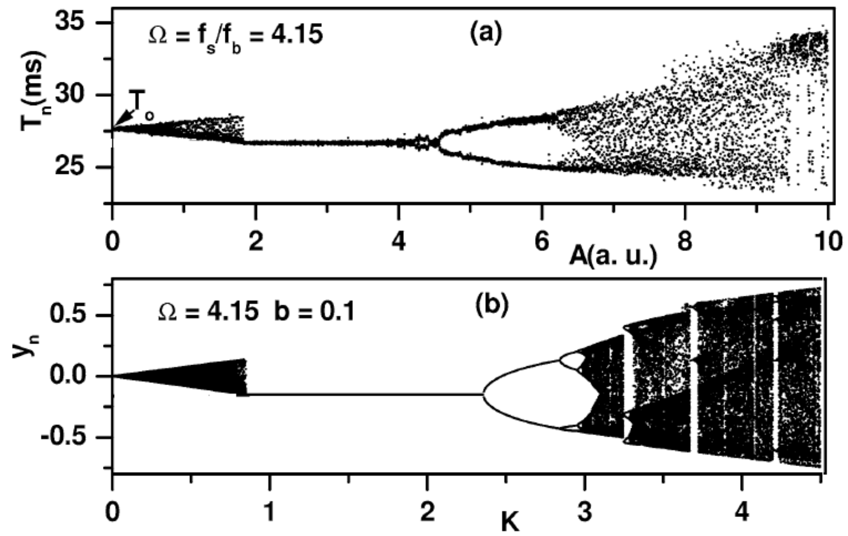

Figure 6. Comparison between the time series obtained from the experiment in (a) and the data obtained from the model of the circle map dynamics in (b).

4 Foam as a dynamical system

One possible definition of liquid foam is a way to store surface energy. We have studied different ways to create foams [13-16].

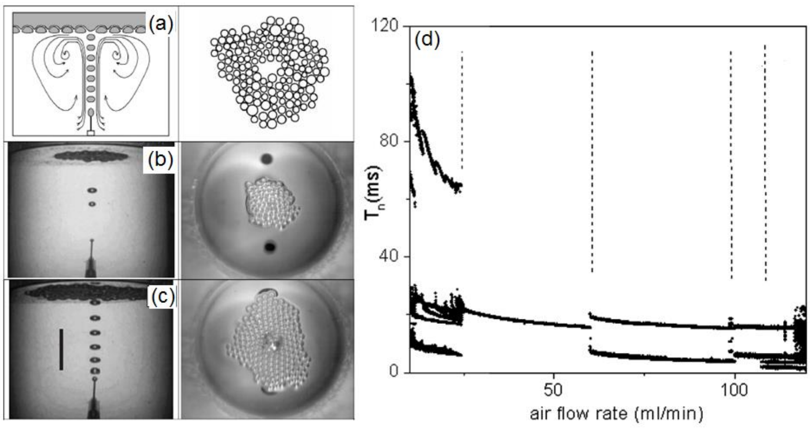

Figure 7. Bubble raft (a), a type of two dimensional foam created adding one bubble at time. In (b), there is a foam obtained from a period 2 bubbling and in (c), a foam obtained from coalesced bubbles. In (d), we have observed foams from different bubbling regimes, from periodic to chaotic behavior.

Injecting air through a submerged nozzle can form air bubbles in a liquid [14]. Once bubbles are formed, each bubble rises toward the liquid surface forming the foam of Fig. 7(a). The bubble population balance equation in a bubble raft with *N* bubbles is given by

$$\frac{dN}{dt} = -DN + f_b, \quad (2)$$

in which *D* is the bubble-bursting rate coefficient and f_b is the bubbling frequency, the transient in this type of foam is

$$N(t) = N_0 e^{-Dt} + \frac{f_b}{D}. \quad (3)$$

For the case of a bubble raft with an stable population we have $dN/dt = 0$ when time tends to infinity, so

$$N = \frac{1}{D} f_b. \quad (4)$$

In this way, the number of the remaining bubbles in the bubble raft also has a linear relation with the bubble frequency f_b, and is determined by the characteristic time scale $1/D$, which is the average value of the bubble lifespan in this type of foam.

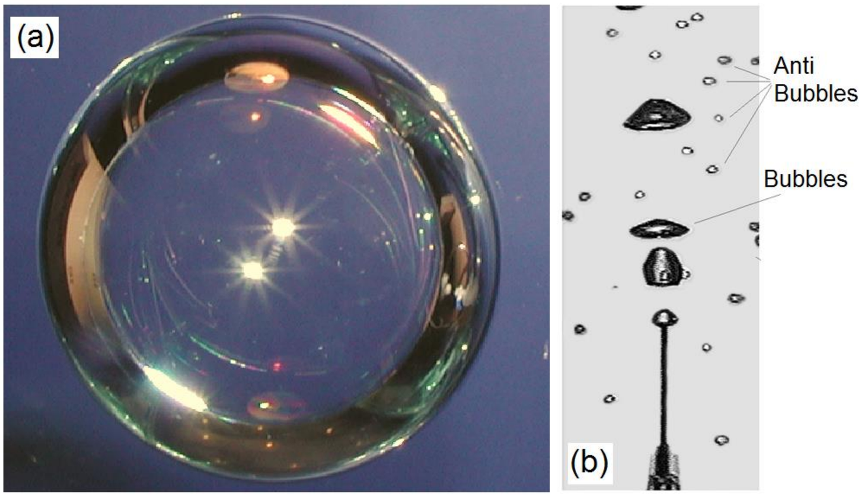

Figure 8. Antibubbles in (a) observed in the experiment when bubble formation presents a intermittent chaotic regime (b). See Ref. [8] for more information.

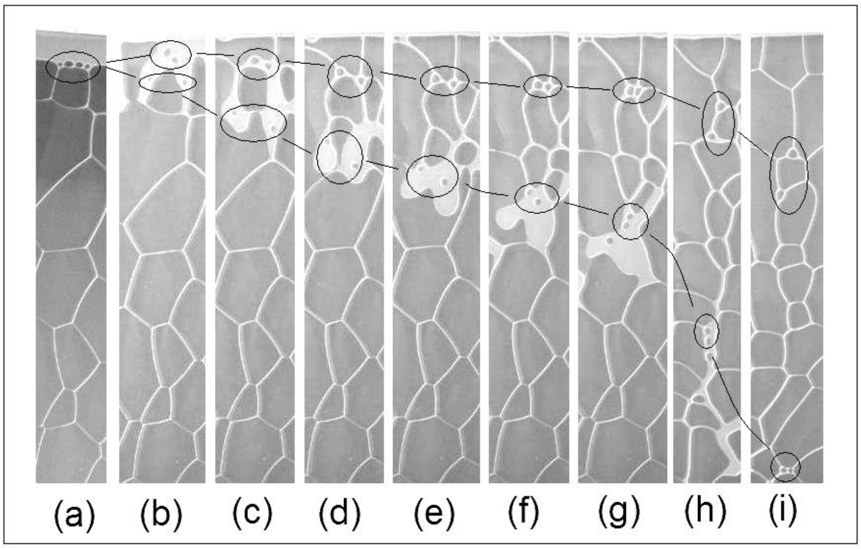

Figure 9. Stretching and folding mechanism observed during the foam formation in a Hele-Shaw cell, representing the ‘butterfly effect”.

Besides the bubble raft formation, we also have observed the formation of antibubbles for the case of the bubbling system in an intermittent regime [8]. An antibubble consists of a spherical shell of air, as it is shown in Fig. 8. The mechanism of antibubble formation is related to droplets, which occur for a bubbling regime of period-4 whenever a spike of liquid penetrates upward inside a large bubble. When some of these droplets encounter the inner bubble surface, a layer of air is trapped around these droplets.

We also have observed the stretching and folding mechanism in foams [15-16], which is popularly known as butterfly effect in Fig. 9, when a liquid containing a surfactant is shaken in the presence of air, a foam is formed by the action of deformation and stretching of the air/liquid interface [15]. Based in our experimental data from the series of the length of the foam from a Hele-Shaw cell, we have found that the soap film length has reached a maximum value as a function of the number of flips. During this stage, we also have found the existence of spatial divergence in the foam elements following neighbor bubbles, in which the distance between bubbles was stretched in the vertical axis, while in the horizontal the distance was folded. Large bubbles present random walk motion in both directions (Figs. 10-11), while small bubbles are scattered like balls in the Galton board dynamics.

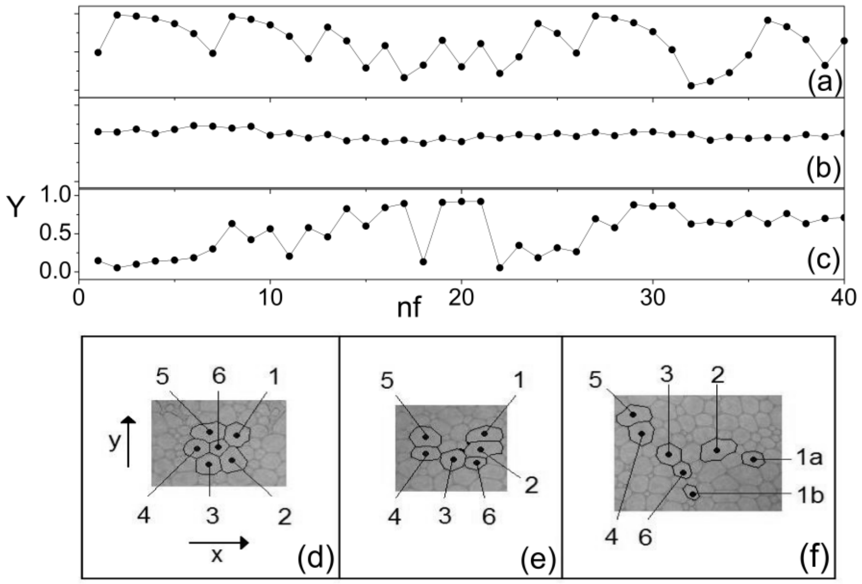

Figure 10. The time series of the baker map (a), the power spectra of Galton board dynamics in (b), and for the random walk in (c). From (d) to (f), the evolution of 6 bubbles in a foam for two flips of the Hele-Shaw cell.

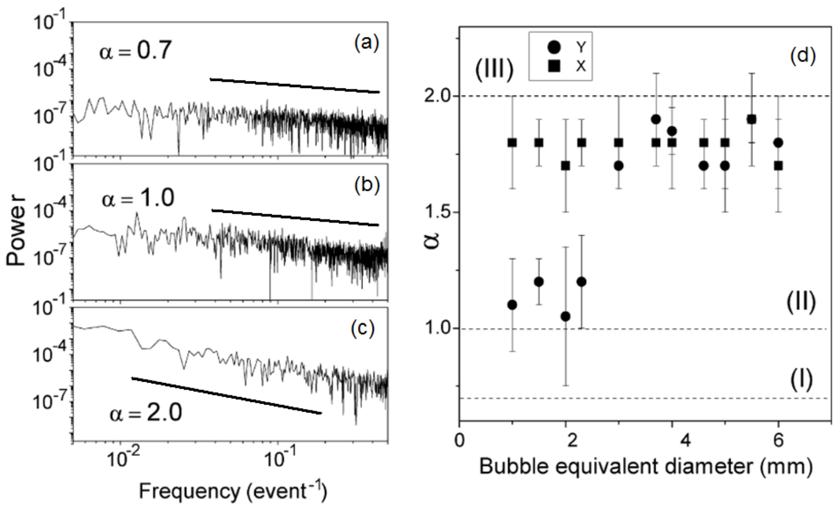

Figure 11. The power spectra for the time series of the baker map (a), the power spectra of Galton board dynamics in (b), and for the random walk in (c). We have applied the same characterization method to obtain the power spectra of the bubble motion for the coordinates (x, y) of some bubbles and obtained their respective values in (d). The dashed lines in (d) represent the values of the decay a ($f^{-\alpha}$) for the baker map (I), for the Galton board dynamics in (II), and for the random walk in (III).

5 Acoustic streaming in drops and bubbles

It is interesting to note that bubbles and drops have different behaviors when they are subjected to vibrations. We have observed experimentally using the diagram of Fig. 12, that drops are attracted to a vibrating membrane, while bubbles are repelled for the same vibrations.

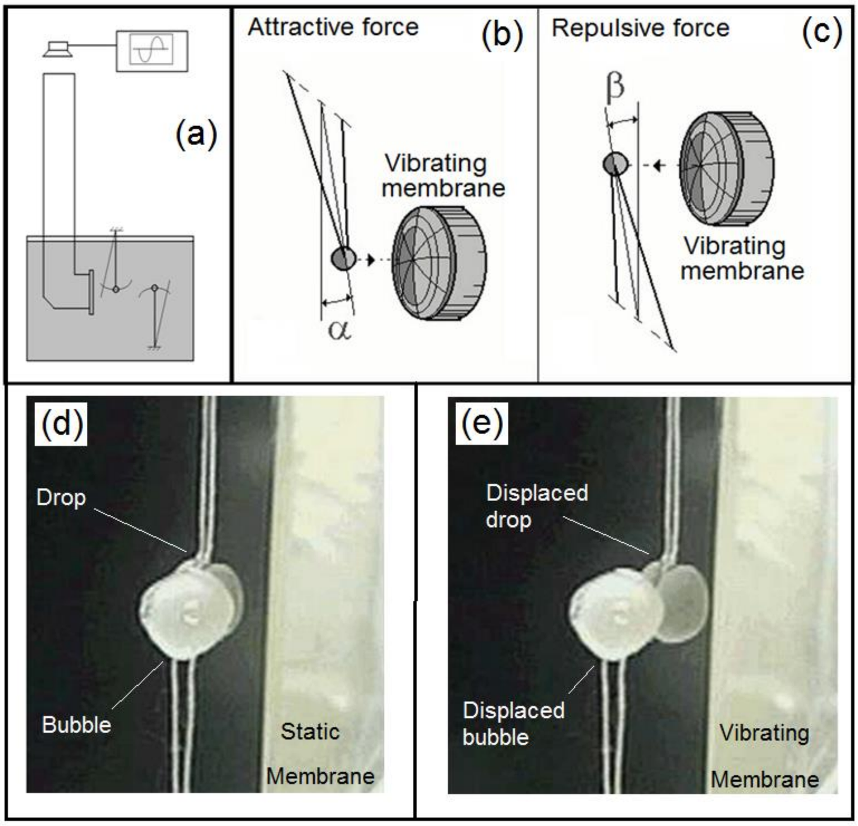

Figure 12. In (a) diagram of the experiment showing attraction of a drop and the repulsion of a bubble in the presence of the vibrations of a membrane using pendulums. In this experiment, the drop is a sphere with a density greater than the liquid, while the bubble is a sphere with density lesser than the liquid [17]. The diagram of the experiment in (b) for the attractive force for a drop represented by a pendulum, and in (c) the case of the repulsive force for the case of a bubble, represented by an inverted pendulum. The picture of the experiment with a steady membrane in (d), and with a vibrating membrane in (e).

Conclusions

We have presented the chaotic behavior of systems involving the formation of bubbles, drops and foams. Under certain conditions, these systems present the main features necessary to show chaotic behavior of dissipative systems, which are the existence of nonlinearities and sensitiveness to initial conditions. In most part of our work, in order to interpret the complex behavior of these two phase flow systems, we have proposed some empirical models using recurrences, differential equations, and metrical and geometrical characterization such as Lyapunov exponents, measuring fractal dimensions, in addition to the study of the topology of the dynamical systems.

Basically, the balance between inertial and dissipative forces defines the dynamics of these systems, however the abrupt changes of surfaces collapse the bubble or drop formation in another dynamical state, causing intrinsic hysteresis. For example, the increasing or decreasing of the control parameter can create different sequence of bifurcations, and bistability is ubiquitous in those systems. Bistability is the coexistence of two periodic regimes for the same value of the control parameter. This phenomenon implies in a shift of the position of the bifurcations points. This behavior is present during the period-adding bifurcations in the bubbling system.

There is a limit to how many bubbles can occupy the foam simultaneously. For example, for the experiment of bubble injection with a nozzle, the number of bubbles in the foam is proportional to the bubble frequency multiplied by the average value of bubble lifetime. For the case of the foam in a Hele-Shaw cell, the soap film length has reached a maximum value as a function of the number of the flips, following the logistic growth. In addition to this, when this foam reaches this stage of length stability, there is a spatial divergence of the bubbles in this foam during a sequence of flips. Large bubbles present random walk motion, while small bubbles present chaotic motion.

When air bubble formation in a nozzle is submitted to sound wave perturbation, we have observed the circle map dynamics. Another interesting case is the emergent phenomenon of antibubble formation. The antibubbling regime is a complex system, in which a heterogeneous amalgam of different things happen, like bubbling following by period doubling, discontinuity in the dynamics characterized by bubble coalescence, inverted dripping as a result of the instability of the Rayleigh jet, and finally the creation of antibubbles as the result of the intermittency and liquid circulation.

To sum up, we consider that the application of Chaos theory in two-phase flows formed by the formation of bubbles, drops, antibubbles, and foams is very good way to understand these systems.

Acknowledgments

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References

1. A. Tufaile, R. D. Pinto, W. M. Gonçalves and J. C. Sartorelli. Simulations in a dripping faucet experiment, Physics Letters A 255, 58{64, 1999.
2. Pierre-Gilles de Gennes, J. Badoz, Fragile Objects: Soft Matter, hard Science, and the thrill of Discovery, Springer, 1996.
3. A. Tufaile and J. C. Sartorelli. Hénon-like attractor in air bubble formation, Physics Letters A 275, 211{217, 2000.
4. M. B. Reyes, R. D. Pinto, A. Tufaile, J. C. Sartorelli. Heteroclinic behavior in a dripping faucet experiment, Physics Letters A 300, 192{198, 2002.
5. A. Tufaile and J. C. Sartorelli, Chaotic behaviour in bubble formation dynamics, Physica A 275, 336{346, 2000.
6. A. Tufaile and J. C. Sartorelli. The circle map dynamics in air bubble formation, Physics Letters A 287, 74{80, 2001.
7. A. Tufaile, M. B. Reyes and J. C. Sartorelli, Explosion of chaotic bubbling, Physica A 308, 15{24, 2002.
8. A. Tufaile and J. C. Sartorelli, Bubble and spherical air shell formation dynamics, Physical Review E 66, 056204, 2002.
9. J. Nagumo, S. Arimoto, S. Yoshizawa, An active pulse transmission line simulating nerve axon, Procedings of IRE, 2061{2070, 1962.
10. V. S. M. Piassi, A. Tufaile and J. C. Sartorelli, Period-adding bifurcations and chaos in a bubble column, Chaos, an interdisciplinary journal of nonlinear science 14, 477{486, 2004.
11. E. Colli, V. S. M. Piassi, A. Tufaile and J. C. Sartorelli, Bistability in bubble formation, Physical Review E 70, 066215, 2004.
12. G. Liger-Belair, A. Tufaile, B. Robillard, P. Jeandet and J. C. Sartorelli, Period-adding route in sparkling bubbles, Physical Review E 70, 066215, 2004.
13. G. Liger-Belair, A. Tufaile, P. Jeandet and J. C. Sartorelli, Champagne experiences various rhythmical bubbling regimes in a flute, Journal of Agricultural and Food Chemistry 54, 6989{6994, 2006.
14. A. Tufaile, J. C. Sartorelli, P. Jeandet and G. Liger-Belair, Chaotic Bubbling and nonstagnant foams, Physical Review E 75, 066216, 2007.
15. A. Tufaile and A. P. B. Tufaile. Stretching and folding mechanism in foams, Physics Letters A 372, 6381{6385, 2008.
16. A. P. B. Tufaile, A. Tufaile and T. A. S. Haddad, Mixing foams and grains in Hele-Shaw cell, Journal of Physics: Conference Series 246, 012017, 2010.
17. C. G. Ragazzo, Dynamics of many bodies in a liquid: Added-mass tensor of compound bodies and system with fast oscillating body, Physics of Fluids 14, 1590{1595, 2002.