Chaotic Modeling and Simulation (CMSIM) 2: 205-214, 2018

Experimental evidence of wave chaos signature in a microwave cavity with several singular perturbations

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Abstract. Wave chaos signature in a cavity quasi-optical cylindrical resonator under the influence of several singular perturbations has been studied experimentally. Earlier the spectrum of a cavity quasi-optical cylindrical resonator with a single singular perturbation as a thin metal rod was considered theoretically and experimentally. It was found that the effect of spectral lines "repulsion". This effect demonstrates the dependence of the probability distribution of inter-frequency intervals for wave spectrum of our resonator. The question is as follows. Does a sufficiently large number of irregularities have the cumulative sufficiently large impact on resonator spectrum chaotic behavior? We have detected experimentally that the sign of quantum chaos in our resonator is getting bigger with the increase of the number of singular perturbations and the dependence of the probability distribution of inter-frequency intervals tends to Wigner distribution.

Keywords: Wave chaos, Cavity quasi-optical cylindrical resonator, Singular perturbation, Probability distribution of inter-frequency intervals, Wigner distribution.

1 Introduction and problem statement

The problem of wave (quantum) chaos covers the study of various quantum systems, classical analogs of which exhibit chaotic behavior. Recently, this problem attracts quite a lot of attention, as evidenced by numerous of its theoretical and experimental studies (see, for example, [1] and the literature cited therein). When studying of wave chaos, model systems are usually used in the form of wave scattering billiards such as Sinai and Bunimovich billiard types, in which wave) chaos signs were discovered. Because of the identity of the stationary Schrödinger equation and Maxwell's wave scalar equation, quasi-optical microwave resonators for this purpose are also used, which are similar in shape to a corresponding scattering billiard.

The wave chaos characteristics are inherent of other systems, such as scattering cylindrical billiards, with a rough lateral boundary [2, 3], billiards filled with random bulk inhomogeneities [4], cylindrical billiards with a lateral

Received: 1 November 2017 / Accepted: 27 March 2018

ISSN 2241-0503

boundary with small smoothness, when at certain points there is no second derivative [5, 6].

The presence of wave chaos features was theoretically predicted also in systems containing one or several small inhomogeneities, each of which can be treated as a singular perturbation [7, 8]. However, despite the fruitfulness of the theoretical method used in these works to describe the influence of such inhomogeneities, a whole series of questions did not receive sufficient clarification in them. In particular, one of the fundamental questions remained unclear: what is the physical reason for the fact that a regular wave system under the influence of a small spatial disturbance acquires distinct signs of wave chaos? There was no answer the question: does wave chaos occur at any level of a singular perturbation, or should this perturbation be sufficiently large? We will consider a closed electrodynamic system in the form of a quasi-optical cylindrical microwave resonator with inhomogeneities, in which there are no signs of spatial symmetry and there is only one obvious integral of the motion-the energy of the system.

Because of the lack of spatial symmetry, such a system is non-integrable (in the classical sense), and because of this it is chaotic, since its state does not depend on the initial conditions (mixing effect [9]). The non-integrability of the system is the basis for the search for wave chaos signs in it. M. Feingold and A. Peres [10] have shown that the series of quantum perturbation theory describing such a system are divergent. It follows that with the help of classical methods of theoretical description, it is unlikely that satisfactory results can be obtained for it. Therefore, more promising, in our opinion, is an experimental study.

The most common method of experimental study of wave chaos in a closed electrodynamic system is the spectral approach, when the properties of the frequency spectrum of a quasi-optical microwave cavity are studied. In this case, the inter-frequency intervals between the nearest spectral lines are measured. L. Landau and Ya. Smorodinsky [11] found that in the spectrum of such a system lines belonging to the same class of symmetry are "pushed". The sense of it is as follows. The probability of very small inter-frequency intervals between the resonance lines in the spectrum is very small. Later it was established that the effect of the "repulsion" of spectral lines, which manifests itself in the form of a Wigner distribution of inter-frequency intervals, is of great stability for systems with wave chaos.

In the Wigner probability *P (s)* distribution of the inter-frequency intervals in the spectrum is determined by the expression, $P_w(s) = s\pi/2\exp(-s^2\pi/4)$, where *s* is the average inter-frequency interval normalized to the mean value. The presence of a Wigner distribution in the chaotic spectrum of the resonator system was also accepted as a characteristic feature of wave chaos [1]. We note here that the existence of the "repulsion" in the spectrum of a non-integrated system was indicated in [11], more than 50 years ago, but its nature has not yet been sufficiently studied.

In this paper, an attempt is made to move forward in elucidating this fundamental question in the concept of wave chaos by the example of a non-integrable system of a cylindrical bulk microwave cavity resonator with inhomogeneities in the form of a set of singular perturbations randomly located in it.

Earlier, a resonator with a single singular inhomogeneity was considered theoretically in [8], in which it was predicted that the "repulsion" effect, which manifests itself in the form of a Wigner distribution of inter-frequency intervals, should appear only for a sufficiently large number of inhomogeneities, when their total effect on the resonator spectrum is large enough. In [12], the effect of a singular perturbation on the resonator spectrum was studied experimentally using the example of a quasi-optical cylindrical microwave cavity. A singular perturbation was created in it by introducing into the resonator an eccentrically located thin metal rod attached to the upper and lower cover of the resonator. The location of the rod in the resonator was random. The rod had electrical contact over the microwave with the upper and lower covers of the resonator. Owing to this and the singular character of the perturbation, it essentially changed the distribution of the electromagnetic field in a small neighborhood at the end

In this paper, we study the effect of a large number of singular inhomogeneities in the resonator on the manifestation of the characteristics of wave chaos. Of great importance in this case is the condition under which electrical contact is achieved over the microwaves between the rods and the metal covers of the resonator. This was achieved with the use of special metal collets, pressing the rods to the covers of the resonator.

In the scattering of modes of resonator oscillations with thin rods, intermode and between different modes scattering also occurs [12]. By analyzing the between different modes scattering operator, it was theoretically established that this interaction is dominant in comparison with intermode one, and it leads to the effect of "repulsion" of the spectral levels. The distribution of inter-frequency intervals in the spectrum due to the influence of singular perturbations created by the rods under certain conditions can become similar to the Wigner distribution. It was previously theoretically established [8] that a thin metal rod eccentrically located in the resonator, in spite of the fact that its thickness d is sufficiently small, $d << \lambda$, where λ is the operating wavelength, creates conditions sufficient for manifesting in the resonator spectrum of the main wave chaos criterion for the Wigner distribution of inter-frequency intervals.

In this connection, the question arises: does the condition for the appearance of the wave chaos depend on the number of singular inhomogeneities? Are there differences in the conditions of the realization of the wave chaos in the case of using not a single, as in [12], but a large number of thin metal rods? Since the singular rods are located at a distance of $l > \lambda$ in the resonator, the interaction between them is sufficiently small, and the answer the question of the influence of a large number of rods on the "repulsion" of the levels in the resonator spectrum cannot be obtained a priori, as well as the question of the correlation between the spectral levels, as well as their spectral rigidity. Since the system of a resonator with singular rods is chaotic (in the classical sense), then, for its theoretical description, the methods of classical dynamics can hardly be used. Therefore, the signs of wave chaos in such a system, we looked for by conducting experimental studies.

2 Experiment and discussion

The working model (see Fig. 1) was a quasi-optical aluminum multimode quasi-optical cylindrical cavity resonator in the form of a cavity of 130 mm in diameter and 16 mm in height. The resonator was designed to operate at frequencies of 26 ... 38 GHz.

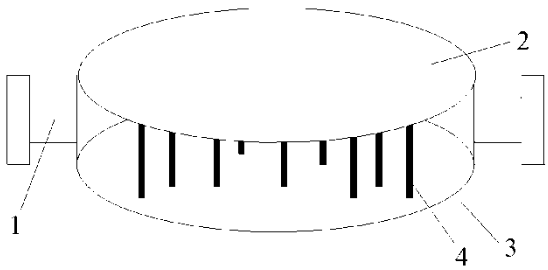

Fig. 1. Schematic representation of a quasi-optical cylindrical microwave resonator with rods (singular perturbations): 1 is the waveguide soldered into a resonator housing with a diffraction antenna for exciting *HE*-oscillations; 2, 3 are upper and lower cover of the resonator; 4 are rods, i.e. singular perturbations (diameter of each rod of 0.7 mm), In the upper and lower cover of the resonator there are collets providing microwave contact of each rod with cavity covers.

To excite electromagnetic microwave oscillations in the resonator, a waveguide diffractive antenna was used in the form of a piece of rectangular standard waveguide sealed in a resonator with a cross section of 7.2*3.4 mm^2. The end of the waveguide entering the cavity of the resonator was covered by a copper diaphragm of 0.1 mm of a thickness with an aperture of 1.8 mm in a diameter. This hole served as an antenna for exciting oscillations in it. For separate excitation of the *E*- and *H*-modes, two different antennas were used. In one of them, designed to excite the *H*-mode, the long side of the rectangular waveguide aperture was directed along the resonator axis. Another antenna that served to excite the *E*-mode had the same structure as the first antenna for exciting the *H*-mode, but differed in that the long side of the waveguide hole was oriented in a perpendicular direction.

Owing to the high conductivity of the aluminum walls of the resonator, the quality factor of the resonator in the millimeter range was sufficiently high, of the order of 10^3, for both the *H*- and *E*- modes of the oscillations. It was found that the widths of the resonance lines in the frequency spectrum differ noticeably not only for each mode, but also within a single mode. This can be explained as follows. The Q-factor of the resonator and, accordingly, the width of the resonance line depends on the ohmic losses in its walls, and also on the interaction between different modes, which is responsible for transferring energy between them. Therefore, the random irregularities of the lateral surface walls, which significantly affect this interaction, can explain the difference in the quality factor of the resonance lines.

To register the spectrum of the resonator with singular perturbations, a wide millimeter wave range meter P2-65 was used, which was connected to a computer. This attachment allowed to record automatically the whole spectrum of the resonator in the specified frequency range with a sufficiently high accuracy and within a relatively short time (40 s). With its help, measurements were made of the characteristics of numerous spectral lines. The total number of observed resonances in this frequency range was more than 80. At the same time, the relative error in measuring the amplitude-frequency characteristic of the resonator did not exceed 10^{-4}, and the quality factor of the resonant lines was 10^{-3}. A fragment of the resonator spectrum with singular inhomogeneities is shown in Fig. 2. When the number of inhomogeneities (the number of singular rods) changes, the shape of the resonator spectrum did not change qualitatively, and the number of resonance lines in it remained practically constant. At the same time, the statistics of the spectrum concerning the distribution of inter-frequency intervals changed significantly.

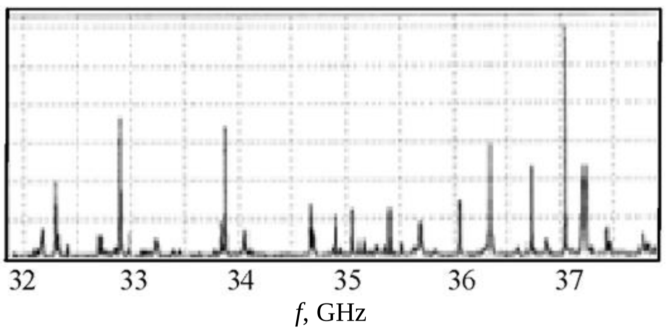

Fig. 2. The fragment of the frequency spectrum of a cylindrical cavity resonator with singular perturbations.

The probability of inter-frequency intervals distribution is close to the Poisson interval distribution for an empty resonator or with small number of inhomogeneities. But with an increase in the number of rods (singularities) the probability of inter-frequency intervals distribution is replaced by a Wigner distribution with a wide maximum of characteristic at a value of the average interval *s* close to 1.

In order to study the spectral properties of the resonator related to wave chaos in the frequency range 27 ... 38 GHz, the resonance frequencies and Q factors of the resonance lines were measured. On the basis of these data, the statistics of the inter-frequency intervals and the correlation of the spectral lines were obtained. The distribution of inter-frequency intervals allowed us to determine the statistical properties of the resonator spectrum with singular perturbations, to compare them with the properties that follow from the theory of random matrices. Measurements of the correlation of the inter-frequency intervals for the spectral lines made it possible to establish how much these lines are statistically related.

Let us consider the results of measurements of the resonator spectrum for various singular perturbations, Fig. 3 and 4. First of all, we note that the probability distribution of inter-frequency intervals, depending on the average distance between the levels *s*, is rather irregular. It follows that the spectrum of a resonator with singular inhomogeneities is random, which is characteristic of a system with wave chaos.

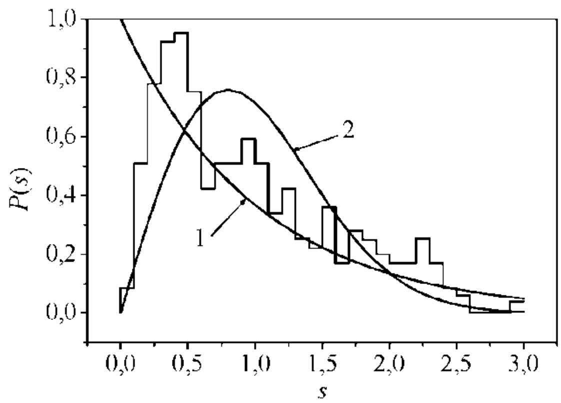

Fig. 3. The histogram for probability distribution of inter-frequency intervals for the resonator spectrum, in which there are no singular perturbations: 1 is Poisson distribution of inter-frequency intervals; 2 is Wigner distribution.

In the absence of singular perturbations, according to the theory, the probability distribution of inter-frequency intervals close to the Poisson interval distribution, $P(s) = \exp(-s)$, should be observed when there is a large probability $P(s)$ density at small intervals *s* and subsequent exponential decay at $s > 0.5$.

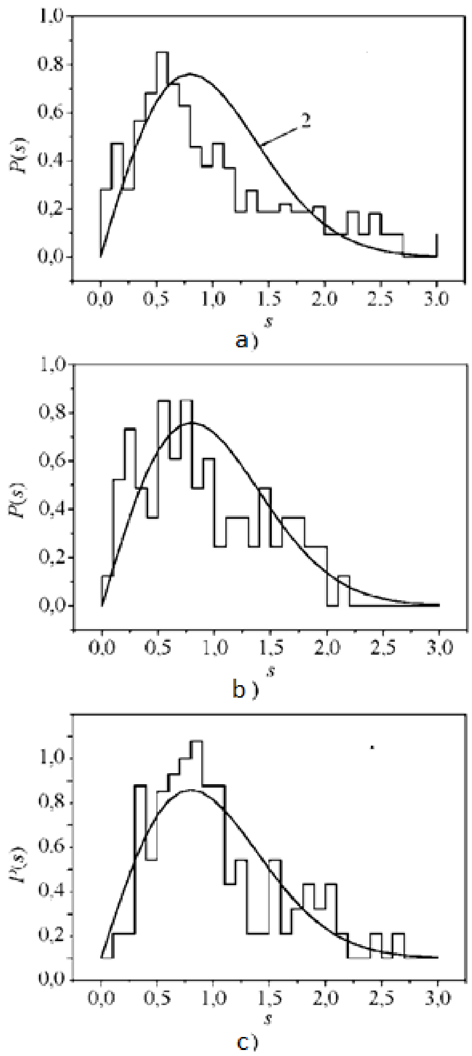

Fig. 4 - Histograms for the resonator spectrum, with singular rods: the case a) - 6 rods; b) - 8 rods; c) - 10 rods, Wigner distribution of inter-frequency intervals (solid curve)

With an increase in the number of rods in the resonator, the maximum probability in the distribution of inter-frequency intervals shifts toward larger s. This can be related to the fact that in this case we are dealing not with a purely chaotic, or purely regular, but with a mixed state of the system, when regular and chaotic motions coexist together. It is important that with increasing number of rods (singular perturbations), the curves approximating normalized histograms approach the Wigner distribution. This means that if the number of perturbations increases, the state of the system of a quasi-optical cylindrical microwave resonator, when the perturbations accumulate, approaches to wave chaos. In other words, to achieve the state of wave chaos, a sufficiently strong singular perturbation is necessary.

Along with the distribution of the inter-frequency intervals, we also built the spectral rigidity dependence of the resonator spectrum under the conditions of the developed wave chaos. The spectral rigidity of the spectrum is an important characteristic of the system from the point of view of wave chaos. To evaluate it, we use the function $\Delta_3(L)$, where L is the length of the spectral interval. The concept of a function $\Delta_3(L)$ for studying the statistical properties of a random spectrum was introduced by Dyson and Mehta [13] and is used to measure the spectral rigidity of a finite sequence of spectral levels, which can be obtained experimentally or by numerical calculations. For such a sequence, a graph of a step function with a constant average distance between them is constructed, which, on average, approaches a straight line.

The statistical dependence of the function $\Delta_3(L)$, gives an estimate of the deviation of this function from the corresponding straight line. To construct the spectral rigidity curve, we used the spectrum processing technique described in [13]. Fig. 5 shows that the experimental dependence $\Delta_3(L)$ is significantly different from the linear one, which is the feature of a system with a regular spectrum. With increasing L, the spectral rigidity curve goes to the plateau, which, along with the Wigner distribution of the inter-frequency intervals, is a sign of wave chaos. Data are also obtained on the correlation between inter-frequency intervals, which are consistent with the theory of random matrices: C (1) = - 0.258 for a resonator under conditions of wave chaos and C (1) = - 0.005 for the same resonator, but in the absence of singular perturbations, when the system is regular.

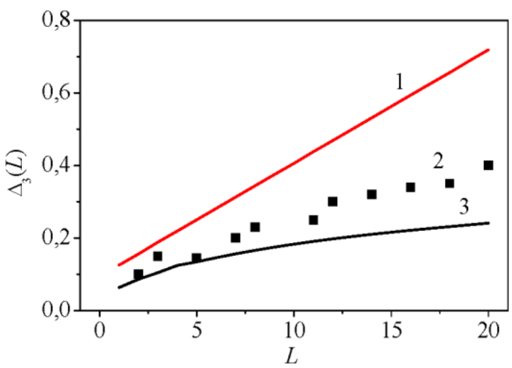

Fig. 5. Spectral stiffness curve $\Delta_3(L)$ for a resonator with a H-mode of oscillations under conditions of wave chaos (10 rods-singular perturbations). L

is the length of the spectral interval for calculating the spectral rigidity; number 1 denotes the regular integrable) system $\Delta_3(L) = L\ /\ 15$; 2 is the experimental data; 3 is the spectral rigidity function for a system corresponding to a Gaussian orthogonal ensemble, $\Delta_3(L) = (1/\pi^2)\ln L - 0.00687$ [14].

Conclusions

The appearance of wave chaos has been studied experimentally in a quasi-optical cylindrical cavity resonator under the influence of singular perturbations on microwaves. Such perturbations were created in the resonator with the help of thin metal rods inserted into it. It is established that the perturbations cause strong changes in the statistical properties of the resonator spectrum. The regular spectrum of the resonator, which has a Poisson distribution of the inter-frequency intervals, under the influence of singular perturbations, is transformed into a Wigner spectrum distribution, which corresponds to the wave chaos. To implement such a transformation, it is necessary that the total singular perturbation be large enough for complete chaotization of the resonator spectrum. We have studied the main spectral properties of the resonator with singular perturbations such as the distribution of inter-frequency intervals, the spectral rigidity, and the correlation between the spectral lines under the conditions of wave chaos. Thus, at the singular perturbations increase in the resonator, the spectrum of the spectrum becomes randomized and the signs of wave chaos appear.

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