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Modified Chaotic Circuit of the Van der Pol-Duffing

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Abstract. In this paper realization of modified chaotic circuit of the Van der Pol-Duffing generator is presented. Modeling and research chaotic behavior as a function of a variable control parameter. The differential equations has been realized using commonly available op amps and the nonlinearity using diodes. The experiments indicate that chaotic behavior indeed emerges through the period doubling route as the parameter is changed.
Keywords: Chaos, Control, Generator, Micro-Cap.

1 Introduction

In recent years, the study of chaotic phenomena in the area of nonlinear periodic self-excited oscillators has attracted much attention and application in different research areas, such as electronic circuits [1, 2], secure communication systems [3], magnetism [4], economic theory [5, 6] etc. Many circuits realize chaotic generators. One of these circuits is Van der Pol-Duffing generator.
Hyperchaos is the non-periodic behavior of deterministic non-linear dynamical system that is highly sensitive to initial conditions with more than one positive Lyapunov exponents. In order to obtain hyperchaotic oscillations from an electronic circuit it should be at least a fifth order one in the case of common passive non-linearity such as cubic non-linear element. The increasing interest in higher order autonomous Van der Pol-Duffing oscillator is simulated by their possible application to secure communication. Therefore, interest in hyperchaos increases due to their possible application in improving secure communication.

By just including an inductor and capacitor in parallel to this third order autonomous Van der Pol-Duffing oscillator circuit, we can realize a very simple fifth order non-linear dynamical system [7-10].

2 Circuit of the chaotic Van der Pol-Duffing generator

Block-scheme of the chaotic Van der Pol-Duffing generator shown in Fig. 1.

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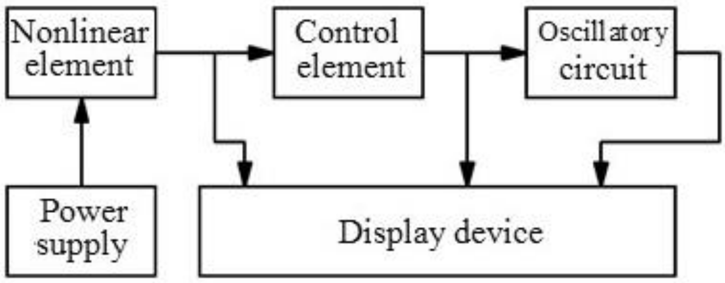


Fig. 1. Block scheme of the chaotic Van der Pol-Duffing generator

Generator includes the following components:

1. the power supply;
2. nonlinear element;
3. control element;
4. oscillatory circuit;
5. display device.

Circuit of the generator shown in Fig. 2.

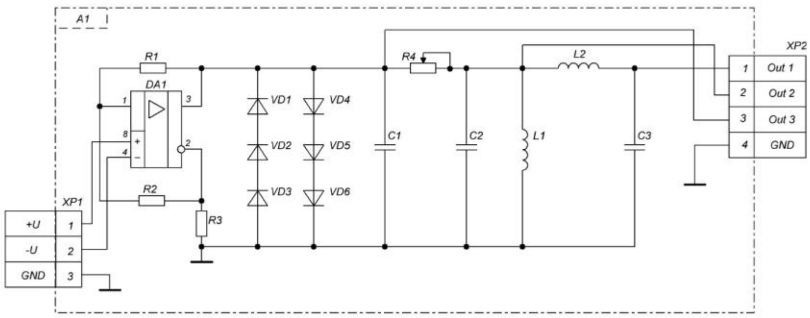


Fig. 2. Circuit of the generator

For circuit have chosen the following typical values: C1 = C3 = 5 nF, C2 = 150 nF, L1 = 11,77 mH, L1 = 62,12 mH, R1 = R2 = R3 = 2 kOhm, variable resistor R4 = 5 kOhm, VD1 = VD6 = 1N4148, DA = TL082. Connector XP1 for connect power supply, connector XP2 for connect display device, for example, oscilloscope.

The higher order autonomous Van der Pol-Duffing oscillator circuit shown in Fig. 2, one of the most simple fifth order autonomous electronic generators of hyperchaotic signal. The circuit contains six linear elements (three capacitors, C_1, C_2, C_3, two inductors, L_1, L_2 and one resistor, R_4) and one active element (cubic non-linear resistor, N_R) that can be built using off-the shift op-amps with six diodes. The chaotic behavior of the circuit was research numerically, confirmed mathematically and realized experimentally.

Reducing the variable resistance R_4, while keeping the other circuit parameter at constant values, one finds that the circuit admits period doubling route to chaos.

For a small range of resistance R, it also exhibits crisis induced hyper chaos. The important requisites for hyperchaos are:

- the minimal dimension of phase space that embeds the hyper chaotic attractor should be at least five, which requires the minimum number of coupled first order ordinary differential equations to be five;
- the number of terms in the coupled equations giving raise to instability should be at least two, of which one should be non-linearity function.

Correspondingly, for hardware implementation:

- the number of energy storage elements (inductors or capacitors) in the circuit should be at least five;
- the number of active elements giving rise to instability should be one or two, of which one should necessarily be a non-linear device.

Applying Kirchoff's laws, the set of five coupled first order differential equations describing the circuit is obtained as:

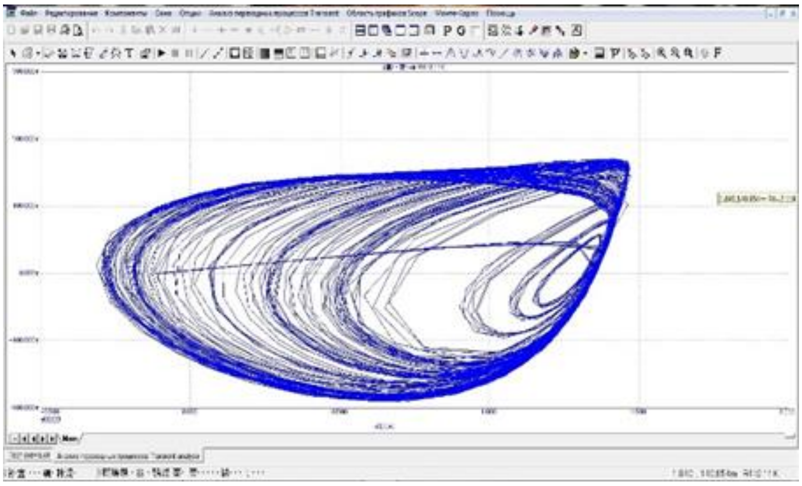
$$
\begin{aligned}
C_1 \frac{dv_1}{dt} &= -\left[\frac{1}{R_4}(V_1 - V_2) + f(V_1)\right]; \\
C_2 \frac{dv_2}{dt} &= \frac{1}{R_4}(V_1 - V_2) - i_{L_1} - i_{L_2}; \\
C_3 \frac{dv_3}{dt} &= i_{L_2}; \\
L_1 \frac{di_{L_1}}{dt} &= V_2; \\
L_2 \frac{di_{L_2}}{dt} &= V_2 - V_3.
\end{aligned}
\tag{1}
$$

While V_1,V_2 and V_3 are the voltages across the capacitors C_1, C_2, C_3, and i_{L_1}, i_{L_2} denote the current through the inductances L_1 and L_2, respectively, the term $f(V_1)$ representing the characteristic of the cubic non-linear resistance can be expressed mathematically:

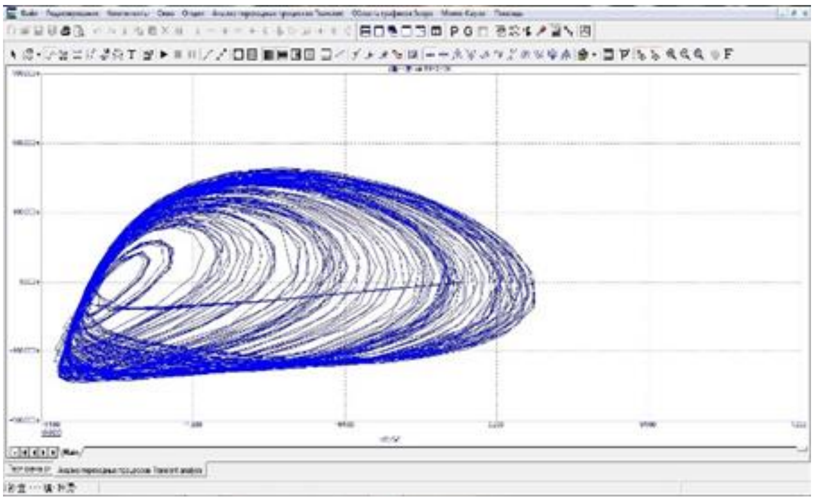
$$f(V_1) = aV_1 + bV_1^3. \tag{2}$$

3 Circuit simulation

For investigate of the generator was using Micro-Cap 9. The simulation results presented in Fig. 3. The intrinsic time constant in computer simulation equal 50 ms.



a



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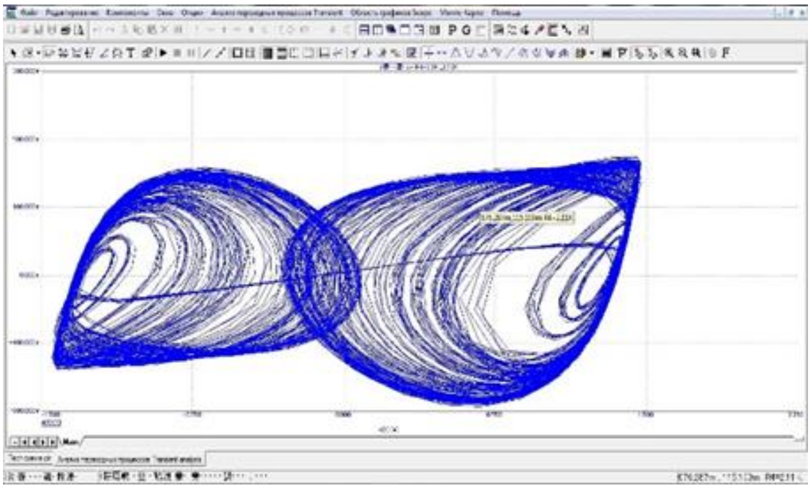
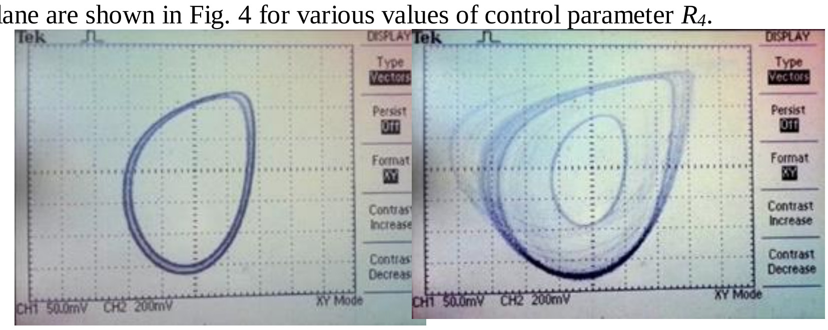


c

Fig. 3. Chaotic attractors of the Van der Pol-Duffing system obtained by computer simulation (phase trajectories on the plane $U_{C1} - U_{C2}$): a – value of resistance R4=2,11 kOhm; b – value of resistance R4=2,13 kOhm; c – value of resistance R4=2,12 kOhm

4 Practical realization

In practice, we change inductors to operational amplifiers, capacitors and resistors. The variable resistor R_4 assumed a control parameter. By decreasing the value of R_4 from 5000 to 0, the circuit behavior of Fig. 2 is found to transit from a periodic limit cycle to chaos and then to hyper chaotic attractor through boundary crisis, etc [11, 12]. The projections of the attractor on the ($U_{C1} - U_{C2}$) plane are shown in Fig. 4 for various values of control parameter R_4.



a

b

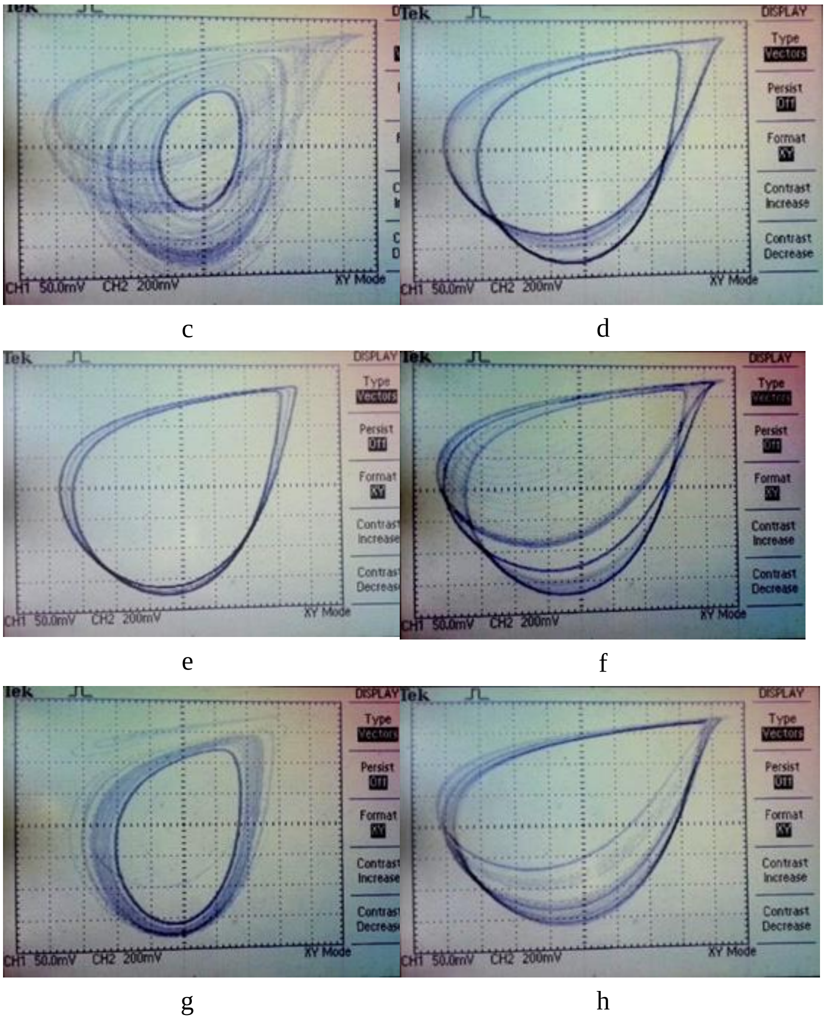


Fig. 4. Chaotic attractors of the Van der Pol-Duffing system experimentally obtained in
(U_{C1} – U_{C2}) plane: a – value of resistance R4=2,10 kOhm; b – value of resistance R4=2,11 kOhm; c – value of resistance R4=2,12 kOhm, d – value of resistance R4=2,14 kOhm, e – value of resistance R4=2,15 kOhm, f – value of resistance R4=2,19 kOhm, g – value of resistance R4=2,13 kOhm, h – value of resistance R4=2,21 kOhm

Conclusions

A higher order autonomous Van der Pol-Duffing oscillator based on fifth order circuit was designed and investigated. Practical research is fully match with the mathematical apparatus. For the first time detected values for control of the chaotic oscillations.

References

1. Rusyn V. B. (2014) *Modeling and Research of Chaotic Rossler System with LabView and Multisim Software Environments*. Bulletin of National Technical University of Ukraine «Kyiv Polytechnic Institute», Series Radiotechnique Radioapparatus Building, Eddition 59, 21-28.
2. Galajda, P., Guzan, M., Petržela, J. (2016) *Implementation of a custom Chua's diode for chaos generating applications.* 26th International Conference Radioelektronika, Radioelektronika 2016, 7477421.
3. Rusyn V., Stancu A., Stoleriu L. (2015) *Modeling and Control of Chaotic Multi-Scroll Jerk System in LabView.* Bulletin of National Technical University of Ukraine «Kyiv Polytechnic Institute», Series Radiotechnique Radioapparatus Building, Edition 62, 94-99.
4. Horley Paul P., Kushnir M. Ya., Morales-Meza M., Sukhov A., Rusyn V. (2016) *Period-doubling bifurcation cascade observed in a ferromagnetic nanoparticle under the action of a spin-polarized current.* Physica B: Condensed Matter, Volume 486, 60-63.
5. Rusyn V., Savko O. (2016) *Modeling of Chaotic Behavior in the Economic Model.* Chaotic Modeling and Simulation. An International Journal of Nonlinear Science, 291-298.
6. Lenka Pribylova. (2009) *Bifurcation routes to chaos in an extended Van der Pol's equation applied to economic models.* Electronic Journal of Differential Equations, No. 53, 1-21.
7. M. Lakshmanan and K. Murali. Chaos in Nonlinear Oscillators: Controlling and Synchronization, World Scientific, Singapore 1996.
8. A. N. Njah and U. E. Vincent. (2008) Chaos, Solitons and Fractals 37, 1356.
9. J. Yu, W. J. Zhang, and X. M. Gao. (2007) Chaos, Solitons and Fractals 33, 1307.
10. Murali K, Tamasevicius A, Mykolaitis G, Namajunas A & Lindberg E. (2000) *Hyperchaotic circuits with unstable oscillators.* Nonlinear phenomena in complex systems, 7.
11. Ott E., Grebogi C. & Yorke J.A. (1990) *Controlling chaos.* Phys. Rev. Lett., Vol. 64, 1196-1199.
12. Rusyn V., Kushnir M., Galameiko O. (2012) *Hyperchaotic Control by Thresholding Method.* Modern Problems of Radio Engineering, Telecommunications and Computer Science - Proceedings of the 11th International Conference, TCSET'2012, 67.