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Chaotic Dynamics of Coupled Nonlinear Circuits in Ring Connection

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Abstract: It is generally difficult to synchronize a ring network that features chaotic behaviour, especially if the system's order is too large. In this paper, we consider a ring network of three identical nonlinear and non-autonomous circuits of fourth order, which are bidirectionally coupled through three coupling linear resistances R_C. We present simulation and experimental results for synchronization of such a network in low frequency area, and derive a sufficient condition for chaotic synchronization of this type of network.
Keywords: Ring connection, Nonlinear circuit, Low frequency area, Chaotic synchronization.

1 Introduction

Synchronization is an important property of chaotic dynamical systems. In the past decades the synchronization in large scale complex networks has attracted lots of attention in various fields of science and engineering [2, 3, 5, 14, 15, 16]. In general, a complex network is a large set of interconnected nodes, where a node is a fundamental unit-joint with detailed contents, which lines intersect or branch.

The nonlinear electric circuits are veritable tools to study the fundamental mechanisms underlying the onset of chaos. A variety of autonomous [7, 8, 12] and non-autonomous [6, 10] circuits have been reported in the literature in recent times. A plethora of bifurcation and chaos phenomena, such as period doubling routes to chaos, intermittency, quasi periodicity, chaotic synchronization and so on, have been studied extensively.

In this paper, theoretical and experimental results of chaos synchronization of three identical non-autonomous circuits, bidirectionally coupled in ring connection network are presented. The system's evolution from non synchronized oscillations to synchronized ones, when its individual circuit exhibits chaotic behaviour, is studied.

2 The Nonlinear, Non-Autonomous Circuit

Chaotic performance of the fundamental non-autonomous circuit has been investigated in the past [4]. It is based on a third order autonomous piecewise

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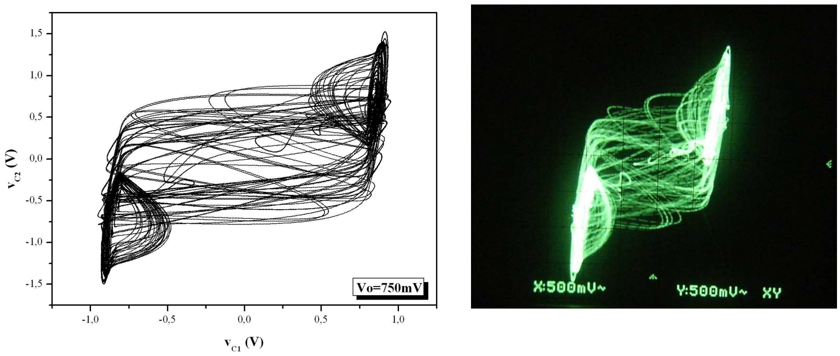
linear circuit, which introduced by Chua and Lin [1], and is capable of realizing every member of Chua's circuit family. A second inductor L_2 has been added in the branch of the voltage source $v_s(t)$, in order to enrich circuit's dynamics. The circuit also consists of two active elements, a nonlinear resistor R_N, which has a v-i characteristic of N-type with G_a=-0.35mS, G_b=5.0mS and B_p=0.8V, and a negative conductance G_n=-0.50mS. In recent papers, circuit's dynamics in low frequency area has been studied extensively [10, 11, 13]. The circuit's parameters are considered unchangeable during our study. More particularly: $L_1=L_2$=100mH, C_1=33nF, C_2=75nF and R_1=1KΩ. We use sinusoidal input signal $v_s(t)$ with amplitude V_o equal to 0.60V or 0.75V, while the frequency f ranges from 30Hz to 50Hz. Using the above parameters circuit exhibits chaotic behaviour. In Figures 1a) and b) theoretical and experimental phase portraits v_{C2} vs. v_{C1} for V_o=0.75V and f=35Hz are presented, respectively. The maximum Lyapunov exponent for the above parameters is positive (LEmax=0.0156), which indicates that the system exhibits chaotic behavior.



a) theoretical b) experimental

Fig. 1. Phase portrait v_{C2} vs. v_{C1} for V_o=0.75V and f=35Hz.

3 Dynamics of Ring Connection Topology

In a recent paper [9] we have seen that chaotic synchronization of two identical non-autonomous, unidirectionally coupled, nonlinear, fourth order circuits is possible. In this work, chaotic synchronization of three bidirectionally coupled circuits in ring connection, as seen in Figure 2, is studied.

More particularly, as illustrated in Figure 2, circuits NA1C1, NA1C2 and NA1C3 are bidirectionally coupled through three identical linear resistances R_C. The connection points are in capacitances C_{2i}, where i=1, 2 and 3 denotes circuit NA1C1, NA1C2 and NA1C3, respectively.

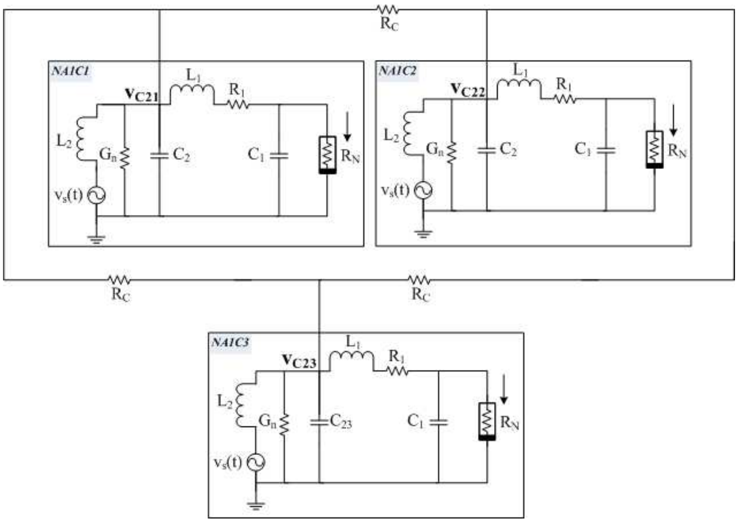


Fig. 2. Three non-autonomous, nonlinear fourth order circuits in ring connection.

The resulting set of system's differential equations is derived using Kirchhoff's circuit laws.

$$\frac{dv_{C11}}{dt}=\frac{1}{C_1}\left(i_{L11}-i_{RN1}\right)$$

$$\frac{dv_{C21}}{dt}=-\frac{1}{C_2}\left(G_n\cdot v_{C21}+i_{L11}+i_{L21}-\frac{v_{C23}-v_{C21}}{R_C}+\frac{v_{C21}-v_{C22}}{R_C}\right)$$

$$\frac{di_{L11}}{dt}=\frac{1}{L_1}\left(v_{C21}-v_{C11}-R_1\cdot i_{L11}\right)$$

$$\frac{di_{L21}}{dt}=\frac{1}{L_2}\left[v_{C21}-R_2\cdot i_{L21}-v_s(t)\right]$$

$$\frac{dv_{C12}}{dt}=\frac{1}{C_1}\left(i_{L12}-i_{RN2}\right)$$

$$\frac{dv_{C22}}{dt}=-\frac{1}{C_2}\left(G_n\cdot v_{C22}+i_{L12}+i_{L22}-\frac{v_{C21}-v_{C22}}{R_C}+\frac{v_{C22}-v_{C23}}{R_C}\right)$$

$$\frac{di_{L12}}{dt}=\frac{1}{L_1}\left(v_{C22}-v_{C12}-R_1\cdot i_{L12}\right)$$

$$\frac{di_{L22}}{dt} = \frac{1}{L_2}\left[v_{C22} - R_2 \cdot i_{L22} - v_s(t)\right]$$

$$\frac{dv_{C13}}{dt} = \frac{1}{C_1}\left(i_{L13} - i_{RN3}\right)$$

$$\frac{dv_{C23}}{dt} = -\frac{1}{C_2}\left(G_n \cdot v_{C23} + i_{L13} + i_{L23} - \frac{v_{C22} - v_{C23}}{R_C} + \frac{v_{C23} - v_{C21}}{R_C}\right)$$

$$\frac{di_{L13}}{dt} = \frac{1}{L_1}\left(v_{C23} - v_{C13} - R_1 \cdot i_{L13}\right)$$

$$\frac{di_{L23}}{dt} = \frac{1}{L_2}\left[v_{C23} - R_2 \cdot i_{L23} - v_s(t)\right]$$

Where the current i_{RNi} through the nonlinear element i, with i=1, 2, 3 for circuit 1, 2 and 3 respectively, and input signal $v_s(t)$ are given by equations:

$$i_{RNi} = G_b v_{C1i} + 0.5\left(G_a - G_b\right)\left(\left|v_{C1i} + B_p\right| - \left|v_{C1i} - B_p\right|\right)$$

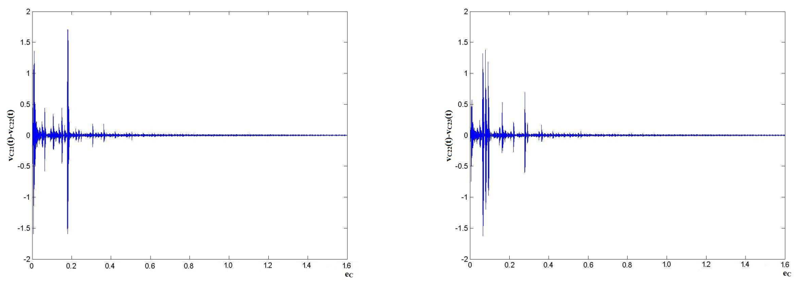
$$v_s(t) = V_o \cos(2\pi f t)$$

In figures 3a) and b) bifurcation diagrams $v_{C21}(t)$-$v_{C22}(t)$ vs. e_C and $v_{C22}(t)$-$v_{C23}(t)$ vs. e_C are presented, where e_C is the coupling parameter and is given by equation.

$$e_C = R_1 / R_C$$

where R_C is the coupling resistance.

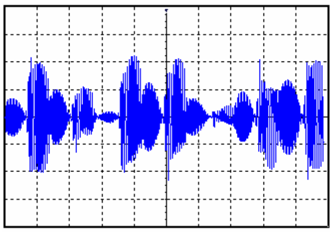
We can see that chaotic synchronization of the three identical circuits in ring connection is observed for coupling parameter e_C>0.568, or for coupling resistance R_C<1.8kΩ



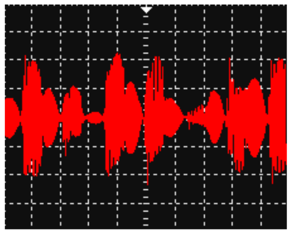
a) $v_{C21}(t)$-$v_{C22}(t)$ vs. e_C b) $v_{C22}(t)$-$v_{C23}(t)$ vs. e_C

Fig. 3. Bifurcation diagrams for V_o=0.75V and f=35Hz.

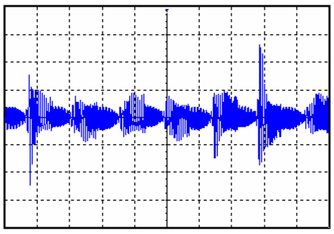
In figure 4 simulation and experimental results of waveforms $v_{C21}(t)$-$v_{C22}(t)$ for various values of coupling resistance R_C are presented. More particularly, in figures 4a), c) and e) simulation $v_{C21}(t)$-$v_{C22}(t)$ for R_C=1.0MΩ ($e_C \to 0$), R_C=10.0kΩ (e_C=0.1) and R_C=1.8kΩ (e_C=0.568) are shown, while in figures 4b), d) and f) experimental $v_{C21}(t)$-$v_{C22}(t)$ for the same parameters are illustrated.



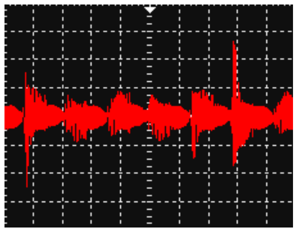
a) R_C=1.0MΩ (simulation)



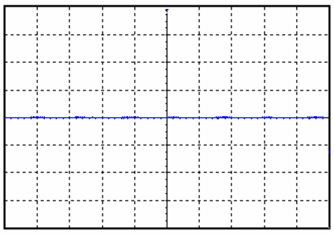
b) R_C=1.0MΩ (experimental)



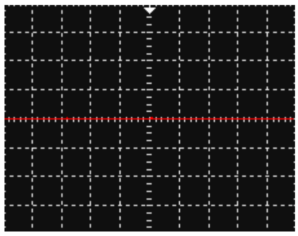
c) R_C=10.0kΩ (simulation)



d) R_C=10.0kΩ (experimental)



e) R_C=1.8kΩ (simulation)



f) R_C=1.8kΩ (experimental)

Fig. 4. a), c), e) Simulation and b), d), f) Experimental waveforms $v_{C21}(t)$-$v_{C22}(t)$ (x: 1ms/ div, y: 1V/ div).

In figures 4e) and f) we can see that chaotic synchronization occurs for coupling resistance R_C=1.8kΩ (e_C=0.568). This threshold synchronization value of R_C is lower than in the case of chaotic synchronization of two bidirectionally coupled identical circuits, with the same circuit's settings, which is R_C=2.28 (e_C=0.479) [9].

In figures 5 and 6 a collection of results are displayed. Specifically, in figure 5 we can see the threshold synchronization value of coupling parameter e_C versus frequency f, for amplitude of the input sinusoidal signal V_o=0.60V and V_o=0.75V. We can see that the values of e_C in the case of V_o=0.60V are lower than in the case of V_o=0.75V. In figure 6 the threshold synchronization value of coupling resistance R_C versus frequency f for the same parameters as in figure 5 is presented.

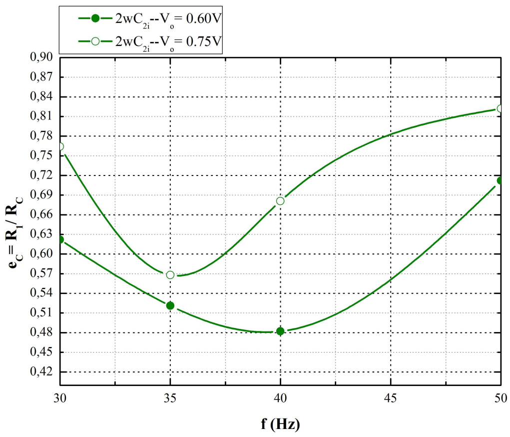


Fig. 5. Threshold synchronization value of coupling parameter e_C vs. f.

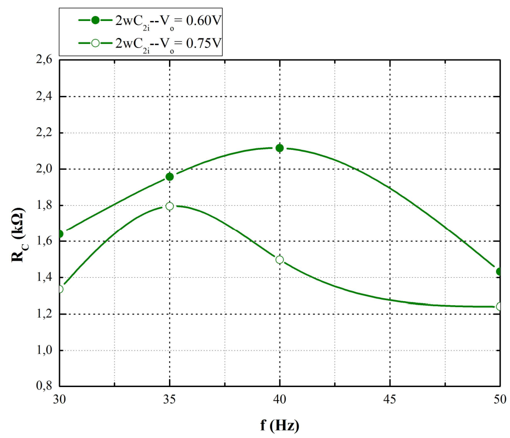


Fig. 6. Threshold synchronization value of coupling resistance R_C vs. f.

4 Conclusions

In this paper, we have studied chaos synchronization of three identical non-autonomous circuits, bidirectionally coupled in ring connection network, in low frequency area. Simulation and experimental results of the system's evolution from non synchronized oscillations to synchronized ones, when its individual circuit exhibits chaotic behaviour, were presented. Both, theoretical calculations and experimental results appear to be in complete agreement. We have seen that the values of threshold synchronization coupling parameter e_C in the case of V_o=0.60V are lower than in the case of V_o=0.75V, for various values of input frequency f, but higher than in the case of two bidirectionally coupled identical circuits with the same setup.

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