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Detection of jet axis in a horizontal turbulent jet via nonlinear analysis of minimum/maximum temperature time series

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Abstract: We have analyzed experimental temperature time series from a horizontal turbulent heated jet, in order to identify the jet axis location using non linear measures. The analysis was applied on both, the original time series as well as on the extreme value (minimum and maximum values) time series. In our analysis we employed mainly non-linear measures such as mutual information and cumulative mutual information. The results show that the analysis of the extreme values time series using cumulative mutual information permits to distinguish the jet axis time series from the rest of the jet, as well as discriminate regions of the jet located close to jet axis or close to the boundaries. Furthermore, it is of interest that the application of simple statistical measures and clustering techniques shows that the use of extremes time series let us distinguish with greater confidence the jet axis than the use of the original one.

Keywords: Non-linear time series analysis, turbulence, mutual information, cumulative mutual information, clustering.

1 Introduction

Jet flow is a very important research subject that has attracted scientific interest due to extensive applications in environmental engineering. So far a large number of investigations have been carried out to locate the trajectory and understand the turbulence properties of the flow using statistical methods which do not necessarily lead to understanding the dynamics of the flow [3, 19].

The transition from laminar to turbulent flow in a jet has been extensively studied as a fundamental non linear dynamical problem [4, 5, 17, 25]. The study of dynamical systems by analysis of the time series of a variable measured in a physical system, is of particular interest over the last decades, and gives the possibility of comprehension the underlying system dynamics. Time series analysis may include linear and non linear methods. The linear analysis includes simple statistical measures such as autocorrelation function and power spectrum, while non linear analysis methods based on the reconstruction of phase through spaces include the mutual information and correlation dimension. For a concise review of these methods one can consult the book by Kantz and Schreiber (1995) [10] and Abarabnel (1996) [1]. These more complex methods

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allow us to extract more detailed characteristics of the underlying dynamical system.
In this paper linear and non linear measures are used to analyze temperature fluctuation time series. Our aim was to study the dynamic characteristics of the temperature fluctuation experiment. More specifically we analyze the temperature fluctuation measurements recorded using fast response thermistors along a horizontal line in order to investigate if one can discriminate time series corresponding to regions close to the jet axis, where conditions of fully developed turbulence are expected, from time series corresponding to regions that are more distant and from those close to the boundary with the ambient water. Horizontal buoyant jet investigations [2, 6, 9, 18] are mainly concerned with the structure of the flow.
The novelty of the present work is that the analysis was applied both on the original time series as well as on the extreme value (minimum and maximum values) time series. The initial time series is reduced to a series (extreme time series) of successive pairs of minimum and maximum values. The objective of our analysis is to investigate whether it is possible that a time series of extreme values can reveal dynamic characteristics of the underlying system in the same or better way as the analysis of the original time series. One can easily understand that the interest is important, since this would permit us to study dynamical systems using reduced information.

The structure of the paper is as follows. In Sec. 2 we discuss briefly the theoretical background and the experimental set-up for the temperature measurements. In Sec.3 we present the methodology employed for data analysis along with the linear and nonlinear measures. The results and discussion are presented in Sec. 4. Finally the conclusions are presented in Sec. 5.

2 Theory and Experimental Set-up

2.1 Theory

A horizontal heated round jet of diameter D and density ρ_o flows out of a nozzle with velocity U in a calm ambient fluid of density ρ_α. The specific volume, horizontal momentum and buoyancy fluxes are defined as

$$\begin{aligned} Q &= \frac{\pi D^2}{4} U \\ M &= QU \qquad and \\ B &= g_o' Q = \frac{\rho_\alpha - \rho_o}{\rho_\alpha} g Q \end{aligned} \tag{1}$$

respectively, where g is the gravitational acceleration and $g_o^{'}$ the effective gravity that will subsequently produce vertical momentum flux. Fisher et al. (1979) [7] have defined two characteristic length scales as:

$$l_Q = \frac{Q}{M^{1/2}} \qquad and \qquad l_M = \frac{M^{3/4}}{B^{1/2}} \tag{2}$$

the ratio of which is the initial jet Richardson number R_o:

$$R_o = \frac{l_Q}{l_M} = \left(\frac{\pi}{4}\right)^{1/4} \frac{\sqrt{g_o^{'} D}}{U} = \left(\frac{\pi}{4}\right)^{1/4} \frac{1}{F_o} \qquad and \qquad F_o = \frac{U}{\sqrt{g_o^{'} D}} \tag{3}$$

where F_o is the initial densimetric Froude number.
The temperature difference between the jet and ambient fluid produces the density deficiency that is responsible for the initial jet specific buoyancy flux. The dilution S at a point of the jet flow field is defined to be the ratio:

$$S = \frac{T_o - T_a}{T - T_a} \tag{4}$$

Where T_o is the initial jet temperature T_a the ambient temperature and the T the local time-averaged temperature. Jirka (2004) [8] has defined the jet axis to be the point of minimum dilution S_c:

$$S_c = \frac{T_o - T_a}{T_c - T_a} \tag{5}$$

where T_c is the maximum time-averaged (centerline) temperature. We define x_c and y_c the horizontal and vertical distances from the nozzle where the jet axis is located. Near the nozzle $(x/l_M < 1)$ the jet trajectory is horizontal, the flow is mainly driven by the initial momentum flux and it is characterized as jet-like [18]. When $(y/l_M > 2)$ the trajectory of the flow is altered to vertical and the flow is characterized as plume-like. The flow regime $(1 < x/l_M < 5)$ is the transition from jet-like to plume-like flow [18], [21].

2.2 Experimental setup

The experiments were performed at the Hydromechanics and Environmental Engineering Laboratory of the University of Thessaly [20]. The dispersion tank is made of 12.5mm thick Lucite with orthogonal horizontal section 0.90m x 0.60m and 0.80m depth. A perspective view of the experimental setup is shown in Fig.1.

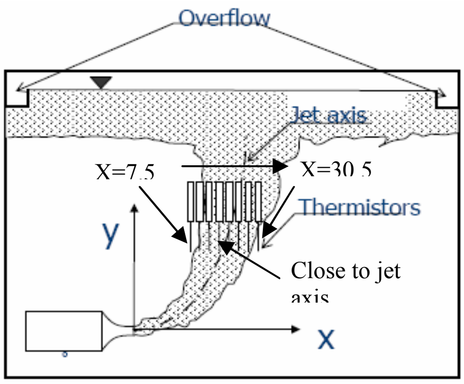


Fig. 1. Perspective view of the experimental setup

The tank was equipped with a peripheral overflow to remove excess water. In this mater the depth of water is fixed at 77 cm. The hot water jet supply consists of a water heater made of stainless steel, which is well insulated and pressurized by air at 2 atm, to provide adequate constant head pressure to drive to jet. During the water heating, a recirculating pump was used to ensure that the hot water is well mixed and there are no temperature gradients. An insulated pipe drives the water from the heater into the jet plenum, through a calibrated flow meter. A jet nozzle of 0.65cm diameter was used. The jet water temperature was around 60 ºC, while the ambient water temperature ranged between 18 to 20 ºC. Temperature measurements were obtained by an array of eight fast response thermistors spaced equally at 1cm apart, positioned at constant elevation from the nozzle, on the plane of symmetry of the buoyant jet. The jet was made visible by means of a slide projector on a semitransparent paper sheet (shadowgraph) in order to place the rake of thermistors properly. In this paper, we use the data recorded at an elevation of 5cm above the nozzle axis. The initial parameters of the flow are shown in table 1. We analyzed 24 recordings of temperature time series, one for each location of measurement, where the sampling time at each location was 30s at a frequency of 200Hz. Comprehensive details about the experimental setup can be found in Karakasidis et al (2009) [11].

Table 1. Experimental conditions

D(cm)	W(cm/s)	T_o(ºC)	T_a(ºC)	M(cm^4/s^2)	B(cm^4/s^3)	Re	l_m(cm)	R_o	y_c/l_m
0.65	29.25	60	17.8	284	149	1646	5.66	0.102	0.883

3. Time series analysis

3.1 Methodology

In an effort to discriminate the jet axis time series from the rest of the jet we used linear and nonlinear measures applied both on the original time series as well as on the extreme value (minimum and maximum values) time series. The initial time series is reduced to a series (extreme time series) of successive pairs of minimum and maximum values following the methodology by D.Kugiumtzis et al., 2006 [14]. The goal of this work was to examine if simple linear and non linear methods such as cumulative mutual information, can discriminate different states of systems from the analysis of the reduced length time series, instead of the full original time series.

3.2 Data set – Extreme time series model

As already mentioned 24 time series of temperatures have been recorded using fast response thermistors along a horizontal line of a fully developed turbulent heated jet. Consequently some of the time series correspond to conditions of turbulent flow (time series derived close to centerline of the jet) while other time series, obtained close to the boundary between the heated jet and the ambient water, have intermittent (laminar and turbulent) flow characteristics. Each time series consist of 6000 observations.

We derived new *extreme time* series of successive maximum and minimum values from each initial time series. Suppose we have a time series of length N, $\chi(t)$, $t = 1,2,....N$. If $y_1 = \chi(t_1)$ the first minimum, $y_2 = \chi(t_{1+...})$ the first turning point (maximum), $y_3 = \chi(t_{2+...})$ the second turning point (minimum) etc we extract from the initial time series the time series $y(t) = y_1, y_2, y_3,.....y_n$ called *extreme time series*. The extracted time series have lengths varying from N=130 to 1500 depending on the structure of the initial time series.

An example of a whole initial and extreme time series is shown in Fig. 2(a). In Fig. 2 (b) a zoom of a segment of the initial temperature time series of Fig 2(a) is presented.

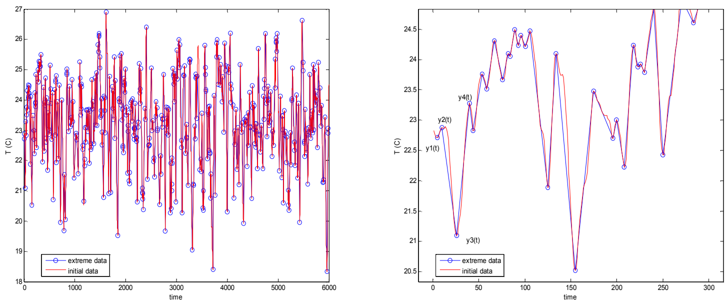


Fig. 2. (a) Initial and extreme time series. (b) Segment of initial and extreme time series.

3.3 Nonlinear measures

The most widely known nonlinear measure, which is used to select the appropriate delay time τ for state space reconstruction is the Mutual Information *I(τ)* and is defined as:

$$\mathrm{I}(\tau)=\sum_{\mathrm{x(ti),x(ti+}\tau)} \mathrm{P(x(ti),x(ti+}\tau))\mathrm{*log}[\frac{\mathrm{P(x(ti),x(ti+}\tau))}{\mathrm{P(x(ti))*P(x(ti+}\tau))}] \tag{6}$$

Where $x(t_i)$ is the ith data point of time series, $\tau = k\Delta t(k = 1,2,.....k_{max})$; $P(x(t_i))$ is the probability density at $x(t_i)$; $P(x(t_i),x(t_i+\tau))$ is the joint probability density at $x(t_i),x(t_i+\tau)$; τ is the delay time.

The delay τ of the first minimum is chosen as a delay time for the reconstruction of phase space.

We also used a new nonlinear measures the *Cumulative Mutual Information* $M(\tau_{max})$, defined as the sum of mutual information *I(τ)* D.Kugiumtzis et al., 2007 [14] for a number of delay τ.

$$M(\tau_{\max}) = \sum_{\tau=1}^{\tau_{\max}} I(\tau) \tag{7}$$

3.4 Clustering analysis using the Cumulative Mutual Information function

Clustering is an important technique that groups together similar data sets. Several studies used clustering methods based on mutual information [23, 24]. In our study we used single linkage hierarchical clustering algorithm in order to classify our data. The clustering techniques applied both on the original time series as well as on the extreme value (minimum and maximum values) time series. As a measure of similarity we used the *Cumulative Mutual Information.*

One of the main advantages of hierarchical clustering is that a dendrogram can be drawn to find the appropriate number of clusters in a dataset. Briefly we propose the following clustering algorithm steps:

- We compute the Euclidean distance y between pairs of objects in *n*-by-*p* data matrix X. Rows of X correspond to observations; columns correspond to variables.
- We create a hierarchical cluster tree z from the distances in y (y is a Euclidean distance matrix or a more general dissimilarity matrix, formatted as a vector).
- Finally we group the data set into clusters. The most dependent data are grouped together.

4. Results and Discussion

During the experiment the jet axis (at elevation 5cm above the nozzle axis) was located by optical measurements nearly at the midpoint between the jet boundaries (x=16.5 – 17.5 cm). This was also supported by the behavior of the average temperatures observed in these time series, as well as from Recurrence Plot analysis [11]. It is well known from the theory of fluid mechanics that turbulence near the center of the jet is fully developed. There appear many short-lived small scale turbulent structures, while near the jet boundary the large scale flow structures live longer.
We calculated the mutual information function for both, the original as well as for the extreme data series and the results are presented in Figs. 3(a)-(b). In Fig. 3 (b) we observe that for the extreme time series reported at x=16.5 cm and x=17.5 cm (points which are near jet axis) the mutual information function clearly attains the lowest values for any value of the time delay, if compared to the results for the rest of the time series. Such behavior is consistent with what we expected since close to the jet centreline the memory of the flow structures is lost fast.

(a)

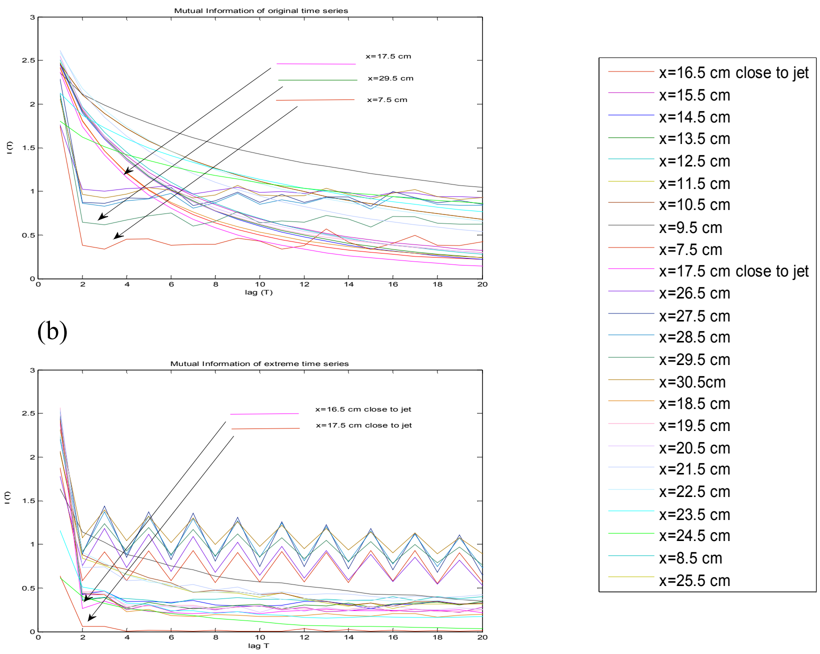
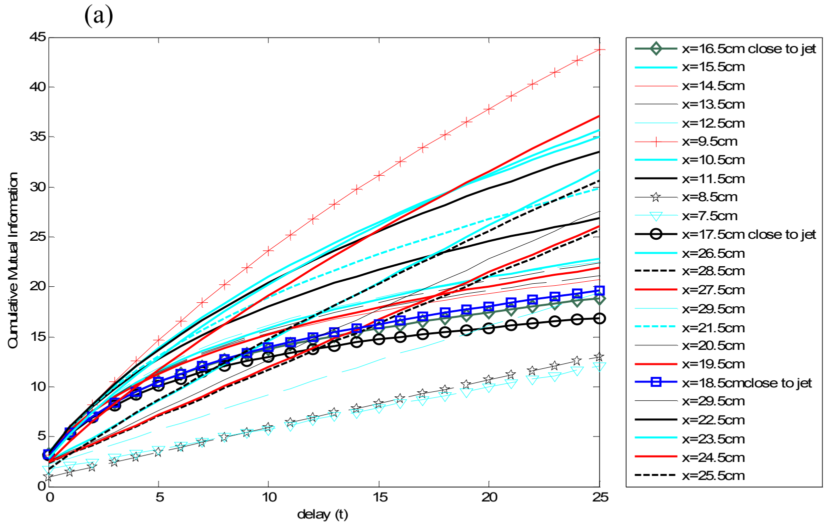


Fig. 3. (a) Mutual information of the Initial time series along the horizontal line.
(b) Mutual information of the Extreme time series along the horizontal line

In Fig. 3(a) we can see that there are time series presenting the smallest local minimum but not the lowest values of average mutual information which correspond in fact to regions in or close to the ambient water, while time series close to the jet axis (close to x=17.5cm) present the lowest values of average mutual information although for larger lags. As we get far from the jet axis ,but always in the turbulent jet region, average mutual information increases and time lags of the minimum are shifted toward larger times. We must however bear in mind that the time lags are not directly compared for the original and the extreme time series, since the distance between successive points varies.

In Fig.4 (a) and (b) we summarize results for the cumulative mutual information for the original and extreme time series. A close look in Fig. 4(b), where the cumulative mutual information for the extreme time series is presented, indicates that we can discriminate three main regions corresponding to time series. The first region corresponds to a set of time series too far from the centerline of jet (x=7.5cm, x=27.5cm, x=28.5cm, x=29.5cm, x=30.5cm). The second region corresponds to a set of time series very close to the center of jet (x=16.5cm, x=15.5cm, x=17.5cm, x=18.5cm, x=19.5cm). The third region corresponds to a set of time series (x=9.5cm, x=21.5cm, x=22.5cm, x=23.5cm) far from the center of jet but not as much as the time series from the first region.

(a)



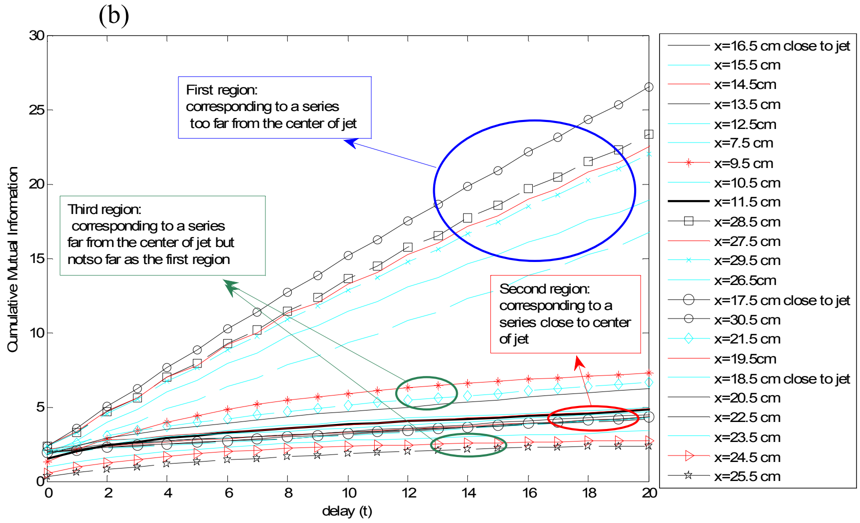
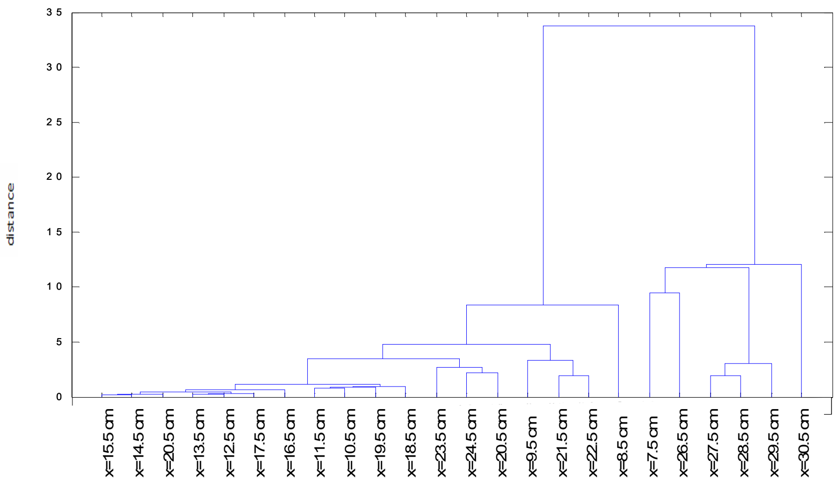


Fig. 4. (a) Cumulative Mutual information of the Initial time series along the horizontal line. (b) Cumulative Mutual information of the Extreme time series along the horizontal line

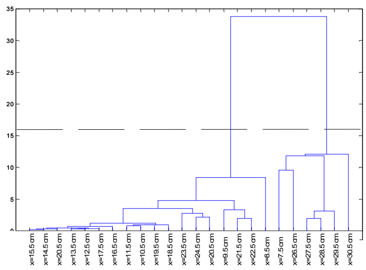
It is of interest to note that such a detailed discrimination of the three regions is not so straightforward in Fig. 4(a) where the cumulative mutual information from the original time series is presented.

Furthermore we evaluate the discriminating power of cumulative mutual information, applying a clustering algorithm to the set of our cumulative mutual information time series. For the clustering we used the algorithm described in paragraph 3.4. The hierarchy built by the clustering algorithm based on cumulative mutual information from reduced and original time series is represented by the dendrograms given in Fig. 5 and Fig.6.

(a)



(b)



(c)

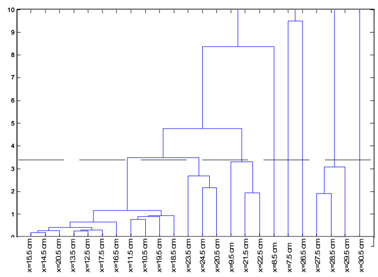


Fig. 5. (a), (b), and (c) Dendrogram Cumulative Mutual information of the Extreme time series along the horizontal line at different cut point

In Figure 5(a) we present the hierarchy clustering of each extreme time series. We decided to make two *´´cuts´´* at the dendrogram at different levels of distance (vertical axis). In Fig. 5(b), the first *´´cut´´* is made at distance~16, where one can clearly see two main partitions. One main group consisted from the time series at x=7.5cm, x=26.5cm, x=27.5cm, x=28.5cm, x=29.5cm, x=30.5cm. This group corresponds to the region too far from the axis of the jet. The second main cluster includes the remaining time series. This first step is important because we can exclude the time series time series corresponding to the edges of the measuring area.

In Fig. 5 (c) we can see the dendrogram which results in from the second *´´cut´´* at distance~3.5. We can see more clearly some major cluster and some smaller. Specifically the time series at x=23.5cm, x=24.5cm, x=20.5cm and at x=9.5cm,

x=21.5cm, 22.5cm join and at x=21.5cm, x=10.5cm, x=19.5cm is joined with x=18.5cm. These above partitions correspond to a set of time series far from the center of jet but not as much as the time series from the first step (x=7.5cm, x=26.5cm, x=27.5cm, x=28.5cm, x=29.5cm, x=30.5cm).
Moreover in Fig. 5 (c) we can distinguish other some small clusters which include the time series at x=15.5cm, x=14.5cm, x=20.5cm and x=13.5cm, x=12.5cm, x=17.5cm. We can notice that the time series at x=16.5cm correspond close to the centerline of jet is separate from other.

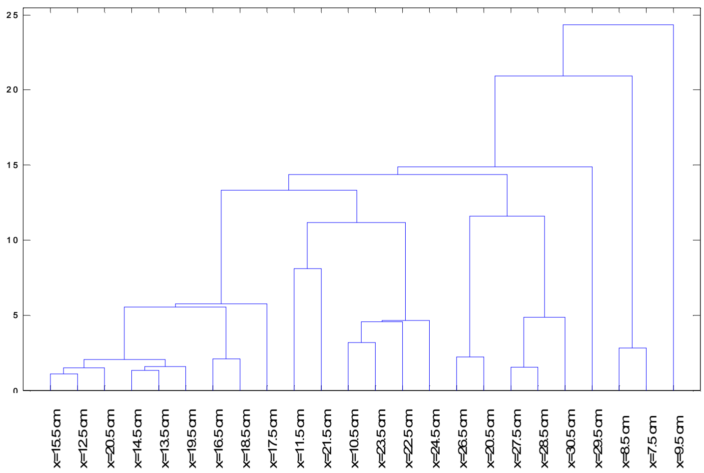


Fig. 6. Dendrogram Cumulative Mutual information of the Initial time series along the horizontal line.

As we can see in Fig 6 where the results for the cumulative mutual information resulting from the analysis of the original time series are presented, there are several clusters without the same discriminating structure observed from the analysis of the extremes time series (Fig.5).

5. Conclusions

In this work we have investigated a new approach in order to detect the jet axis of temperature time series derived from experimental data. The novelty of this study is that the analysis was applied both on the original time series as well as on the extreme value (minimum and maximum values) time series. More specifically we focus to a new measure the *Cumulative Mutual Information*, and we showed that it can discriminate the underlying dynamics from one time series to another. Another important issue is that the performance of the *Cumulative Mutual Information* was applied to a reduced length time series (extreme time series) and showed that it has higher discriminating power than in the original time series. This issue is very important if we take into account the size of the computational analysis of original data due to the length of the time series.

6. References

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