Chaos Control Applied to Mechanical Systems

Aline S. de Paula[1] & Marcelo A. Savi[2]

[1] Department of Mechanical Engineering, Universidade de Brasília, Brasília, DF, Brazil. E-mail: alinedepaula@unb.br
[2] COPPE – Department of Mechanical Engineering, Universidade Federal do Rio de Janeiro, Rio de Janeiro, Brazil. E-mail: savi@mecanica.ufrj.br

Abstract: Chaos is a kind of nonlinear system response that has a dense set of unstable periodic orbits (UPOs) embedded in a chaotic attractor. The idea that chaotic behavior may be controlled by small perturbations applied in some system parameters allows this kind of behavior to be desirable in different applications. This paper considers different chaos control methods, including discrete and continuous, to stabilize some desired UPOs of a mechanical system. Essentially, a control rule is of concern and each controller needs to follow this rule. Noisy time series is treated establishing a robustness analysis of control methods. The main goal is to establish a comparative analysis of chaos control methods evaluating the capability of each one of them to stabilize a desired UPO analyzing its performance.
Keywords: Chaos, control, noise, nonlinear dynamics, pendulum.

1. Introduction

Chaos is a kind of nonlinear system response that has a dense set of unstable periodic orbits (UPOs) embedded in a chaotic attractor. This set of UPO constitutes the essential structure of chaos. Besides, chaotic behavior has other important aspects as sensitive dependence to initial conditions and ergodicity. The idea that chaotic behavior may be controlled by small perturbations applied in some system parameters allows this kind of behavior to be desirable in different applications.

In brief, chaos control methods may be classified as discrete and continuous methods. Semi-continuous method is a class of discrete method that lies between discrete and continuous method. The pioneer work of Ott *et al.* [1] introduced the basic idea of chaos control proposing the discrete OGY method. Afterwards, Hübinger *et al.* [2] proposed a variation of the OGY technique considering semi-continuous actuations in order to improve the original method capacity to stabilize unstable orbits. Pyragas [3] proposed a continuous method that stabilizes UPOs by a feedback perturbation proportional to the difference between the present and a delayed state of the system.

This article deals with a comparative analysis of chaos control methods that are classified as follows: OGY methods – that includes discrete and semi-continuous approaches [1,2]; multiparameter methods – that also includes discrete and semi-continuous approaches [4,5]; and time-delayed feedback

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methods that are continuous approaches [3,6]. In order to consider a system with high instability, a nonlinear pendulum treated in other references is considered [5,7,8].

2. Chaos Control Methods

Control of chaos can be treated as a two-stage process. The first stage is called learning stage where it is performed the identification of UPOs and system parameters necessary for control purposes. A good alternative for the UPO identification is the close return method [9]. This identification is not related to the knowledge of the system dynamics details. The estimation of system parameters is done in different ways for discrete, semi-continuous and continuous methods. After the learning stage, the second stage starts promoting the UPO stabilization.

2.1. OGY Method

The OGY method [1] is described by considering a discrete system of the form of a map $\xi^{n+1} = F(\xi^n, p^n)$, where $p \in \Re$ is an accessible parameter for control. This is equivalent to a parameter dependent map associated with a general surface, usually a Poincaré section. Let $\xi_C^{n+1} = F(\xi_C^n, p_0)$ denote the unstable fixed point on this section corresponding to an unstable periodic orbit in the chaotic attractor that one wants to stabilize. Basically, the control idea is to monitor the system dynamics until the neighborhood of this point is reached. When this happens, a proper small change in the parameter p causes the next state ξ_{n+1} to fall into the stable direction of the fixed point. In order to find the proper variation in the control parameter, δp, it is considered a linearized version of the dynamical system in the neighborhood of the equilibrium point.

2.1.1. Semi-continuous Method

The semi-continuous method (SC) lies between the continuous and the discrete time control because one can introduce as many intermediate Poincaré sections, viewed as control stations, as it is necessary to achieve stabilization of a desired UPO [2]. Therefore, the SC method is based on measuring transition maps of the system. These maps relate the state of the system in one Poincaré section to the next.

2.2. Multiparameter Method

Proposed by De Paula & Savi [4,5], the multiparameter chaos control method (MP) was developed based on the OGY approach. Different from the original idea, this procedure considers Np different control parameters, p_i (i=1,…,Np). Two important points considered in the formulation of MP method are: only one of the control parameters actuates in each control station; and system response to all control parameters actuations is given by a linear combination of its individual effect. Moreover, two approaches are considered, the coupled and the uncoupled approach.

The difference between the multiparameter method (MP) [4] and the semi-continuous multiparameter method (SC-MP) [5] is that the first considers only one control station per forcing period while the other considers as many control stations as necessary to stabilize the system per forcing period. Therefore, the SC-MP is the general case that can represent the MP when only one control station per period is of concern. In the same way, the OGY can be seen as a particular case when only one control station and only one control parameter are considered.

2.3. Time-delayed Feedback Methods

Continuous methods for chaos control were first proposed by Pyragas [3] and are based on continuous-time perturbations to perform chaos control. Socolar *et al.* [6] proposed a control law named as the extended time-delayed feedback control (ETDF) considering the information of time-delayed states of the system represented by the following equations:

$$B(t) = K\left[(1-R)S_\tau - x\right]$$

$$S_\tau = \sum_{m=1}^{\infty} R^{m-1} x_{m\tau}$$

where $K \in R^{n\times n}$ is the feedback gain matrix, $0 \le R < 1$, $S_\tau = S(t-\tau)$ and $x_{m\tau} = x(t - m\tau)$.

An important difference between continuous and discrete methods is that in continuous methods it is not necessary to wait the system to visit the neighborhood of the desired orbit. Another particular characteristic related to the learning stage is that, besides the UPO identification common to all control methods, it is necessary to establish proper values of the control parameters for each desired orbit. In ETDF method this choice is done by analyzing Lyapunov exponents of the UPO, establishing negative values of the largest Lyapunov exponent. De Paula & Savi [7] discussed a proper procedure to evaluate the largest Lyapunov exponents necessary for the controller parameters.

3. Comparative Analysis

As an application of the general chaos control methods, a system with high instability characteristic is of concern: a nonlinear pendulum actuated by two different control parameters discussed in De Paula *et al.* [10]. The mathematical model for the pendulum dynamics describes the time evolution of the angular position, ϕ, assuming that ϖ is the forcing frequency, I is the total inertia of rotating parts, k is the spring stiffness, ζ represents the viscous damping coefficient and μ the dry friction coefficient, m is the lumped mass, a defines the position of the guide of the string with respect to the motor, b is the length of the excitation arm of the motor, D is the diameter of the metallic disc and d is the diameter of the driving pulley. The equation of motion is given by:

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{kd^2}{2I} & -\dfrac{\zeta}{I} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} +$$

$$\begin{bmatrix} 0 \\ \dfrac{kd}{2I}(\Delta f(t) - \Delta l_1) - \dfrac{mgD\,\mathrm{sen}(x_1)}{2I} - \dfrac{2\mu}{\pi I}\arctan(q x_2) \end{bmatrix}$$

where $\Delta f(t) = \sqrt{a^2 + b^2 + \Delta l_2{}^2 - 2ab\cos(\varpi t) - 2b\Delta l_2 \sin(\varpi t)} - (a - b)$ and Δl_1 and Δl_2 correspond to actuations. Numerical simulations of the pendulum dynamics are in close agreement with experimental data by assuming parameters used in De Paula *et al.* [10]: a=1.6×10^{-1}m; b = 6.0×10^{-2}m; d = 4.8×10^{-2}m; D = 9.5×10^{-2}m; m = 1.47×10^{-2} kg; I=1.738×10^{-4}kg.m2; k=2.47 N/m; ζ=2.368×10^{-5}kg.m^2.s^{-1}; μ=1.272×10^{-4}N.m; ω=5.61rad/s.

Due to system instability, some OGY methods are not capable to perform the system stabilization. Thus, the comparative analysis deals with only four different controllers: SC, SC-MP and TDF methods. A control rule is defined for the stabilization of 4 different UPOs in the following sequence considering 500 periods for each orbit: a period-5, a period-3, a period-8 and a period-1.

Figure 1(a) shows the desired trajectory, and the system time evolution at control station (CS) #1 controlled by parameter Δl_1, while Figure 1(b) presents the same results by assuming parameter Δl_2. It should be noticed that in both procedures only three of the four UPOs are stabilized. Moreover, before the stabilization of UPO is achieved it can be observed a region related to chaotic behavior that corresponds to the wait time that system dynamics takes to reach the neighborhood of desired control point.

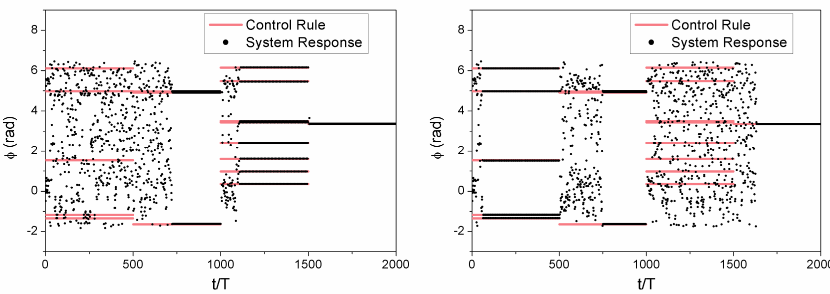

Fig. 1. System controlled using SC with parameter at CS #1: (a) Δl1; (b) Δl2.

The coupled and the uncoupled approaches of the SC-MP are now employed using both control parameters. Figure 2(a) shows the desired trajectory and system time when applying the coupled approach, while Figure 2(b) presents the same results for the uncoupled approach.

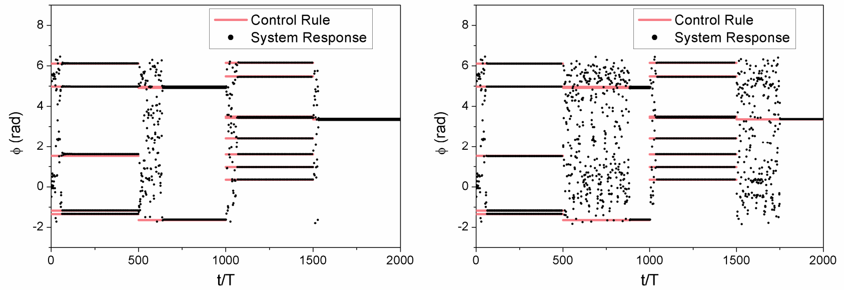

Fig. 2. System controlled using SC-MP at the CS #1: (a) Coupled approach; (b) Uncoupled approach.

Finally, the ETDF method is employed to follow the control rule considering the use of parameter Δl_1. Figure 3 shows the desired trajectory and the system time evolution at control station #1. Note that the ETDF is not able to stabilize the first and the third orbits of the control rule. Besides, the second orbit is different from the stabilized orbit.

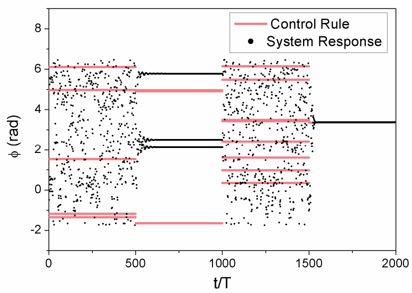

Fig. 3. System controlled using ETDF at the control station #1.

3.1. Chaos Control Performance Considering Noisy Signals

Since noise contamination is unavoidable in experimental data acquisition, it is important to evaluate its effect on chaos control procedures. In general, noise can be expressed as follows:

$$\begin{cases} \dot{x} = Q(x,t) + \mu_d \\ \dot{y} = P(x,t) + \mu_o \end{cases}$$

where x represents state variables, y represents the observed response and $Q(x,t)$ and $P(x,t)$ are nonlinear functions. μ_d and μ_o are, respectively, dynamical and observed noises. Notice that μ_d has influence on system dynamics in contrast with μ_o. In this work, it is considered only an observed noise, simulating noise in experimental data due to instrumentation apparatus and, therefore, noise does not have influence in system dynamics.

The noise level can be expressed by the standard deviation, σ, of the system probability Gaussian distribution, that is parameterized by the standard deviation of the clean signal, σsignal, as follows:

$$\eta(\%) = \frac{\sigma}{\sigma_{\text{signal}}} \times 100$$

A different control rule is assumed in order to compare the control methods performance considering noisy signals. This control rule is defined in order to choose orbits that can be stabilized by all control methods for an ideal signal: a period-6, a period-2, a period-3 and, finally a period-1.

By considering a noisy signal with 1% of amplitude all analyzed methods can achieve the stabilization of some orbits. When increasing the noise level to 2% few methods have a satisfactory performance. Considering this noise level, Figure 4 shows the desired trajectory imposed by the control rule and the system time evolution at CS #1 when the SC is employed considering the isolated actuation performed by the parameters Δl_1 and Δl_2. Figure 5 presents the same pictures for the SC-MP, coupled and uncoupled approaches, while Figure 6 presents results of the ETDF.

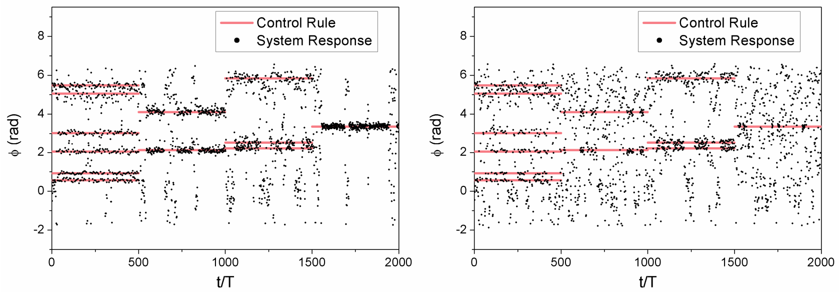

Fig. 4. System controlled using SC at the CS #1 with η=2%: (a) Δl_1; (b) Δl_2.

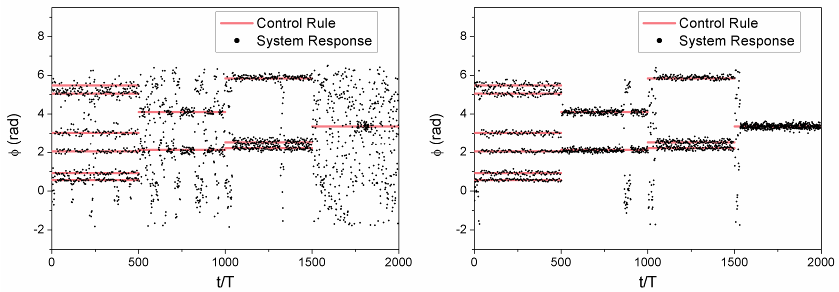

Fig. 5. System controlled using SC-MP at CS #1 with $\eta = 2\%$: (a) Coupled approach; (b) Uncoupled approach.

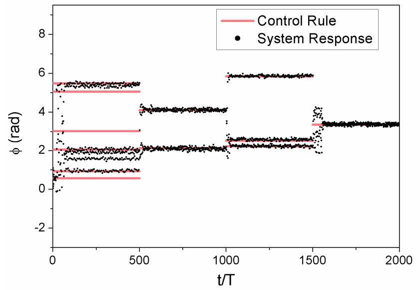

Fig. 6. System controlled using ETDF at the CS #1 with $\eta = 2\%$.

Note that with 2% of noise level the single-parameter SC and the coupled approach SC-MP do not have a good performance. The uncoupled SC-MP presents better results when compared with the preceding methods and the ETDF successfully stabilize all UPOs of the control rule.

3. Conclusions

This paper presents a comparative analysis of chaos control methods performances, including OGY, multiparameter and time-delayed feedback methods. In general, systems with high instability need a greater number of actuations which makes the semi-continuous and continuous methods more effective for chaos control. By defining efficacy as the capability to stabilize desired orbits, the coupled and the uncoupled approaches of the SC-MP method are more effective to perform system stabilization. The continuous methods present low efficacy but avoid the wait time necessary in the case of discrete methods. Moreover, continuous methods present a difficulty for the stabilization of orbits with high instability and of high periodicity since different orbits can be stabilized instead of the desired one. Results from comparative analysis point that the SC methods present good performance for ideal time series, free of noise. When noisy time series is of concern, continuous methods present greater robustness being associated with better performances; however, the uncoupled approach of the SC-MP method also presents a good performance.

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