Utilisation of the Perturbation Theory without Presuppositions. A Repeating Equation for Azimuthal Velocities into Cylindrical and Magnetized Plasma

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Abstract: As the understanding of the chaotic state increases, it becomes clearer that the definition and the theoretical elaboration of the chaos is not a simple hypothesis. In addition, it is a commonplace fact that the mathematical representation of the chaos theory on the whole is very difficult to be given. This means that there is not any mathematical equation capable of describing and solving a nonlinear and chaotic problem. So, as every case is unique, our work has to contribute to the chaotic topic both mathematically and experimentally. Magnetized argon plasma is produced into a metallic cylinder. A coaxial antenna is used for the r-f energy importation and the plasma maintenance consequently. This device has a complete cylindrical symmetry and the mathematical elaboration in the cylindrical system is carried out. An attempt to show a repeating relation for ion velocities of magnitude of every order is presented as our new work. In addition, it is well known that the perturbation theory can be used to extend the linear theory of plasma waves into the nonlinear regime and, thus, give an explanation of many nonlinear phenomena. This nonlinear perturbation theory of small amplitude plasma waves and their interactions is well developed; on the contrary, the perturbation theory of large-amplitude plasma waves is still being developed. In the present paper, a generalization of the perturbation theory is attempted with the division of the perturbed magnitude and the use of the repeating estimation. Computational results and experimental findings are in a very satisfactory accordance.

Keywords: Chaos theory, Nonlinear problem, Cylindrical system, r-f plasma production, Repeating relation, Perturbation theory, Loop on the repeating relation.

1. Introduction

The stability and instability of the plasmatic state was an old problem for researchers during the last decades. Especially, in the early 60’s many plasma
instabilities have been observed taking wavy forms into the plasma [1-4]. These waves absorb the plasma energy, then the plasma temperature is consequently reduced and the removal of the thermonuclear fusion conditions is resulted. So, the wavy instabilities are considered to be a serious obstacle to the nuclear fusion process and their study has been carried out constantly and in detail during the last decades [5,6]. Many special books constitute the Plasma Physics Literacy [7-10], list and study all the waves from the low frequency region [1-6] to the high frequency one [11]. In the Plasma Laboratory of the Center “Demokritos” an adequate amount of experience has been gained, especially on the low frequency electrostatic waves [12-14] and their effect on the plasma conductivity [13]. The chaotic behavior of the plasma waves has been studied as well [15,16]. It is well known that the plasma can easily pass from a steady state into a chaotic one, which was repeatedly published in our previous papers [17,18]. In the present work an attempt takes place to compare the experimental data with the computation results, and so, our theory may be confirmed. A mathematic relation, which connects the different order velocities, was found and may be used as a repeating relation showing the chaotic behavior of the plasma. The relation is valid under the condition that the perturbed qualities are small in comparison with the unperturbed one [8-10]. In the present work a calculative trial using the relation as a repeating one may bring it into the function conditions and the perturbed theory can be therefore extended. Although the experimental results are in a satisfactory agreement with the calculation, the subject remains open as a chaotic state one and requires further study. In the next research of ours, the influence of the initial conditions on the computational results is planed to be studied.

A brief description of the experimental devices is given in Sec.2, since the experimental results are presented in the following Sec.3. In Sec. 4 a full mathematical elaboration and the computational results are curried out. The confirmation between theory and experiment and conclusions are included in Sec. 5. A more detailed mathematical elaboration is provided in the Appendix at the end of the paper.

2. Description of the Experimental Set-Up

It is well known that the predominant direction of the external magnetic field over the Q-machine is well matched with the cylindrical geometry of the device, when the cylinder axis and magnetic field coincide. As our experience on the magnetized argon plasma is concentrated on full cylindrical symmetry, the same geometry is used at the present study as well, since the low frequencies of plasma waves are persistent [1-4, 12-14]. A cylindrical cavity made of steady...
steel is located with its’ axis along the external magnetic field \( B \). The cavity is 60 cm long with 6 cm internal diameter and, in the center of the first disk-like base, the 25 cm rf power antenna is mounted; in the other disk-like base a 25 cm external driving wave antenna is mounted as well, which enables us to affect and control the plasma waves. Electrostatic Langmuir probes were fixed to move radially, azimuthally and axially with the ability to detect the plasma waves that appear and measure their physical quantities (wave frequency, wave amplitude, plasma temperature, plasma density, plasma potential et c.). Furthermore, a disk-probe was fixed to move radially and around its’ axis, which allows, apart from the above quantities, the measuring of the azimuthal electron drift current. In Fig.1 (a) the plasma column cut is shown, whereas an extensive drawing of the cavity’s position into the magnetic field is presented in Fig.1 (b).

![Diagram](image)

Fig.1 (a), the plasma column is shown. Fig.1 (b), the cavity’s placing into the magnetic field is presented (ground plan).

The argon entrance, its’ outlet to the pump and a suitable window are placed on the curved surface of the cylinder, as they are represented at the Fig.1 (b).

### 3. Experimental Data

The existence of the electrical waves into the argon plasma is confirmed once more. These low frequency waves are divided into three frequency regions with a quasi-same behavior in many instances. An extensive study of these waves was carried out at the Plasma Laboratory of NCSR “Demokritos” previously, and two of them were absolutely identified [12, 14]. A stable dependence of the gas pressure on the waves has been presented and measured.
again. This influence consists of the simultaneous decrease of the waves’ amplitude and frequency as the gas pressure increases. Figures 2 and 3 give a middle frequency region wave indicatively with its’ spectrum of frequencies where the upper harmonics appear; the first with a high value of the gas pressure and the second with a low one.

The plasma is lit into an wide space of the external plasma parameters (gas pressure $p$, magnetic field $B$ and rf field absorbed power $P$) and results in a wide region of plasma quantities as well; these quantities include the plasma temperature $T$, the plasma density $n$, the plasma potential $\Phi$, and all the wave parameters. Table 1 shows some typical values of the plasma parameters.

![Fig. 2. A typical wave spectrum in the middle frequency region with high gas pressure.](image)

![Fig. 3. A typical wave spectrum in the middle frequency region again with low gas pressure.](image)
### Table 1: The plasma parameters ranging values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Minimum value</th>
<th>Maximum value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argon pressure $p$</td>
<td>$0.001 \text{ Pa}$</td>
<td>$0.1 \text{ Pa}$</td>
</tr>
<tr>
<td>Argon number density, $n_g$</td>
<td>$2 \times 10^{15} \text{ m}^{-3}$</td>
<td>$2 \times 10^{17} \text{ m}^{-3}$</td>
</tr>
<tr>
<td>Magnetic field intensity, $B$</td>
<td>$10 \text{ mT}$</td>
<td>$200 \text{ mT}$</td>
</tr>
<tr>
<td>Microwaves’ power, $P$</td>
<td>$20 \text{ Watt}$</td>
<td>$120 \text{ Watt}$</td>
</tr>
<tr>
<td>Frequency of the rf power (standard value)</td>
<td>$2.45 \text{ GHz}$</td>
<td></td>
</tr>
<tr>
<td>Electron density, $n_0$</td>
<td>$2 \times 10^{15} \text{ m}^{-3}$</td>
<td>$4.6 \times 10^{15} \text{ m}^{-3}$</td>
</tr>
<tr>
<td>Electron temperature, $T_e$</td>
<td>$1.5 \text{ eV}$</td>
<td>$10 \text{ eV}$</td>
</tr>
<tr>
<td>Ion temperature, $T_i$</td>
<td>$0.025 \text{ eV}$</td>
<td>$0.048 \text{ eV}$</td>
</tr>
<tr>
<td>Ionization rate</td>
<td>$0.1%$</td>
<td>$90%$</td>
</tr>
<tr>
<td>Electron drift velocity, $u_e$</td>
<td>$1 \times 10^4 \text{ m/s}$</td>
<td>$1.7 \times 10^4 \text{ m/s}$</td>
</tr>
<tr>
<td>Electron-neutral collision frequency, $v_e$</td>
<td>$1.2 \times 10^7 \text{ s}^{-1}$</td>
<td>$3 \times 10^9 \text{ s}^{-1}$</td>
</tr>
</tbody>
</table>

The experimental part of the present paper consists of the following steps:

i) By using the radial moving probe, the plasma potential $\Phi(r)$ is measured along the cylinder radius and then, from the relation $\varepsilon = -\frac{\Delta\Phi}{\Delta r}$ the plasma electric field $\varepsilon$ is calculated. Figure 4 is shows the radial potential and radial electric field along the cylinder radius.

![Graph showing plasma potential and electric field](image)

Fig. 4 shows the plasma potential $\Phi(r)$ and the electric field $\varepsilon$ along the cylinder radius.
It must be noted that the electric field $\varepsilon$ remains nearly constant in the middle of the radius, where the wave rises and its’ amplitude constantly increases [6, 12-14]. The measurement has been done by $B < B_0$.

ii) The perturbed electric field $E$ must be measured, consequently. This measurement may be a result of the wave amplitude as it appears along the cylinder radius. Figure 5 is shows the wave amplitude (in Volts) and the perturbed field $E$ correspondingly. The measurement was repeated for values of the magnetic field $B$, under and above the upper cyclotron resonance $B_{res}$.

![Graph showing wave amplitude and perturbed electric field](image1.jpg)

Fig. 5 shows the wave amplitude and the perturbed electric field $E$ along the cylinder radius.

iii) The measurement of the azimuthal electron drift velocity $u_\theta$ is the next step. This is obtained by using the disk probe as it moves around its’ axis. Figure 6 indicates the method of the measurement of the azimuthal electron current $I_\theta$, which requires two simple movements: the orientation of the probe surface perpendicularly to the electron drift course, and after, in the opposite direction of the electrons’ motion.

![Diagram showing measurement of electron drift current](image2.jpg)

Fig. 6, the electron drift current measurement
The next relations \( I_1 = I_{\theta} + I_{th} \) and \( I_2 = -I_{\theta} + I_{th} \) are valid and result in the relation below,
\[
I_{\theta} = \frac{I_1 - I_2}{2}
\]
Taking into consideration that the relation \( I_{\theta} = e n_e u_e A \) is valid (with \( A \) the probe surface area), the azimuthal electron drift can be found. Measurements and estimations are listed in Table 2, since the electron drift velocity \( u_e \) and the perturbed velocity \( \nu_{\theta} \) are presented in Fig. 7, as well.

### Table 2 The azimuthal electron drift current and the drift velocity along a cylinder radius

<table>
<thead>
<tr>
<th>Radius ( r ) (cm)</th>
<th>Drift current ( I_e ) (( \mu A ))</th>
<th>Plasma density ( n_e ) (x10^{15} m^{-3})</th>
<th>Drift velocity ( u_e ) (x10^3 m/s)</th>
<th>( E/\varepsilon )</th>
<th>Perturbed velocity ( \nu_{\theta} ) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>20</td>
<td>6.0</td>
<td>4.43</td>
<td>-0.1</td>
<td>-443</td>
</tr>
<tr>
<td>0.6</td>
<td>26</td>
<td>6.2</td>
<td>5.57</td>
<td>-0.05</td>
<td>-278</td>
</tr>
<tr>
<td>0.9</td>
<td>30</td>
<td>6.5</td>
<td>6.17</td>
<td>-0.067</td>
<td>-413</td>
</tr>
<tr>
<td>1.2</td>
<td>37</td>
<td>7.0</td>
<td>7.02</td>
<td>0.04</td>
<td>281</td>
</tr>
<tr>
<td>1.5</td>
<td>42</td>
<td>6.8</td>
<td>8.20</td>
<td>0.037</td>
<td>303</td>
</tr>
<tr>
<td>1.8</td>
<td>46</td>
<td>6.2</td>
<td>9.85</td>
<td>0.25</td>
<td>2462</td>
</tr>
<tr>
<td>2.1</td>
<td>35</td>
<td>5.9</td>
<td>7.92</td>
<td>0.1</td>
<td>792</td>
</tr>
<tr>
<td>2.4</td>
<td>25</td>
<td>5.6</td>
<td>5.93</td>
<td>0.1</td>
<td>593</td>
</tr>
<tr>
<td>2.7</td>
<td>16</td>
<td>5.5</td>
<td>3.86</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

iv) The perturbed velocity \( \nu_{\theta} \) is impossible to be found directly by an experimental measuring, but it can be estimated from the relation \( \nu_{\theta} = \frac{E}{\varepsilon} u_{\theta} \), as the quantities \( \varepsilon, E \) and \( u_{\theta} \) have been measured above.

The results for the \( \nu \) are given again in Table 2.
4. Mathematical Elaboration - Computational Results

Perturbed velocities’ study

In the Appendix it is proved that the drift and perturbed velocities are related to the electrical fields with the Eq. (A. 9),

\[ u_r = \frac{E}{\varepsilon} u_r \quad \text{and} \quad u_\theta = \frac{E}{\varepsilon} u_\theta \]  

(A. 9)

A. When the perturbed electric field \( E \) is very small, then the relations \( u_r \approx u_r \) and \( u_\theta \approx u_\theta \) are valid.

B. If the relation \( u \approx u \) is valid, the electric fields must have the same behavior as \( \varepsilon \approx \varepsilon \). This means that the wave amplitude undergoes some big changes along the cylinder radius.

With the replacement of the quantity \( \Pi \), the Eqs (A. 7) are written:

\[ u_\theta = \frac{E}{B} \frac{\omega_0^2}{\omega_0^2 + [j(ku - \omega) + v]^2} \quad \text{and} \quad u_r = \frac{E}{B} \frac{\omega_0 [j(ku - \omega) + v]}{\omega_0^2 + [j(ku - \omega) + v]^2} \]

(1)

The azimuthal perturbed velocity \( u_\theta \) is of the most interest: by taking \( \Pi^2 = v^2 - (ku - \omega)^2 + j2\nu(ku - \omega) \) and limiting the real part only, the first of Eqs (1) is rewritten as following,
\[ v = \frac{E}{B} \frac{\omega_c^2}{\omega_c^2 + \nu^2 - (ku - \omega)^2} \quad (2) \]

with \( \nu_0 = \nu \) for simplicity; this may be used as the repeating relation.

It must be reminded that the Eq. (A. 9) was produced with the presupposition that the relation \( \nu \ll u \) is valid. This consists a necessary condition for the linearization of Eq. (A. 1) (perturbation theory).

Now, if we seek a solution for \( \nu = u \), approximately, by separating the perturbed velocity \( \nu \) into small parts \( \nu_1, \nu_2, \nu_3, \ldots, \nu_i \) with \( i = 10, 100, 1000, \ldots \) and every part \( \nu_i \ll \nu \), the perturbation theory condition is satisfied.

Taking \( u = u_0 \) and \( \nu = \nu_1 < \frac{\nu}{10} \), the Eq. (2) is written,

\[ \nu_1 = \frac{E}{B} \frac{\omega_c^2}{\omega_c^2 + \nu^2 - (ku_0 - \omega)^2} \quad (3) \]

With the addition \( u_1 = u_0 + \nu_1 \), the above equation gives the term \( \nu_2 \),

\[ \nu_2 = \frac{E}{B} \frac{\omega_c^2}{\omega_c^2 + \nu^2 - (ku_1 - \omega)^2} \]

If it is taken \( u_2 = u_1 + \nu_2 \), the Eq. (2) gives the term \( \nu_3 \),

And so on, with \( u_{i+1} = u_i + \nu_{i+1} \) the repeating relation,

\[ \nu_{i+1} = \frac{E}{B} \frac{\omega_c^2}{\omega_c^2 + \nu^2 - (ku_i - \omega)^2} \quad (4) \]

is obtained.

**Repeating equation study**

It is evident that the minimum value of the term \( (ku_i - \omega)^2 \) is zero, and then the denominator in the Eq. (4) takes the maximum value. Then, we conclude that, at
the value \( u_j = \frac{\omega_j}{k} \), the \( \nu_j \) has the minimum value, \( \nu_j = \frac{E_j}{B_j} \frac{\omega_j^2}{\omega_j^2 + \nu_j^2} \), which is

the same as if the quality \( \Pi = j(\nu_j - \omega_j) + \nu_j \equiv \nu \), is taken.

Another significant result is obtained if the relations \( u_1 = u_0 + \nu_1 , \) \( u_2 = u_1 + \nu_2 , \) .. \( u_3 = u_2 + \nu_3 , \) .. \( u_i = u_{i-1} + \nu_i \), .. are added by parts, when the relation

\[ u_j = u_0 + \nu \]  

(5)

is obtained

with \( \nu = \nu_1 + \nu_2 + \nu_3 + .. + \nu_i \) the whole-total large perturbed velocity.

The relation \( u_j = u_0 + \nu \) must be confirmed experimentally.

**Computational results**

The experiment leads to the following calculations;

\[
\frac{E}{B} \geq 100V.m^{-1} 
\]

\[
\Rightarrow E \approx 1430 \frac{m}{s^2}
\]

\[
\omega_j^2 = \left( \frac{eB}{m_j} \right)^2 = \left( \frac{1.6.10^{-19} \times 7.10^{-2}}{9.1.10^{-31}} \right)^2, \quad \Rightarrow \omega_j^2 \approx 1.5.10^{20} s^{-2}
\]

\[
\nu_j^2 \approx 10^{19} s^{-2}
\]

When it is taken \( ku_0 - \omega = 0 \), then \( u_0 = \frac{\omega_0}{k} = \frac{\omega_0 R_0}{l} \Rightarrow u_0 = \frac{\omega_0 R_0}{l} \).

Taking \( \omega = 2\pi f = 2\pi \times 7.10^4 s^{-1} \Rightarrow \omega = 1.4 \pi \times 10^5 s^{-1}, \ l = 1 \) and

\( R = 2.10^{-2} m \) then

\[
u_0 = 1.4 \pi \times 10^5 \times 2.10^{-2} \frac{m}{s} \quad \Rightarrow u_0 \approx 0.88 \times 10^4 \frac{m}{s}
\]

In the above case the perturbed velocity \( \nu_1 \) is minimized at the value, (see eq. 3),
\[ \nu_1 = \frac{1430 \cdot 10^{20}}{10^{20} + 10^{10}} \text{ m/s} \implies \nu_1 \approx 1300 \text{ m/s} \]

On the other hand, the relation \( \nu = \frac{E}{c} \) gives,

\[ \nu = \frac{2V \cdot \text{cm}^{-1}}{20V \cdot \text{cm}^{-1} \cdot \text{m}} \implies \nu = 880 \text{ m/s} \]

Now, with \( \nu_1 = u_0 + \nu_1 = (8800 + 880) \text{ m/s} \implies \nu_1 = 9680 \text{ m/s} \), the perturbed \( \nu_2 \) from the repeating equation below can be calculated,

\[ \nu_2 = \frac{\nu}{B} \cdot \frac{\omega^2}{\omega_c^2 + \nu^2 - (k\nu - \omega)^2} \]

or

\[ \nu_2 = 1430 \cdot \frac{1.5 \cdot 10^{10}}{16 \cdot 10^9 - (5.0968 - 4.4)^2} \]

or

\[ \nu_2 = 1340, 62499998377843750019 \]

Now it is taken, \( u_2 = u_1 + \nu_2 \), \( \Rightarrow u_2 = 9680 + 1340, 62499998377843750019 \)

\( \Rightarrow u_2 = 11020, 62499998377843750019 \), and so on.

5. Explanation- Conclusions

The existence of the low frequency waves into the argon magnetized plasma was observed in our early experiments at the Plasma Laboratory of Demokritos. A satisfactory explanation about it was given as well \[12,14\]. In previous publication the possibility for development of the low frequency waves has been presented. Two kinds of these waves have been identified already \[12, 14\]. The cylindrical symmetry of the plasma column gives them azimuthal propagation, whereas the boundaries cause for standing waves formation. By using the perturbation theory on the two fluids model, the relation,

\[ \nu = \frac{E}{B} \cdot \frac{\omega^2}{\omega_c^2 + \nu^2 - (k\nu - \omega)^2} \]
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is obtained under the conditions \( v \ll u \).

The validity of the above equation is attempted to be proved with the present experiment. So, the value of the perturbed velocity \( v \) was found at first experimentally and then by estimation from the above equation. The direct measurement of the perturbed velocity \( v \) is impossible to be carried out as it is added on the drift velocity \( u \), resulting in the inability to be distinguished from it. For this reason, the relation \( v = \frac{E}{\varepsilon} u \) was used, which requires the measurement of the quantities \( E, \varepsilon \) and \( u \). As Figs 4 and 5 show, the electric fields \( E \) and \( \varepsilon \) are maximized in the middle of the radius, where the wave is developed, and the ratio \( \frac{E}{\varepsilon} \) is very close to the perturbation theory condition.

Furthermore, from Table 2 the values of the drift electron velocity are taken. Figure 7 gives the measured values of the perturbed velocity \( v \). Afterwards, the calculated values from the repeating equation are taken. Despite the inevitable inclinations of the measurements, the two results are satisfactory close, and may have the certainty that the suggested calculation method is right. Another significant observation is that, because of the use of the equation as a repeating one, the values of the perturbed velocity are slightly affected from the drift velocity enlargement. On the contrary, the drift velocity enlargement strengthens the function condition \( v \ll u \).

**Appendix**

The momentum equation on the two fluids theory based on a non-local slab is written as,

\[
N_a m_a \left[ \frac{\partial}{\partial t} + \vec{V}_a \nabla \right] \vec{P}_a = N_a q_a \vec{E} + N_a q_a \frac{\vec{V}_a \times \vec{B}}{c} - N_a m_a \vec{V}_a - \vec{V} p
\]

where the indicator \( \alpha \) is given for both kinds of the charged particles, electrons and ions. In the following elaboration, the \( \alpha \) is omitted for simplicity and the momentum equation for either electrons and ions becomes,

\[
N m \left[ \frac{\partial}{\partial t} + \vec{V} \nabla \right] \vec{V} = Nq(\vec{E} + \vec{E}) + Nq \frac{\vec{V}_x \vec{B}}{c} - Nm \vec{V} p - \vec{V} p \quad (A.1)
\]

where \( N = n_0 + n(\vec{r}, t), \vec{E}_{\text{tot}} = \vec{E} + \vec{E}(\vec{r}, t), \) and \( \vec{V} = \vec{u}_0 + \vec{v}(\vec{r}, t), \) and \( n(\vec{r}, t), \vec{E}(\vec{r}, t), \) and \( \vec{v}(\vec{r}, t), \) the perturbed qualities with harmonic influence \( \alpha e^{i(k_0 \vec{x} - \omega t)} \).
When no perturbation exists, the drift velocity $u_0$, is obtained;

$$0 = n_0 q \frac{\vec{e}}{m} + n_0 q \frac{\vec{B}_0 x \vec{B}}{m} - n_0 m v \vec{u}_0$$  \hspace{1cm} (A. 2)

With the separation on the $\vec{r}$ and $\vec{\theta}$ axis the drift components are given,

$$u_\theta = \frac{q e}{m} \frac{\omega_c}{\omega_c^2 + \nu^2} \quad \text{and} \quad u_r = \frac{q e}{m} \frac{\nu}{\omega_c^2 + \nu^2}$$  \hspace{1cm} (A. 3)

3) (drift velocities are represented by the 0th-order equation).

i) If the perturbation is taken into account, eq. (A. 1) gives,

$$n \frac{\partial}{\partial t} + n_0 \vec{B}_0 \vec{\nabla} \vec{V} = n_0 \frac{\vec{E}}{m} + n \frac{\vec{B}_0 x \vec{B}}{m} + n_0 q \frac{\vec{B}_0 x \vec{B}}{m} - n \vec{u}_0 - n_0 \vec{V}$$  \hspace{1cm} (A. 4)

( the 1st order equation)

$\beta)$ $n_0 \vec{u}_0 \vec{\nabla} \vec{V} + n \frac{\partial \vec{V}}{\partial t} + n_0 \vec{B}_0 \vec{\nabla} \vec{V} = \frac{nq \vec{E}}{m} + nq \frac{\vec{B}_0 x \vec{B}}{m} - n \vec{V} - \frac{\vec{V}}{m} \hspace{1cm} (A. 5)$

(the 2nd order equation)

$\gamma)$ And finally,

$$n_0 \vec{u}_0 \vec{\nabla} \vec{V} = 0 \hspace{1cm} (A. 6)$$

(the 3rd order equation).

From the equilibrium state (zero order equation), the drift velocity components are easily obtained,

$$u_\theta = \frac{q e}{m} \frac{\omega_c}{\omega_c^2 + \nu^2} \quad \text{and} \quad u_r = \frac{q e}{m} \frac{\nu}{\omega_c^2 + \nu^2}$$

From the first order equation, the perturbant velocity components may be given as,

$$v_\theta = \frac{q E}{m} \frac{\omega_c}{\omega_c^2 + \nu^2 + \Pi^2} = \frac{E}{B} \frac{\omega_c^2}{\omega_c^2 + \nu^2 + \Pi^2} \quad \text{and} \quad v_r = \frac{q E}{m} \frac{\nu}{\omega_c^2 + \nu^2 + \Pi^2}$$  \hspace{1cm} (A. 7)

with $\Pi = j(ku - \omega) + \nu$

A combination of drift and perturbed velocities components gives,
\[ \nu_{\theta} = \frac{E}{\varepsilon} \frac{\omega_e^2 + v^2}{\omega_e^2 + \Pi^2} \mu_{\theta} \quad \text{and} \quad \nu_{r} = \frac{E}{\varepsilon} \frac{\omega_e^2 + v^2}{v \omega_e^2 + \Pi^2} \mu_{r} \]

(A. 8)

If it is considered that \( ku - \omega \ll v \), then it is taken \( \Pi \equiv v \) and the perturbed velocity components (eq. A. 7) become,

\[ \nu_{\theta} = \frac{E}{B} \frac{\omega_e^2}{\omega_e^2 + v^2} \quad \text{and} \quad \nu_{r} = \frac{E}{B} \frac{\omega_e v}{\omega_e^2 + v^2} \]

as the drift velocity components by replacing the dc electric field \( \varepsilon \) with the perturbed one \( E \).

If \( ku - \omega \ll v \), then \( \Pi \equiv v \) is taken likewise and from Eqs (A. 8) the below relations (A. 9) are obtained,

\[ \nu_{r} = \frac{E}{\varepsilon} u_{r} \quad \text{and} \quad \nu_{\theta} = \frac{E}{\varepsilon} u_{\theta} \quad \text{(A. 9)} \]

References


