Monte Carlo Modelling of Soliton Pulse Timing Jitter in Silicon Nanowire Waveguides

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Abstract: The purpose of this effort is to study changes in the amplitude noise and timing jitter of an optical pulse chain from a mode-locked laser, as it undergoes soliton propagation through a nonlinear silicon nanowire waveguide. A numerical model was developed using the Non-Linear Schrödinger Equation to model the soliton formation with two-photon absorption. The amplitude noise was modeled as a separate noise envelope, and the phase noise and timing jitter was modeled using Monte-Carlo simulations of jitter-induced phase-shifts. It was observed that while increased pulse energy will result in increased amplitude and phase noise, the presence of two-photon absorption, which attenuates optical nonlinearities in the waveguide, results in a reduction in phase noise at the output of the silicon waveguides.
Keywords: Noise, Phase Noise, Timing Jitter, Monte-Carlo, Non-Linear Schrödinger Equation, Silicon, Photonics, Soliton, Dispersion, Waveguides, Self-Phase Modulation, Kerr, Nonlinear Optics,

1. Introduction

One of the challenges that must be overcome for the practical implementation of optical data transfer is the issue of noise, particularly phase noise, amplitude noise, and timing jitter. Practical optical data communication often requires pulse repetition rates of tens of gigahertz (GHz), and therefore timing jitter on the order of femtoseconds (fs) is often necessary to ensure a low bit-rate error in the data. This paper investigates numerically the effects of soliton pulse propagation within silicon nanowire waveguides, and the effects of these nonlinearities on noise and jitter, for the purpose of applied optical data communications.

Much research has previously been conducted on the effects of optical propagation through a dispersive waveguide on the phase noise, timing jitter, and amplitude noise [1-2]. This research to date has predominantly focused on

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optical fibers [3], photonic crystal fibers [4], and mode-locked lasers [5]. The purpose of this paper is to investigate optical soliton propagation [6-8] through silicon nano-waveguides. Silicon waveguides are of interest to the scientific community for their high-nonlinearities and tight optical confinement. Compared to optical fibers, silicon nano-waveguides have much smaller length scales, and offers many applications at the chip-scale level for all-optical data transfer, information manipulation, and computing.

2. Simulations

It has been previously observed that the noise can often be attributed as a separate envelope [2,9] of much weaker intensities than the undisturbed pulse input:

$$A(z,t) = (P_0^{1/2} + a(z,t))*\exp(-j*\varphi(z)) \qquad a(z,\omega) = \int_{-\infty}^{\infty} a(z,t)*\exp(-i*\omega t)$$

With this assumption, the NLSE can be linearly separated, and a separate NLSE for the noise can be derived:

$$(j/2)*\beta_2*\omega^2*a + (j/6)*\beta_3*\omega^3*a + j*\gamma*P_0*\{a+a^*\}*\exp(-\alpha*z) = -\,\partial a/\partial z$$

The noise can be assumed to be an independent envelope propagating through the waveguide, and analyzed as a separate NLSE problem, propagating concurrently with the pulse.

In the time domain, $a(z,t) = a_r(z,t) + j*a_i(z,t)$, where $a_r(z,t)$ and $a_i(z,t)$ are real functions. By substituting these terms into the noise-NLS equation, one gets a simple relationship for the real and imaginary components of the noise function in the spectral domain:

$$\partial a_r(z,\omega)/\partial z = \rho * a_i(z,\omega)$$
$$\partial a_i(z,\omega)/\partial z = -\{\rho + (2*j*\gamma*P_0*\exp(-\alpha*z))\} * a_i(z,\omega)$$
$$\rho = (\beta_2*\omega^2/2) + (\beta_3*\omega^3/6)$$

Using these assumptions, with a given noise input, one can estimate the change in the power spectral density after optical soliton propagation through a given distance increment of a waveguide [9] by using the following equations:

$$\Phi\,(L,\omega) = ½*\Phi(0,\omega)*\exp(-\alpha*z)*(2*|M_{11}(\omega)|^2 + |M_{12}(\omega)|^2 + |M_{21}(\omega)|^2)$$
$$M_{11}(\omega) = \cos(\delta(\omega)*L)$$
$$M_{12}(\omega) = (\rho/\delta)*\sin(\delta(\omega)*L)$$
$$M_{21}(\omega) = -(\delta/\rho)*\sin(\delta(\omega)*L)$$
$$\delta = [\rho^2 + 2*\rho*\gamma*P_0]\hat{}½$$

Using these terms and incorporating them into the NLSE numerical simulation, an accurate prediction of the changes in the frequency noise after propagation through a silicon waveguide could be obtained.

Many NLSE simulations were conducted in order to complement the experimental silicon waveguide used in this experiment. The silicon waveguide parameters include a length of 4.1 mm, an effective area of 250 nm by 450 nm, a Kerr coefficient of $4.4 * 10^{-18}$ m^2/W, an effective index of 2.5, a group index of 4.5, and a 2nd and 3rd order GVD of 4.5 ps^2/m and 0.01 ps^3/m, respectively. The model took into account both two-photon absorption (TPA), free-carrier absorption (FCA), and linear loss of the pulse envelope. Because the noise is assumed to be substantially weaker compared to the pulse envelope, only linear loss is applied to the noise envelope.

For the initial simulations, the wavelength was set at 2543 nm, so that there would be no effects of TPA or FCA. Simulations were run repeatedly for various input pulse energies ranging from 1 pJ to 500 pJ; these energies are far in excess of the fundamental soliton energy for the 2.3 ps hyperbolic secant pulse. As the lasers timing jitter was in excess of the pulse duration, the simulation assumed a constant noise envelope for the temporal window analyzed. It was observed that at lower input pulse powers, the noise would decrease after propagation through the waveguide, but this loss would decrease with increasing powers. After an input pulse energy of 250 pJ, it was found that the energy would in fact increase exponentially with increasing energy. This is expected, as previous work in glass photonic crystal fibers [4] has also noticed an increase in jitter from solitons not subjected to TPA.

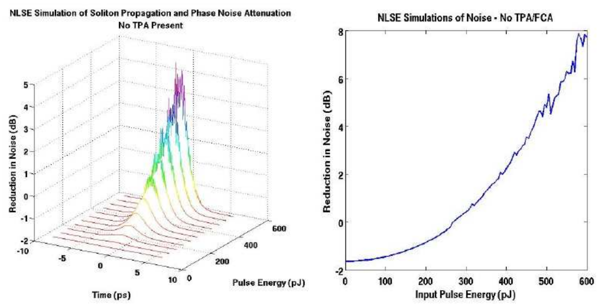

Figure 1 – Results of NLSE simulations of noise after propagation in the silicon waveguide, with a wavelength of 2543 nm that is not subjected to the nonlinear effects of two-photon and free-carrier absorption.

The simulation was then conducted for optical pulses at 1543 nm, which are now subjected to a considerable amount of TPA at this wavelength [10,11]. It was observed numerically that for optical soliton propagation in a silicon waveguide, the noise would consistently be reduced from 1.6 to 1.4 dB; this reduction would decrease with increasing input pulse energies within the waveguide. After 1 nJ of energy, which is far more than will be practically realized experimentally, the noise decrease will plateau, and there will be little change with increasing power.

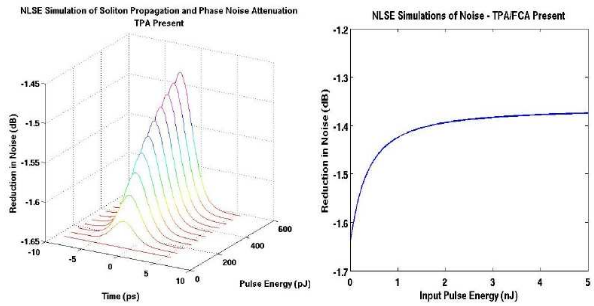

Figure 2 – Results of NLSE simulations of noise after propagation in the silicon waveguide, with a wavelength of 1543 nm that is subjected to the nonlinear effects of two-photon and free-carrier absorption.

3. Monte-Carlo Analysis of Soliton Timing Jitter

One of the challenges of performing a numerical analysis on the effects of optical soliton propagation on phase noise and timing jitter is the fact that such noise can reasonably be assumed to be random jitter. Even though most of this jitter is deterministic and repeatable, the variation of each pulse can still have a significant amount of randomness involved. Therefore, in an effort to numerically model the changes in phase noise after soliton propagation, Monte-Carlo simulations of pulse phase-shifts will be used in conjunction with the Non-Linear Schrödinger Equation (NLSE) solver.

The goal of this solver is to determine the change in timing jitter after propagation through a silicon waveguide for various energies and wavelengths. Input pulse energies from 5 pJ to 5 nJ were studied, and the wavelengths of 1550 nm and 2300 nm were analyzed. At each pulse-energy being studied, the program first solves the NLSE for a transform-limited hyperbolic secant squared pulse with no chirp; the output pulse shape and phase of the NLSE simulation will be used for comparison against a number of random trial simulations of jitter-shifted pulses. Before propagating these pulses, the same hyperbolic secant-squared input pulses are phase-shifted to represent the timing jitter. The phase shift is as follows:

Phase Shift = exp[i*(2*f*Jitter)*((2*rand)-1)]

where *f* is the frequency of the mode-locked laser (39.11 MHz), *Jitter* is the RMS of the input timing jitter (this study used 20 ps), and *rand* is a random number from zero to 1. The code is written so that the phase shift varies up to twice the specified average jitter, and can be either positive or negative.

After applying the random phase shift, the pulse was analyzed with the NLSE solver. The new output pulse phase was compared to the original non-shifted phase, the difference in phase was converted to timing jitter, and the RMS of the jitter was calculated. As Monte-Carlo simulations require many repeated random terms to be statistically significant, the simulation was repeated 1,000 times at each energy level, for a total of over 400,000 separate NLSE simulations. The raw data of the results can be seen in Figure 3, which shows the output timing jitters as a function of input pulse-energy.

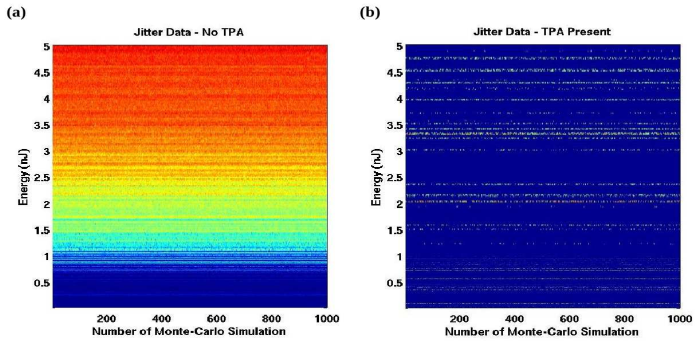

Figure 3 – Raw Data of simulations, (a) λ = 2300 nm and (b) λ = 1550 nm.

After all of the simulations were completed, in order to remove any statistical outliers, the code went through and factored out all simulations greater than 2 standard deviations away from the mean jitter. The RMS of this noise was then collected, and a final output timing jitter was given for each energy level. The data of the timing jitter as a function of energy was cleaned up of statistical outliers, and averaged out to obtain the trend of output timing jitter as a function of energy.

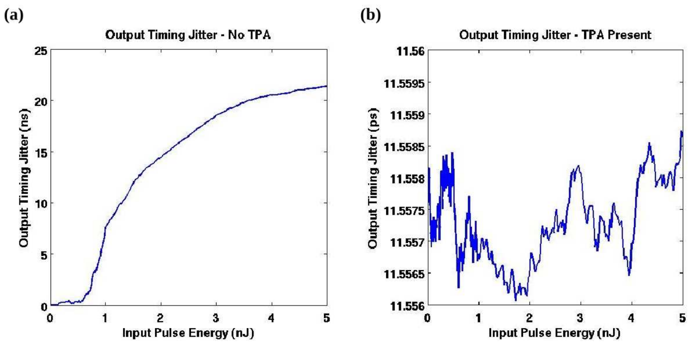

Figure 4 – Output timing jitter as a function of pulse energy, for (a) λ = 2300 nm and (b) λ = 1550 nm.

In the study of the 2300 nm pulse without TPA, the simulation clearly demonstrated the timing jitter growing exponentially with increasing pulse energy, just as the NLSE simulation of the separate noise envelope has demonstrated. In the case of the 1550 nm pulse subjected to TPA, the Monte-Carlo simulations showed the output timing jitter to consistently decrease from 20 ps RMS to 11.6 ps of RMS timing jitter. Just as observed with the study of the NLSE of the phase-noise envelope, the presence of TPA has attenuated the jitter, rather than allowed it to develop with increasing energies. It is therefore concluded, based on these two separate simulations, that, soliton propagation in the presence of TPA will result in a decrease in phase noise and timing jitter.

4. Conclusion

The numerical simulations have demonstrated that an optical pulse propagating in the optical C-band within a silicon waveguide will see an attenuation of the amplitude noise and timing jitter due to the presence of the two-photon absorption. The two-photon absorption has the property of attenuating the pulse proportionally to the intensity, which acts to inhibit the self-phase modulation and thus soliton compression. If this attenuation were not present, an increase in intensity will result in an increase in nonlinear effects and thus an increase sensitivity to jitter-induced phase-shifts; for this reason high optical intensities have shown to increase the timing-jitter in the simulations of longer wavelengths not subjected to two-photon absorption. In the presence of two-photon absorption, however, less variation in the pulse phase-shifts can be expected as a result the reduction in two-photon absorption. For this reason, it is concluded that optical soliton propagation in the presence of two-photon absorption has the ability to attenuated the phase noise and timing jitter of a mode-locked optical pulse.

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