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Complex Dynamics of Multiparticle System Governed by Bounded Rationality

Arkady Zgonnikov[1] and Ihor Lubashevsky[2]

[1] University of Aizu, Fukushima, Japan
(E-mail: arkady.zgonnikov@gmail.com)
[2] University of Aizu, Fukushima, Japan
(E-mail: i-lubash@u-aizu.ac.jp)

Abstract. We consider a system of interacting elements that mimic certain properties of human perception, namely, the bounded capacity of ordering events, actions, etc. according to their preference. Previously this feature was described by the notion of dynamical traps, which is modified in the present work in order to take into account the imperfectness of human perception of their own actions. Numerically we demonstrate that the considered system under the presence of dynamical traps of a new type exhibits complex dynamics, including highly irregular motion.
Keywords: Complex dynamics, multiparticle systems, dynamical traps.

1 Introduction

The employment of various physical models in social sciences could be observed during last decades. Among the models that are used widely in studying cooperative phenomena in social systems are multi-particle dynamical models (see, e.g., Helbing and Mólnar[1], Ohnishi[2]). Advances in this field, though, face the fact that human beings indeed differ in their basic properties from the objects of the inanimate world described by Newtonian mechanics. This fact may lead one to the problem of development of new physical notions that should be introduced in addition to the well-studied ones of the modern physics in order to reflect the essential aspects of human behavior in social systems.

Mathematical notion of equilibrium points is one of the cornerstones of the modern physics; it is also widely used in social psychology (see, e.g., Vallacher[3]). However, human as a key acting element of the dynamical systems is often not capable to clearly recognize the desired equilibrium position among a certain set of its neighboring points in the corresponding phase space. This feature of human cognition is referred to as bounded or fuzzy rationality (Dompere[4]). The application of the dynamical traps notion as a mathematical formalism for describing human fuzzy rationality was investigated by Lubashevsky[5]. To briefly review this concept, let us appeal to the car following theory and consider hypothetical dynamical system controlled by an operator

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whose purpose is to maintain the system near the equilibrium point set to the origin. The system of equations describing the system dynamics under the control of the operator take the following form

$$\begin{aligned} \dot{x} &= v, \\ \dot{v} &= \Omega(x,v) F(x, v, a_{opt}(x,v)). \end{aligned} \tag{1}$$

Here x and v are the system coordinate and velocity, respectively; a_{opt} is optimal in some sense control strategy chosen by the operator. The cofactor $\Omega(x,v)$ equals unity for all values of (x,v) that are far enough from the equilibrium point and $\Omega(x,v) \ll 1$ in a certain neighborhood $\mathbb{Q}_{tr}$ of the equilibrium point. In order to explain the meaning of the cofactor $\Omega(x,v)$ we consider the behavior of the operator who is approaching desired phase space position $(x = 0, v = 0)$. Let us assume that if the current position is far from the origin, the operator perfectly follows the optimal control strategy. If the current position is recognized by the operator as "good enough" $((x,y) \in \mathbb{Q}_{tr})$ (though it may be not strictly optimal) due to her fuzzy rationality, she halts active control over the system so that the system dynamics is stagnated in a certain vicinity of the desired position (in case of stable equilibrium). Therefore, $\mathbb{Q}_{tr}$ is called the area of dynamical traps.

Previous studies on the dynamical trap effect in chains of particles governed by equations of form (1) have shown that it may cause complex cooperative phenomena to arise in the systems under the presence of white noise (Lubashevsky *et al.*[6]), as well as in the systems without the influence of stochastic factors (Lubashevsky[5]). However, it should be taken into account that in the real world the operator cannot usually affect the system velocity directly as prescribed by equations (1), e.g., in the car following the operator is not able to directly affect the speed of the car and in fact controls only the acceleration (Lubashevsky[7]).

It should also be noted that the operator perception of her own actions is not perfect, and could also be described in terms of fuzzy rationality. Namely, the value of the actual control effort could be treated as an acceptable by the operator if its deviation from the optimal strategy is of low magnitude. Therefore, in order to take into account the issues discussed above, in present work we introduce the dynamical trap model of a new type. While previously the dynamical trap region was referred to as two-dimensional region in the "coordinate-velocity" phase space, we propose the concept of the dynamical trap in the "space" of behavior strategies as a certain neighborhood of the optimal one.

The purpose of the current paper is to demonstrate that bounded rationality of human cognition in perceiving their own actions could be responsible for intrinsic cooperative phenomena in the systems of interacting elements under the control of human operators.

2 Model

Let us consider the chain of N motivated particles (Fig. 1) moving along parallel vertical axes; the motion of each particle is characterized by its coordinate x_i,

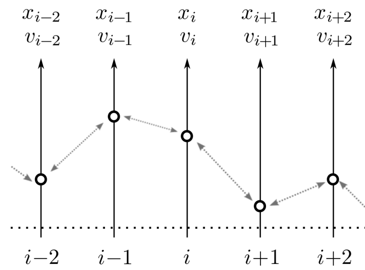

Fig. 1. The chain of N motivated particles moving along parallel axes. Terminal particles $i = 0$ and $i = N + 1$ are fixed at $x = 0$. Dotted arrows indicate the interaction between neighboring particles.

velocity v_i and acceleration a_i. Each particle tends to minimize the absolute values of its relative coordinate and velocity with rescept to its neighbors, namely, $\eta_i = x_i - \frac{1}{2}(x_{i-1} + x_{i+1})$ and $\vartheta_i = v_i - \frac{1}{2}(v_{i-1} + v_{i+1})$. Two terminal particles are assumed to be fixed: $x_0(t) \equiv x_{N+1}(t) \equiv 0$. The dynamics of such system could be described by the following equations

$$\begin{aligned} \dot{x}_i &= v_i, \\ \dot{v}_i &= a_i, \\ \dot{a}_i &= \Omega_a\big(a_i, a_{opt}\big(\eta_i, \vartheta_i, v_i\big)\big)\big(a_{opt}(\eta_i, \vartheta_i, v_i) - a_i\big), \end{aligned} \tag{2}$$

for $i = \overline{1, N}$. Here

$$a_{opt}(\eta, \vartheta, v) = -\Omega_\vartheta(\vartheta)(\eta + \sigma\vartheta + \sigma_0 v) \tag{3}$$

is the optimal strategy of the operator behavior which is considered to depend mainly on the current values of the relative position η and velocity ϑ. σ could be treated as a relative weight of the velocity variations as a stimulus causing operator actions (with respect to the first stimulus η_i); $\sigma_0 v_i$ stands for the friction force which characterizes the physical properties of the environment where the system is placed ($\sigma_0 \ll 1$). The dynamical trap effect in system (2), (3) is modelled by cofactors Ω_ϑ and Ω_a defined as follows

$$\begin{aligned} \Omega_\vartheta(\vartheta) &= \frac{\Delta_\vartheta + \vartheta^2}{1 + \vartheta^2}, \\ \Omega_a(a, a_{opt}) &= \frac{\Delta_a + (a_{opt} - a)^2}{1 + (a_{opt} - a)^2}, \end{aligned} \tag{4}$$

where parameters $0 \leq \Delta_\vartheta, \Delta_a \leq 1$ determine the intensity of dynamical traps: the less these parameters, the stronger the effect of corresponding dynamical traps.

It should be pointed out that we assume the former dynamical trap cofactor Ω_ϑ not to depend on particle coordinate; it could be explained in such a manner that the control over system relative velocity ϑ is of prior importance for the operator comparing to the control over position η. Thus, if the relative velocity becomes sufficiently small, the operator prefers to retard the correction of the coordinate in order not to make the velocity variations take undesirably large values (Lubashevsky[5]).

The cofactor Ω_a in (2) stands for the dynamical trap effect of a new type which was not studied previously. Assuming $\Omega_a = 1$, one could easily see that the last equation in (2) in fact implies the equality $a_i = a_{opt}()$. However, we consider that the operator, first, is hardly able to precisely implement the strategy a_{opt} defined by (3), and, second, cannot distinguish between the strategies that are close in some sense to the optimal one. Therefore, one may think of a certain neighborhood of the optimal strategy in the space of all possible strategies, such that each strategy from this region is treated as the optimal one by the operator. So in case the operator feels that current control regime is optimal, she just keeps maintaining the current value of the control effort constant so that $\dot{a} \approx 0$. When the operator realizes that the current strategy is far from the optimal one, she starts adjusting it to the desired value which means that $\dot{a} \sim (a_{opt} - a)$.

These speculations led us to the system (2)–(4) as a model that may reflect some of mentioned properties of human bounded rationality. The rest of the paper is devoted to the analysis of anomalous cooperative phenomena that could be observed in such system for various values of system parameters.

3 Numerical simulation

In the current work we present the results of the preliminary analysis of system (2)–(4). The scope of the future work should comprise certain extensions of the proposed model; to be specific, the characteristic time scale of the system dynamics should be taken into account, as well as the thresholds of the velocity and acceleration perception. Here we consider all these parameters to take values equal to unity.

We analyze numerically the collective behavior of the particle chain by solving equations (2)–(4) using the standard $(4, 5)$-Runge-Kutta algorithm. Due to the fact that the behavior of the studied system significantly varies depending on the number of interacting particles, the below analysis is divided into three parts according to the cases $1) N = 1, 2$; $2) N = 3$; $3) N \geq 4$. We should specify that all of the following results were obtained for small values of parameters Δ_ϑ and Δ_a, namely 0.001, which correspond to the strong effect of dynamical trap. Below all phase space portraits depict projections of 3-dimensional phase trajectories on the "coordinate-velocity" plane generated by the system motion during the time interval of $T = 10^4$ given small randomly assigned initial disturbances. In case of multi-particle chains the middle particles trajectories are represented; particle motion structure is similar for all particles in the given ensemble, however, particles in the center of the chain have slightly larger fluctuations amplitude.

The numerical simulation of the single particle oscillating between its two fixed neighbors ($N = 1$) figures out that the combination of two dynamical traps causes the limit cycle to arise in the system phase space, while without the dynamical trap effect the system has single stable fixed point $(x = 0, v = 0, a = 0)$. Also it is notable that the previous studies discovered the stable behavior of the single oscillator under the presence of the single dynamical trap characterizing the fuzzy rationality in perceiving the velocity variations (Lubashevsky[5]).

First let us consider the case of the single particle oscillating between two fixed neighbors. The phase portrait and phase variables distributions of the system motion are depicted on Fig.2*a*-*c*. The chain of two interacting particles exhibits the similar behavior patterns (see Fig.2*d*-*i*), except for the phase trajectories assymetry caused by the introduction of the second oscillator. In both cases the structure of the limit cycles is stable with respect to variations of the system parameters. Namely, the found pattern remains for the following values of system parameters: $\sigma = 1,\, 3$; $\sigma_0 = 0,\, 0.01,\, 0.1$.

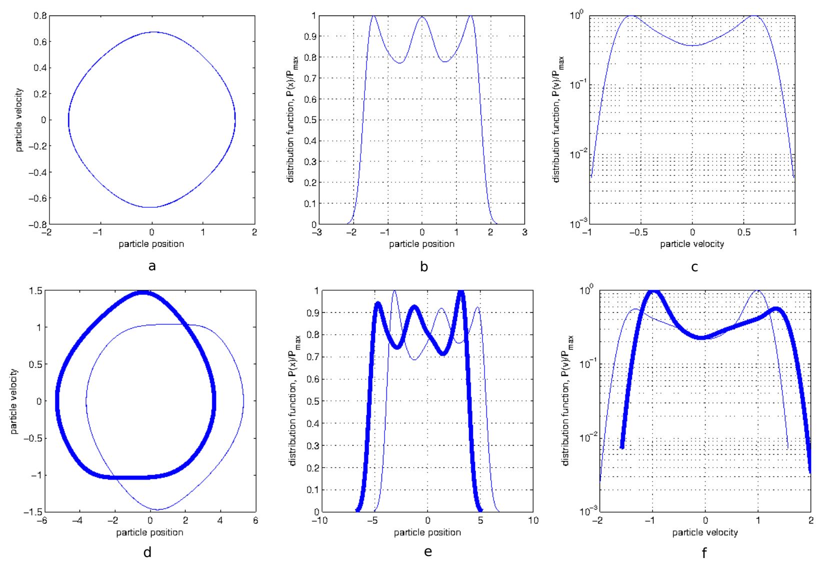

Fig. 2. The phase trajectory projections of system (2)–(4) for $N = 1$ (a) and $N = 2$ (d) on the "coordinate-velocity" plane. The right four frames show corresponding phase variables distributions. On figures (d)–(f) thin and thick lines are introduced in order for one to distinguish between two moving particles. Parameters used for simulation are $\sigma = 1$, $\sigma_0 = 0.01$.

From Fig.2 it could be seen that the dynamical trap effect causes the instability of the single particle motion; the limit cycle emerges. The similar phenomena could be observed in almost the same form for each particle in the pair of coupled oscillators. The situation dramatically changes when the ensemble of three particle is taken into consideration. Adding just one more oscillator to the system causes the anomalous cooperative phenomena to emerge, particularly, complex 3-dimensional attractor arises in the system phase space (see Fig.3*a*-*c*).

Notably, unlike the previous cases ($N = 1, 2$), introducing the external friction force ($\sigma_0 \neq 0$) causes the attractor to become significantly blurred (see

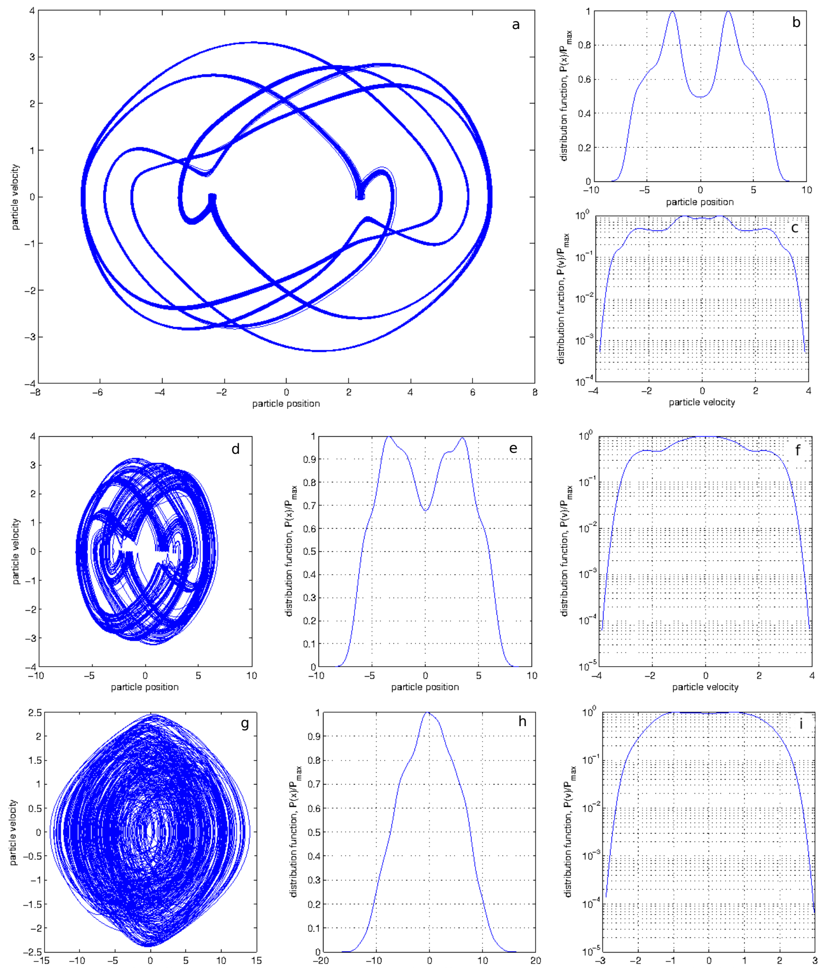

Fig. 3. The phase trajectory projections of the middle particle from the ensemble (2)–(4) and corresponding phase variables distributions for $N = 3$. Frames a-c illustrate the case $\sigma = 1, \sigma_0 = 0$, frames d-f depict the case $\sigma = 1, \sigma_0 = 0.01$, frames g-i are for the values of parameters $\sigma = 3, \sigma_0 = 0$

Fig.3d-f), while increasing the relative weight of the particle velocity as the stimulus for the operator actions makes the particle dynamics to take form of chaotic oscillations (Fig.3g-i).

In case of the relatively large number of interacting elements the system dynamics becomes highly irregular. The chain of four particles demonstrate the oscillatory behavior as could be seen on Fig.4a-c. It is worth underlining that the well-defined attractor (Fig.3a) could be destructed just by adding one particle to the ensemble (Fig.4a) without changing any of the system parameters.

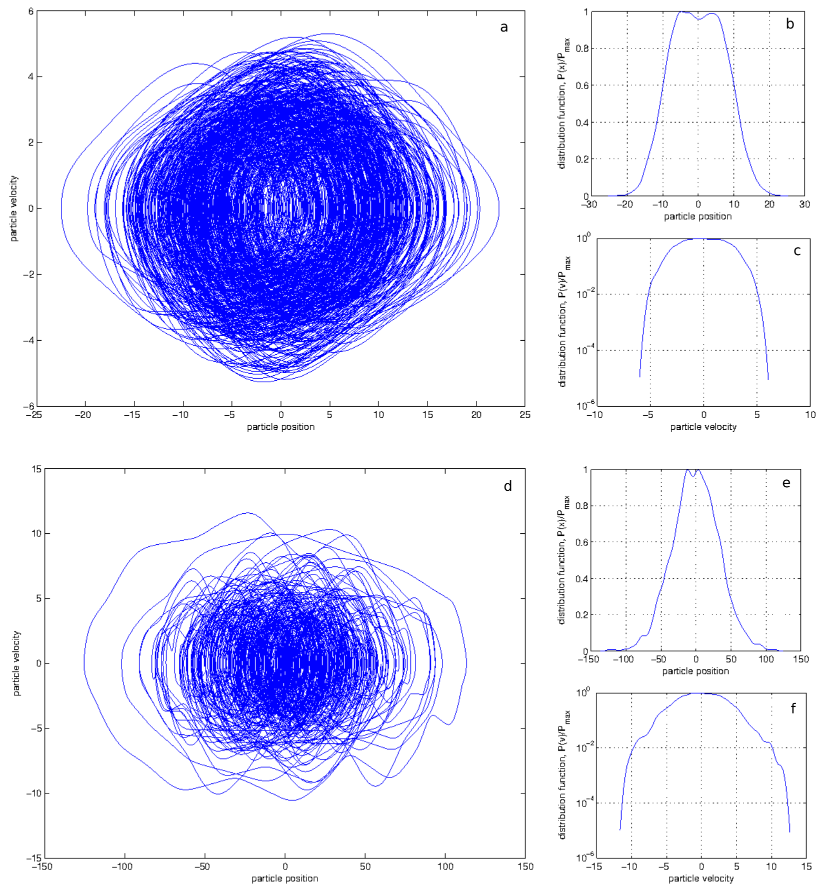

Fig. 4. The phase trajectory projections and phase variables distributions of the middle particle from the chain (2)–(4) for $N = 4$ (figures *a*-*c*) and $N = 15$ (figures *d*-*f*). Parameters used for simulation are $\sigma = 1$, $\sigma_0 = 0$.

The system motion trajectories for $N = 15$ (Fig.4*d*-*e*) are of even greater irregularity due to the increased number of particles and corresponding cooperative effect. For larger N the system motion exhibits the patterns of similar structure, but the amplitude of the fluctuations increases with N).

4 Conclusion

In the present paper we discuss the new type of the dynamical trap – a model describing human bounded rationality. The standard "coordinate-velocity" phase space inherited from the Newtonian mechanics is proposed to be extended by the acceleration variable. By analyzing the behavior of the motivated particles chain governed by bounded rationality we demonstrate that the multi-particle system under the presence of the dynamical trap of a new type exhibits intrinsic cooperative behavior. The various complex patterns of the system motion are shown to arise depending on the system parameters. First, it is demonstrated that the dynamical trap effect of a new type can cause the instability in the single oscillator dynamics which was not observed in the previous studies on the dynamical traps model. Second, the system dynamics patterns are shown to take the complex 3-dimensional structure in case of three-particle ensemble. Third, we demonstrate that with the increasing number of elements the system motion becomes significantly irregular, for large N exhibiting chaotic oscillations. The obtained results confirm that the system under consideration could exhibit anomalous behavior; however, the proposed model require more detailed analysis.

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