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Stochastic modeling of hydraulic operating parameters in pipeline systems

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Introduction

The problems of calculating hydraulic operating parameters are the basic problems in the analysis of operating conditions of pipeline systems when designed, operated, and controlled. These problems are traditionally solved using models and methods, which, however, do not allow us to quantitatively assess the satisfiability of operating conditions when consumption is random, which is typical of many practical situations. This is explained by high complexity and dimensionality of pipeline systems (heat-, water-, gas supply systems, etc.) as modeling objects, excessive efforts necessary to apply general methods of stochastic modeling (such as the Monte-Carlo method), and difficulties in obtaining initial statistical data.

The paper presents an approach, a set of mathematical models and methods for modeling the operating parameters of pipeline systems that were developed in terms of stochastics and dynamics of consumption processes and the established rules of their control, which make it possible to rationally combine the adequacy of modeling and its high computational efforts [1, 2].

Problem statement of the probabilistic calculation of hydraulic operating parameters. Probabilistic description of definite hydraulic operating parameters is reduced to the probability density function, which is denoted here by $p(R,\phi_R)$, where R – the value of a random vector of operating parameters (pressure, flow rate, etc.); ϕ_R – distribution parameters. Most of the practical cases allow us to use the hypothesis about normal distribution of R. Then $\phi_R = \{\bar{R}, C_R\}$ and the probabilistic description of hydraulic operating parameters can be reduced to the specification of values of mathematical expectation ($\bar{R}$) and covariance matrix (C_R) for value *R*.

Not every combination of R components is acceptable, since they

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should satisfy the equations of flow distribution model $U(R)=0$ (where U – non-linear vector function). These equations result from general physical conservation laws, and hence should be solved deterministically.

The traditional deterministic model of steady hydraulic operating parameters in a pipeline system as a hydraulic circuit with lumped parameters can be represented as [3]

$$U(R)=U(X,Y)=U(x,Q,P,\alpha)=\begin{pmatrix} Ax-Q \\ A^{\mathrm{T}}P-f(x,\alpha) \end{pmatrix}=0. \qquad (1)$$

Here the first subsystem of equations represents the conditions of material balance at the nodes of hydraulic circuit (equations of the first Kirchhoff law); the second subsystem – the equations of the second Kirchhoff law; X – boundary conditions; Y – unknown operating parameters; T – transposition sign; A – $m \times n$ - incidence matrix with elements $a_{ji}=1(-1)$, if node j is the initial (end) node for branch i, $a_{ji}=0$, if branch i is not incident to node j; *m, n* – number of nodes and branches of the hydraulic circuit; x – *n*-dimensional vector of flow rate in branches, Q,P – *m*-dimensional vectors of nodal pressures and flow rates, $f(x,\alpha)$ – *n*-dimensional vector-function with components $f_i(x_i,\alpha_i)$, reflecting the laws of hydraulic flow for the branches; α – n_α -dimensional vector of parameters of these characteristics. For instance, if $f_i(x_i,\alpha_i)=s_i x_i\,|\,x_i\,|-H_i$, then $\alpha_i=\{s_i,H_i\}$, where x_i – flow rate in the *i*-th branch; s_i – hydraulic resistance of the branch; $H_i>0$ – increase in pressure in the case of an active branch (e.g. a branch representing a pumping station); $H_i=0$ in the case of a passive branch (e.g. a branch representing a pipeline section). If in (1) all parameters s_i, H_i, $i=\overline{1,n}$ are set deterministically, then $R=(x^{\mathrm{T}},\bar{Q}^{\mathrm{T}},\bar{P}^{\mathrm{T}})^{\mathrm{T}}$.

Thus, the probabilistic model of steady flow distribution can be represented as $U(R)=0,\quad R \sim N_r(\bar{R},C_R)$, where N_r – *r* – dimensional normal probability distribution; r – dimensional of vector *R*. In the case of normal distribution of X, if we neglect the non-linear distortion of distribution $p[Y(X),\phi_{Y(X)}]$ (where $Y(X)$ – implicit function given by the flow distribution equations), the problem can be reduced to the determination of $\varphi_R=\{\bar{R},C_R\}$ with the given function $\varphi_X=\{\bar{X},C_X\}$ and under condition $U(R)=U(X,Y)=0$. Moreover, the composition of X should provide solvability of equations $U(X,Y)=0$ with respect to Y, i.e. $\dim(Y)=\dim(U)=\operatorname{rank}(\partial U/\partial Y)$, where $\partial U/\partial Y$ – Jacobian matrix (of partial derivatives) under fixed boundary conditions X^* in the neighborhood of the solution point Y^*, $\dim(\cdot)$ – vector dimensional, $\operatorname{rank}(\cdot)$ – matrix rank.

Methodological approach. Let $\xi_X = (X - \bar{X})$ be a random deviation of possible realization of boundary conditions from its mathematical expectation $\bar{X}$. After linearizing function $Y(X)$ in the neighborhood of $\bar{X}$, we obtain $Y \approx Y(\bar{X}) + (\partial Y / \partial X)\xi_X$, where $\partial Y / \partial X$ is derivative matrix at point $\bar{X}$. Since $E(Y) = \bar{Y}$ and $E(\xi_X) = 0$, where E is the operation of mathematical expectation, then $\bar{Y} = Y(\bar{X})$. Thus, the mathematical expectation of unknown operating parameters ($\bar{Y}$) is the function of flow distribution equations under boundary conditions $\bar{X}$. Correspondingly,

$$\bar{R} = \begin{pmatrix} \bar{X} \\ \bar{Y} \end{pmatrix} = \begin{pmatrix} \bar{X} \\ Y(\bar{X}) \end{pmatrix} \tag{2}$$

and $$C_R = E\left[\begin{pmatrix} \xi_X \\ \xi_Y \end{pmatrix}\begin{pmatrix} \xi_X \\ \xi_Y \end{pmatrix}^{\mathrm{T}}\right] = \left[\begin{array}{c:c} C_X & C_{XY} \\ \hdashline C_{YX} & C_Y \end{array}\right],$$

where $$C_Y = E\left[\xi_Y \xi_Y^{\mathrm{T}}\right] \approx E\left[\frac{\partial Y}{\partial X}\xi_X \xi_X^{\mathrm{T}}\left(\frac{\partial Y}{\partial X}\right)^{\mathrm{T}}\right] = \frac{\partial Y}{\partial X} C_X \left(\frac{\partial Y}{\partial X}\right)^{\mathrm{T}},$$

$$C_{XY} = C_{YX}^{\mathrm{T}} = E(\xi_X \xi_Y^{\mathrm{T}}) = E\left(\xi_X \xi_X^{\mathrm{T}}\left(\frac{\partial Y}{\partial X}\right)^{\mathrm{T}}\right) = C_X \left(\frac{\partial Y}{\partial X}\right)^{\mathrm{T}},\ \xi_Y = (Y - \bar{Y}).$$

Thus, *the general scheme for solving the problem of probabilistic calculation of hydraulic parameters* is reduced to the following: 1) to obtain vector $\bar{Y}$ by traditional methods for calculating the flow distribution with the given $\bar{X}$; 2) to determine matrix C_R, whose individual blocks are determined using the known matrix C_X and derivative matrix $\partial Y / \partial X$ at point $\bar{X}$.

Here two main questions arise: 1) based on what do we set the distribution parameters of boundary conditions ($\varphi_X = \{\bar{X}, C_X\}$); 2) what is the final form of relationships for the resultant covariance matrices in different variants of the division of R into X and Y, since in the traditional methods for the flow distribution calculation the derivatives $\partial Y / \partial X$ are not calculated in explicit form, which represents a separate problem.

Probabilistic description of consumer loads. A typical example of pipeline systems operating under the conditions of stochastic consumer loads is water supply systems. The approach applied to the probabilistic description of these stochastic conditions is based on the use of the queuing theory methods and on results of the studies [4, 5, etc.], which found their reflection in the regulatory documents [6]. According to these results, the probability of using plumbing units (p_{hr}) can be described by "Erlang formulas", which demonstrate a discrete limit distribution of used channels, depending on the characteristics of the flow of requests and the performance of the queuing system.

The suggested technique for calculating the mathematical expectation of consumer flow rates ($\overline{q}_{hr}$) and their variances ($\sigma^2_{q,hr}$) consist in the following:

1. Knowing the number of plumbing units at the consumption node (N) and the probability of using them p_{hr} [6], we can calculate $m = \overline{m}_{hr}$ such that maximum value ($p_{\max}(m)$) acquires the probability

$$p(m) = \left(\frac{\left(N\, p_{hr}\right)^m}{m!} \right) \Big/ Z \ , \ m = 0,1,...,N \ , \qquad (3)$$

where $Z = \sum_{k=0}^{N} \frac{\left(N\, p_{hr}\right)^k}{k!}$, m is the number of simultaneously used plumbing units; Np_{hr} is their usage rate.

2. We should determine the average hourly flow rate $\overline{q}_{hr} = \overline{m}_{hr}\, q_{0,h}$, where $q_{0,h} = q_{0,hr} / 1000$ – hourly water flow rate by one device, m^3/h; $\overline{q}_{hr}$ – can be interpreted as the mathematical expectation of flow rate at the consumption node; $q_{0,hr}$ – standardized value, l/h.

3. When approximating the discrete Erlang distribution by the continuous normal distribution, we should calculate the equivalent variance by formula $\sigma^2_{m,hr} = 1/2\pi\, p^2_{\max}(m)$.

4. The variance of the average hourly flow rate will be determined as $\sigma^2_{q,hr} = q^2_{0,h}\ \sigma^2_{m,hr}$.

Figure 1 presents a diagram of function (3), where N=270 and p_{hr} =0.023. The diagram shows that the maximum probability density function corresponds to $\overline{m}_{hr}$, whose average hourly flow rate is $\overline{q}_{hr}$.

General scheme of obtaining the covariance matrix consists of three stages: 1) to linearize system (1) at point $\overline{X}$; 2) to reduce linearized system $\frac{\partial U}{\partial R}\xi_R = 0$ to $\xi_Y = \frac{\partial Y}{\partial X}\xi_X$; 3) to obtain covariance matrix of the vector of unknown operating parameters C_R using the operation $E\left[\begin{pmatrix}\xi_X \\ \xi_Y\end{pmatrix}\begin{pmatrix}\xi_X \\ \xi_Y\end{pmatrix}^{\mathrm{T}}\right]$.

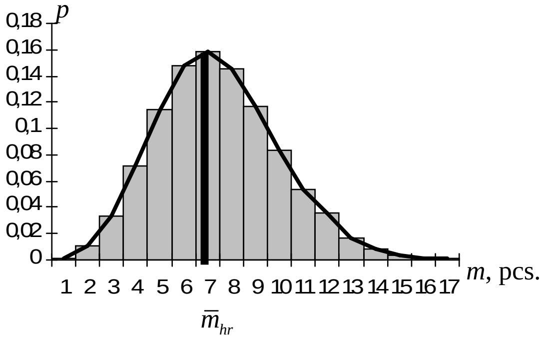

Fig. 1. Continuous approximation of Erlang distribution for the probability of simultaneously used devices for the case where N =270 and p_{hr} =0.023.

Thus, for the case, where $X = Q$, $Y = \begin{pmatrix} x \\ P \end{pmatrix}$, $P_m = \text{const}$, $\alpha = \text{const}$:

$$\frac{\partial U}{\partial R} = \left[\begin{array}{c|c} A & 0 \\ \hline f' & A^{\mathrm{T}} \end{array}\right]; \begin{pmatrix} \xi_x \\ \xi_P \end{pmatrix} = \left[\begin{array}{c} (f_x')^{-1} A^{\mathrm{T}} M^{-1} \\ \hline M^{-1} \end{array}\right] \xi_Q;$$

$$C_R = \left[\begin{array}{c|c|c} C_Q & C_{Qx} & C_{QP} \\ \hline C_{xQ} & C_x & C_{xP} \\ \hline C_{PQ} & C_{Px} & C_P \end{array}\right] =$$

$$= \left[\begin{array}{c|c|c} C_Q & C_Q M^{-1} A (f_x')^{-1} & C_Q M^{-1} \\ \hline (f_x')^{-1} A^{\mathrm{T}} M^{-1} C_Q & (f_x')^{-1} A^{\mathrm{T}} M^{-1} C_Q M^{-1} A (f_x')^{-1} & (f_x')^{-1} A^{\mathrm{T}} M^{-1} C_Q M^{-1} \\ \hline M^{-1} C_Q & M^{-1} C_Q M^{-1} A (f_x')^{-1} & M^{-1} C_Q M^{-1} \end{array}\right],$$

where f_x' – diagonal matrix with elements $\partial f_i(x_i, \alpha_i) / \partial x_i$; C_Q – known covariance matrix of nodal flow rate; C_P, C_x – covariance matrix of nodal pressure and covariance matrix of flow rate in branches; $C_{Qx} = C_{xQ}^{\mathrm{T}}$ – covariance matrix of nodal flow rate and flow rate in branches; $C_{PQ} = C_{QP}^{\mathrm{T}}$ – covariance matrix of nodal pressure and flow rate; $C_{Px} = C_{xP}^{\mathrm{T}}$ – covariance matrix of nodal pressure and flow rate in branches. Thus, knowing $C_X = C_Q$, we can calculate C_R. No special requirements are imposed on matrix C_Q, however, in practice it is usually taken as a diagonal matrix from considerations of statistical independence of consumer loads. This means that $\mathrm{cov}(Q_j, Q_t) = \sigma^2_{Q_j}$ for $j = t$, and $\mathrm{cov}(Q_j, Q_t) = 0$ for $j \neq t$.

Covariance matrix for the general case of setting boundary conditions $X = (Q_X^{\mathrm{T}}, P_X^{\mathrm{T}}, \alpha_X^{\mathrm{T}})^{\mathrm{T}}$, where at each node we can set either the flow rate or the pressure, and each branch is characterized by $n_{\alpha,i}$-dimensional vector (e.g. $\alpha_i = \{s_i, H_i\}$, $n_{\alpha,i} = 2$) of hydraulic parameters, which is specified in the probabilistic form in full or partially [1, 2].

Divide the set of nodes in the design scheme into subsets of nodes with the given flow rate (J_Q) and pressure (J_P), and the set of branches into subsets of branches with hydraulic parameters given in the probabilistic (I_V) and deterministic (I_D) forms. We omit the conclusion and give the finite expressions for the covariance matrix of unknown operating parameters:

1) Covariance matrix of unknown nodal pressure

$$C_{PY} = \mathrm{E}\left[\xi_{PY},\ \xi_{PY}^{\mathrm{T}}\right] = \frac{\partial P_Y}{\partial Q_X} A_{QV} \frac{\partial x_V}{\partial \alpha_V} C_{\alpha V} \frac{\partial x_V}{\partial \alpha_V} A_{QV}^{\mathrm{T}} \left(\frac{\partial P_Y}{\partial Q_X}\right)^{\mathrm{T}} +$$

$$+ \frac{\partial P_Y}{\partial Q_X} C_{QX} \left(\frac{\partial P_Y}{\partial Q_X}\right)^{\mathrm{T}} + \frac{\partial P_Y}{\partial P_X} C_{PX} \left(\frac{\partial P_Y}{\partial P_X}\right)^{\mathrm{T}};$$

2) Covariance matrix of flow rate in the branches with deterministically specified characteristics

$$C_{x,D} = \mathrm{E}\left[\xi_{x,D}, \xi_{x,D}^{\mathrm{T}}\right] = \frac{\partial x_D}{\partial P_Y} C_{PY} \left(\frac{\partial x_D}{\partial P_Y}\right)^{\mathrm{T}} + \frac{\partial x_D}{\partial P_X} C_{PX} \left(\frac{\partial x_D}{\partial P_X}\right)^{\mathrm{T}};$$

3) Covariance matrix of flow rate in the branches with probabilistically specified characteristics

$$C_{xV} = \mathrm{E}\left[\xi_{XV}, \xi_{XV}^{\mathrm{T}}\right] = \frac{\partial x_V}{\partial P_Y} C_{PY} \left(\frac{\partial x_V}{\partial P_Y}\right)^{\mathrm{T}} +$$

$$+ \frac{\partial x_V}{\partial P_X} C_{PX} \left(\frac{\partial x_V}{\partial P_X}\right)^{\mathrm{T}} + \frac{\partial x_V}{\partial \alpha_V} C_{\alpha V} \left(\frac{\partial x_V}{\partial \alpha_V}\right)^{\mathrm{T}};$$

4) Covariance matrix of unknown nodal flow rates

$$C_{QY} \equiv \mathrm{E}\left[\xi_{QY}, \xi_{QY}^{\mathrm{T}}\right] = A_{PD} C_{xD} A_{PD}^{\mathrm{T}} + A_{PV} C_{xV} A_{PV}^{\mathrm{T}},$$

where A_{QD} – $(m_Q \times n_D)$-dimensional incidence matrix with elements a_{ji}, $j \in J_Q$, $i \in I_D$; A_{QV} – $(m_Q \times n_V)$- dimensional incidence matrix with elements a_{ji}, $j \in J_Q, i \in I_V$; A_{PD} – $(m_P \times n_D)$-dimensional incidence matrix with elements a_{ji}, $j \in J_P$, $i \in I_D$; A_{PV} – $(m_P \times n_V)$-dimensional incidence matrix with elements a_{ji}, $j \in J_P$, $i \in I_V$; $\frac{\partial x_D}{\partial P_Y} = \left(\frac{\partial f_{xD}}{\partial x_D}\right)^{-1} A_{QD}^{\mathrm{T}}$, $\frac{\partial x_D}{\partial P_X} = \left(\frac{\partial f_{xD}}{\partial x_D}\right)^{-1} A_{PD}^{\mathrm{T}}$,

$\frac{\partial x_V}{\partial P_Y} = \left(\frac{\partial f_{xV}}{\partial x_V}\right)^{-1} A_{QV}^{\mathrm{T}}$, $\frac{\partial x_V}{\partial P_X} = \left(\frac{\partial f_{xV}}{\partial x_V}\right)^{-1} A_{PV}^{\mathrm{T}}$, $\frac{\partial x_V}{\partial \alpha_V} = \left(\frac{\partial f_{xV}}{\partial x_V}\right)^{-1} \frac{\partial f_{xV}}{\partial \alpha_V}$ – matrices of partial derivatives of the corresponding combinations of parameters, which implicitly depend on three matrices only: $\frac{\partial f_{xD}}{\partial x_D}$, $\frac{\partial f_{xV}}{\partial x_V}$ and $\frac{\partial f_{xV}}{\partial \alpha_V}$, whose structure is determined by the type of branch characteristics. Moreover, the first two of them are diagonal, and therefore, easily invertible.

Thus, based on the given relations, we can sequentially calculate the covariance matrices of all the operating parameters, if we know the covariance matrices of nodal flow rate set in the probabilistic form (C_{QX}), nodal pressure (C_{PX}), and hydraulic characteristics of branches ($C_{\alpha V}$).

Probabilistic calculation of dynamics of hydraulic operating parameters. Stochastic boundary conditions initiate the change in hydraulic operating parameters with time. As a result we face the problem of probabilistic modeling and analysis of operating parameter dynamics $R(t)$, $0 \le t \le T$ as a random process for the calculation period T.

Figure 2 presents the graphs of realization-frequency distribution of two hydraulic operating parameters (the nodal flow rate and the nodal pressure). The first parameter can be considered as a disturbance, the second – as a response. Figure 2a shows the graph of water flow rate frequencies for an

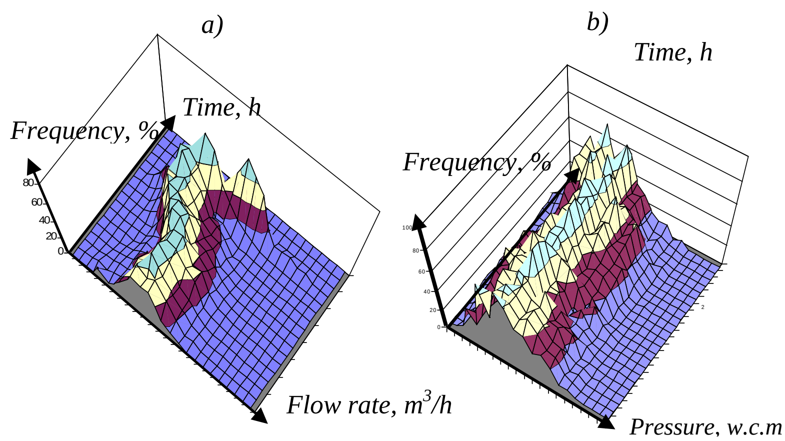

Fig. 2. Daily change in the frequency distribution of hydraulic operating parameters

a) For the nodal flow rate, b) For the nodal pressure individual residential building in the water supply system that is constructed based on the experimental data. Figure 2b shows the graph of pressure frequencies at the connection node of the reservoir in the water supply system in one of the Irkutsk districts that is obtained by processing the data of the dispatching department for 490 days.

Analysis of both processes in Fig. 2 indicates that: 1) the frequency distribution at any cross-section of both processes is approximated by the normal (Gaussian) distribution satisfactorily enough; 2) the variance of every process (σ^2) is practically invariable. The root-mean-square deviation (σ) for daily water flow rate changes negligibly, i.e. within 10 per cent (Table 1), for pressure – within 7 per cent; 3) the mathematical expectation for both processes changes during a day (Fig. 3a); 4) the autocorrelation function stabilizes at the zero value (for the nodal value in Fig. 3b) fast enough.

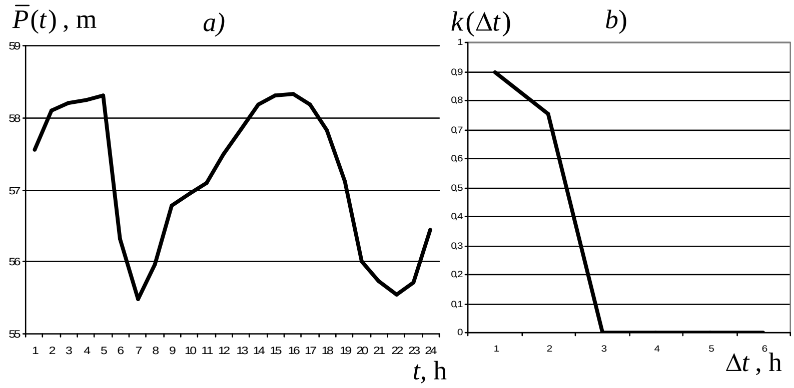

Fig. 3. Statistical characteristics of change in the nodal pressure as a random process
a) Dynamics of mathematical expectation;
b) Graph of the autocorrelation function of pressure in the reservoir

The hydraulic operating parameters vary in time in response to three main disturbing actions (boundary conditions): 1) random actions of regular character (consumer loads); 2) deterministic actions of regular character (control actions); 3) random actions of irregular character (fires, accidents). The second type of disturbances is taken into account algorithmically on the basis of the specified control rules. Analysis of the consequences of relatively rare disturbances of the third type is the subject of the reliability theory of pipeline systems and is not carried out here.

Table 1. Values of mathematical expectations and root-mean-square deviations of the nodal flow rate during day hours for the conditions in Fig. 2a.

Day hour	$\bar{Q}$, m³/h	σ	$\frac{\sigma}{\bar{\sigma}}100\%$	Day hour	$\bar{Q}$, m³/h	σ	$\frac{\sigma}{\bar{\sigma}}100\%$
1	4.60	2.14	3.11	13	11.97	2.25	8,41.
2	2.32	1.94	6.52	14	11.77	2.2	6.00
3	1.88	1.99	4.12	15	11.28	2.15	3.59
4	1.66	1.94	6.52	16	11.16	2.07	0.26
5	1.87	1.84	7.97	17	11.53	2.0	3.63
6	3.28	2.2	6.00	18	12.32	2.09	0.70
7	7.88	2.02	2.67	19	12.35	2.17	4.56
8	10.80	2.08	0.22	20	13.34	2.05	1.22
9	10.88	1.96	5.56	21	13.68	2.04	1.71
10	12.40	2.26	8.89	22	14.34	2.02	2.67
11	12.48	2.02	2.67	23	12.51	1.85	10.86
12	12.13	2.28	9.86	24	9.10	2.18	5.04

Dynamics of hydraulic operating parameters $R(t),\ 0 \le t \le T$ may be considered as a random process with the discrete time (a quasidynamic approach). At each time instant of the process the operating parameters obey the normal distribution. Variation of the operating parameters at the adjacent instants may be considered as insignificant and the flow distribution – as steady. Thus, the problem of probabilistic calculation of hydraulic operating parameter dynamics is reduced to the determination of $\bar{\mathbf{R}} = [\bar{R}(0)^{\mathrm{T}}, \bar{R}(1)^{\mathrm{T}}, ..., \bar{R}(T)^{\mathrm{T}}]^{\mathrm{T}}$ and $\mathbf{C_R} = E\left[\xi_R \xi_R^{\mathrm{T}}\right]$ based on the specified parameters $\bar{\mathbf{X}} = [\bar{X}(0)^{\mathrm{T}}, \bar{X}(1)^{\mathrm{T}}, ..., \bar{X}(T)^{\mathrm{T}}]^{\mathrm{T}}$, $\mathbf{C_X}$ and the conditions $A(t)x(t) = Q(t,P)$, $\bar{A}(t)^{\mathrm{T}} \bar{P}^{\mathrm{T}}(t) = y(t)$, $y(t) = f\left(x(t), \alpha(t)\right)$, $t = 0,...,T$. In this case the suggested analytical probabilistic models and the calculation methods can be applied to each calculation instant, which will sharply decrease computational efforts. The computing experiments in Table 2 have shown the decrease in running time by tens of times.

Table 2. Time required for probabilistic calculation by the Monte Carlo and analytical methods [7]

Number of scheme nodes and branches	Time for method		$t_{M\text{-}C} / t_{Analyt.}$
	Analytical	Monte Carlo	
6 nodes and 8 branches	3.2 s	3 min	56.25
12 nodes and 19 branches	4.8 s	28.5 min	356.25
12 nodes and 29 branches	16.2 s	1.25 h	277.77

In some cases such as availability of reservoirs it is important to take account of the lagging factor of internal responses of pipeline systems, when the successive operating condition depends on the prehistory of conditions. Availability of reservoirs can be taken into account by using the additional dynamic relation $P_{j,k} = P_{j,k-1} + \rho g(\Delta t / F_j) Q_{k,j}$, where Δt – duration of the k-th condition; F_j – liquid surface area in the reservoir; j – index of the node with a reservoir; g – gravitational acceleration; ρ – liquid density. The reservoir operation can be modeled by insertion of a dummy branch connected to a dummy node with zero (or air) pressure. The hydraulic characteristic of such a branch has the form: $y_{i,k} = s_{i,k} x_{i,k} - H_{i,k}$, where $H_{i,k} = P_{j,k-1}$, $s_{i,k} = \rho g \Delta t / F_j$. Let H_k^f be a vector of dummy pressure rises in the branches that represent all the reservoirs. The covariance matrix of vector H_k^f that is used at the k-th calculation step will have the form: $C_{H_f}(t_k) = C^*_{PY}(t_{k-1})$, where $C^*_{PY}(t_{k-1})$ – block of covariance matrix C_{PY} that was calculated at the previous step and is attributed to the pressures at the nodes with reservoirs.

Calculation of probabilistic operating parameters of pipeline systems. The suggested approach to the calculation of statistical parameters of pipeline system operation offers an opportunity to obtain probabilistic estimates of virtually any operating parameters of pipeline systems depending on their operating conditions by the known formulas of the probability theory. For example the probability that any "nondegenerate" subset of operating parameters belongs to a given range at the time t_k will be determined by the formula

$$p_{Rk} = \frac{1}{\sqrt{(2\pi)^n \left| C_{Rk} \right|}} \int_{\underline{v}_n}^{\overline{v}_n} \ldots \int_{\underline{v}_1}^{\overline{v}_1} \exp\left\{ -\frac{1}{2} \left(R_k - \overline{R}_k \right)^{\mathrm{T}} \mathrm{C}_{Rk}^{-1} \left(R_k - \overline{R}_k \right) \right\} dR_1 \ldots dR_n, \quad (4)$$

where R_k – n-dimensional vector (subvector) of operating parameter values at the time instant t_k; $\overline{R}_k$ – n-dimensional vector of mathematical expectation R_k; C_{Rk} – ($n \times n$)-dimensional covariance matrix for R_k; p_{Rk} – probability that R_k belongs to a specified range $[\overline{v},\ \underline{v}]$; $\overline{v} = [\overline{v}_1,...,\overline{v}_n]^{\mathrm{T}}$ and $\underline{v} = [\underline{v}_1,...,\underline{v}_n]^{\mathrm{T}}$ – vectors of upper and lower boundaries of the studied range, whose components can take infinite values to take account of one-sided intervals or their absence.

The assessment of probability that R_k belongs to a specified range $[\overline{v}_r,\ \underline{v}_r]$ during period T will be determined by the formula

$$p_{RT} = \sum_{k=1}^{K}\left(p_{Rk}\Delta t_k\right)\Big/\sum_{k=1}^{K}\Delta t_k = \sum_{k=1}^{K}\left(p_{Rk}\Delta t_k\right)\Big/T\ , \qquad (5)$$

where K – the number of calculated periods over period $T = \sum_{k=1}^{K}\Delta t_k$; Δt_k – duration of the k -th condition.

Equations (4) and (5) can be applied to estimate the operation of pipeline system, its fragments or individual components in a definite operating condition or over the period of time, for example in terms of the extent to which they are loaded, consumer demand is satisfied, or process constraints are met, etc.

Numerical example

Let us consider a numerical example of calculating the stochastics of the hydraulic operating parameters for the network presented in Fig.4. The network consists of 7 nodes and 11 branches of which: one node has a fixed pressure; two nodes have lumped loads; two nodes are nonfixed loads depending on pressure; one branch represents a pumping station with an increasing head H_0=21 m; one dummy branch simulates a reservoir (water level in the reservoir H^f=16.4 m); two dummy branches simulate nonfixed loads, their resistances are random values. Thus, this example illustrates the possibilities of the suggested approach in terms of the random composition of boundary conditions.

The input information specified in the probabilistic form is: $X = (Q_X^{\mathrm{T}}, P_X^{\mathrm{T}}, \alpha_X^{\mathrm{T}})^{\mathrm{T}} = (\overline{Q}_4, \overline{Q}_5, \overline{P}_7, \overline{s}_9, \overline{s}_{10})$ = (5.2, 1.8, 0, 0.30359, 1.2407); C_X – a diagonal matrix with nonzero elements (1.065, 0.3969, 0.0001, 0,059, 0.51564). Resistances in the dummy branches 9 and 10, that simulate nonfixed flow rates at consumers are determined by the formula [1, 2] $\overline{s}_i = P_j^r/(Q_j^r)^2$, and variances – $\sigma_{s,i}^2 = \left(4(P_j^r)^2/(Q_j^r)^6\right)\sigma_{Qr,j}^2$, where P_j^r, Q_j^r – design (required) pressures and flow rates for this consumer, j – index of the initial node of the i-th branch. Correspondingly in the example $Q_2^r = 7.7$, $\sigma_2^2 = 9.61$, $P_2^r = 18$, $Q_3^r = 7.11$, $\sigma_3^2 = 0.81$, $P_3^r = 12$.

Resistances in the branches that were specified deterministically are: $s_1 = 0.00257$, $s_2 = 0.8996$, $s_3 = 0.00408$, $s_4 = 0.095$, $s_5 = 0.67$, $s_6 = 0.067$, $s_7 = 0.0957$, $s_8 = 0.00646$, $s_{11} = 0.014$.

The calculation results for nodes are presented in Table 3 and for branches – in Table 4.

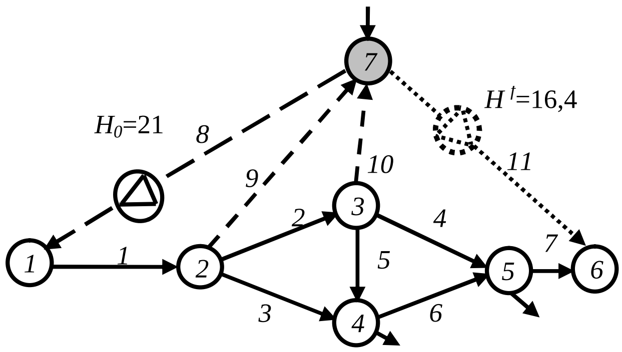

Fig.4. Example of the calculated scheme of the pipeline system for the general case of boundary conditions

Real section; dummy branch simulating nonfixed consumer loads; dummy branch simulating reservoir; dummy branch simulating pumping station; 4 node with the specified nodal loads; 7 node with the specified pressure.

Table 3. Calculation results for nodes

j	Parameters			
	P_j, *Mwc*	$\sigma_{P,j}$	Q_j, m^3/h	$\sigma_{Q,j}$
1	18.22	0.89	–	–
2	17.11	1.25	9.19	4.03
3	14.96	1.21	6.48	1.08
4	16.70	1.25	–	–
5	16.01	0.83	–	–
6	16.37	0.02	–	–
7	–	–	22.67	9.07

Table 4. Calculation results for branches

i	Parameters	
	x_i, m^3/h	$\sigma^2_{x,i}$
1	20.75	3.29
2	1.54	0.02
3	10.01	0.03
4	-3.32	0.59
5	-1.61	0.02
6	3.20	0.98
7	-1.92	2.21
8	20.75	3.29
9	9.19	4.03
10	6.48	1.08
11	1.82	0.66

Figure 5 presents a graphical interpretation of the calculated probability of providing consumers with a required flow rate. For example for the consumer at the second node $p(0<\overline{Q}_2<Q_2^r)\approx 0.3442$ or $p(Q_2^r<\overline{Q}_2<+\infty)\approx 0.64446$, and at the third node $p(0<\overline{Q}_3<Q_3^r)\approx 0.71914$ or $p(Q_3^r<\overline{Q}_3<+\infty)\approx 0.28083$, where Q^r is the required flow rate.

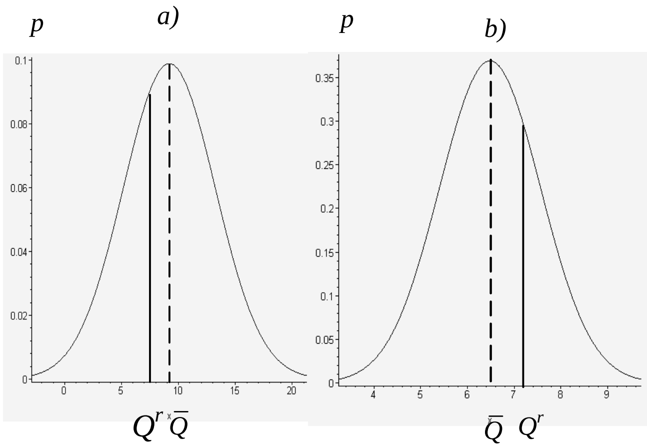

Figure 5. Illustration to the calculation of probability of providing consumer with a required flow rate: a) at node 2, b) at node 3.

$\bar{Q}$ – Calculated value of mathematical expectation of consumer flow rate considering its dependence on nodal pressure, Q^r– required value of consumer flow rate.

Conclusions

1. The paper presents:

- a technique for apriori calculation of statistical characteristics of a probabilistic process of the transported medium consumption as a queuing process;
- a general scheme for probabilistic calculation of pipeline system hydraulic operating parameters. The calculation suggests determining statistical characteristics of the operating parameters by specified characteristics of boundary conditions and flow distribution model. It is shown that such a calculation is reduced to solving a traditional problem of flow distribution at the point of mathematical expectation of boundary conditions in combination with an additional procedure for calculating covariance matrices of operating parameters;
- a technique for obtaining the analytical expressions for covariance matrices of operating parameters as well as the expressions for the general case of specifying boundary conditions;
- a technique for probabilistic modeling of changes in the hydraulic operating parameters on the basis of developed analytical probabilistic flow distribution models. This technique provides a considerable reduction in computational efforts against the known methods of simulation modeling.

2. The suggested technique for modeling pipeline systems provides the possibility of obtaining probabilistic estimates of practically any pipeline system operating parameters that depend on operating conditions.

3. A numerical example of probabilistic calculation of the steady flow distribution in the pipeline system is given for the general case of boundary conditions. The example illustrates the suggested probabilistic approach.

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