Mathematical Modeling on the Exponential Changed Plasma Quantities leads to the more Persuasive Answers.

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Abstract. It is admissible that most of the plasma literature is concerned with the plasma instabilities and the inevitable plasma waves, which remain standard obstacles to the thermonuclear fusion process. Many experimental data on the plasma waves (growth or damping) and their accompanied theoretical interpretations have been published during the last five decades; lots of them have been identified and justified as well, some not yet. Among them our previous research on the plasma waves is included, which originates in the early 80's at the Plasma Physics Laboratory of the NCSR ‘’ Demokritos’’. As the wave rising is defined by the growth rate (or the damping on the extinguishment), these important wavy quantities have been studied in detail in the present paper. Three examples have been used from our previous theoretical results, and the first observation reveals that the involved quantities are complicated enough to be studied themselves. So, the use of suitable approaching models, which may interpret the experimental wavy quantities, is the central idea of the present attempt. Furthermore, calculations with a little change of the initial conditions have been repeated, to determine that the plasma behaves as a chaotic medium.

1. Introduction

It is common experience that the plasma wave growth rate or damping has almost always a complicated form [1-3], as the involved physical quantities are multi-parametric and very hard to be considered as separate, and also influence one another through the feed-back process [4,5]. Such plasma waves have been observed in the early 60's [6-9] and their growth has been studied as well; as time passed their wavy properties have been studied extensively and the plasma waves have been recognized and classified as electrostatic waves [10,11], drift waves [12,13], Alfven waves [14], short-wavelength electron plasma waves [15], long-wavelength waves [10-14, 16], ion-sound waves [17], e.t.c. All the above mentioned cases have been researched and their results have been

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carried out by considering and finding an exponential change on time of the plasma quantities (plasma density, plasma potential, ions and electrons velocity e.t.c.). If the thoughts are extended in the other areas of Physics, then we can find many examples with exponential change on the time usually and, sometimes, on the space dimension as well. For the first case the extinguishing of the oscillations by considering a resistance proportional to the vibrated mass velocity [18, 19], the charge and discharge law of the capacitor from a d.c. generator through a resistance and the establishment or interruption of the d.c. current on a wrapper (the known time-circuits), the radioactive conversions Law from the Nuclear Physics [20] are mentioned. Afterwards, for the second case it is enough to mention the absorption law of the radiation from an absorbent material.
The need for an approaching solution of the differential equations for every problem, which describes the change on time or space, is indispensable and the proposed mathematical models have the ambition to alleviate the problem. In the first approach, the solution of such kind of problems is limited in the exponential known forms-functions, where the equalization factor is considered as ‘’constant’’, and this convenient and easy acceptance results in direct deductions. However, the stability of this kind of ‘’constants’’ must be put under scrutiny as some results of this acceptance may rise doubts. So, the equalization factor must not be considered as a constant, but as lightly changeable in different ways.
In the present work three such examples have been given [21, 22], although the purpose of our team is to shape a full list of the models, which may be useful and easy for the experimental and theoretical researchers. So, the completion of the model list is the immediate future work, since the experimental confirmation is the difficult part of the completed research; this difficulty is caused by the little amount of time in the growth establishment, which is very large at the nuclear decays, as the making of the measurements must be methodical.
These examples that are mentioned above were selected from the previous work on the plasma waves, which has been carried out at the our Plasma Laboratory [23-25] and presented as the first involvement with the topic.
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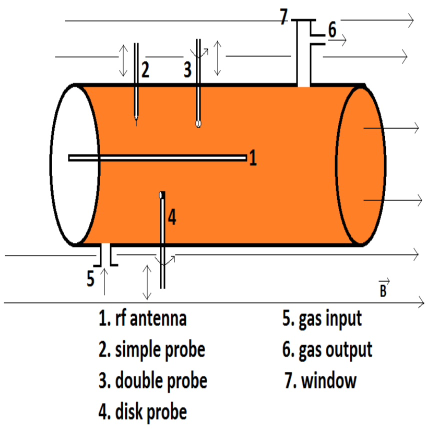
The paper is organized as following: A brief description of our experimental device, the plasma production and the wave appearance is given in Sec.2. In Sec.3 the weakness of the simple radioactivity problem are given in detail. Afterwards, three characteristic models are studied in Sec.4, whereas the discussions and conclusions are made in Sec.5. Finally, in the two Appendix sections more details of the mathematical elaboration are given.

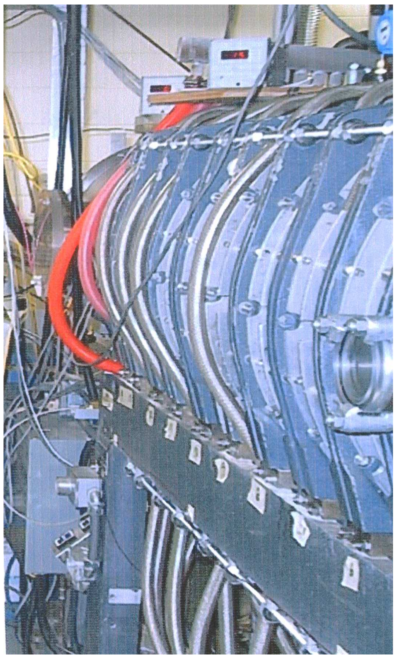
2. **Plasma production – Waves Appearance**

A. Experimental Set-Up Description

A nearly 4m long semi-Q machine has been installed in the Plasma Physics Laboratory of the NCSR ‘Demokritos’ since four decades ago and many studies on the rf produced plasma have been carried out [21-25]. A steady steel cylindrical cavity of 6 cm internal diameter, with its’ length adaptable to any purpose, is used almost always, as it is preferable due to its’ cylindrical symmetry simplicity. The argon-plasma is usually produced due to the argon atoms inertia and its’ low penetration. A d.c. generator supplies the Q-machine with constant current into a wide value region and with high accuracy. So, the produced magnetic field along the cylindrical cavity axis has an inclination from the constant value smaller than 4% if the Q-machine electro-magnets are placed correctly.
A low power Magnetron generator operates at constant value of the signal frequency

($2.45GHz$) and supplies the plasma production with the indispensable energy into a wide region of the external magnetic field values (Table 1).
Electrical probes, disk probes, double probes and probe arrays, which can be moved accordingly or not, provide the possibility of measuring the plasma quantities (plasma density, plasma temperature, plasma potential, plasma wave form, e.t.c.) in every point of the plasma column. Figure 1(a) shows a drawing of the Set-Up for better understanding and Fig.1 (b) presents a photograph of a similar experimental device.





(a) **(b)**

Fig.1 The plasma cavity with probes is presented in (a), whereas a photo of the experimental device is shown in (b).

B. Plasma production-Plasma Waves

By using a combination of a rotary and a diffusion pump (Balzers type) connected with the cylindrical cavity, the argon pressure can be adjusted in order for the plasma to light within a wide region of values. In a previous publication [25], a complete study of the plasma external parameters , such as gas pressure, rf wave power and magnetic field intensity, has been given. In the present paper, the external parameters and the plasma quantities are summarized in Table 1.

Table 1. The plasma parameters and plasma quantities ranging values

Parameters	Minimum value	Maximum value
Argon pressure p	$0.001\,Pa$	$0.1Pa$
Argon number density, n_g	$2\times10^{15}\,m^{-3}$	$2\times10^{17}\,m^{-3}$
Magnetic field intensity, B	$10mT$	$200\,mT$
Microwaves' power, P	$20Watt$	$120Watt$
Frequency of the rf power (standard value)	$2.45GHz$	
Electron density, n_0	$2\times10^{15}\,m^{-3}$	$4.6\times10^{15}\,m^{-3}$
Electron temperature, T_e	$1.5eV$	$10eV$
Ion temperature, T_i	$0.025\,eV$	$0.048\,eV$
Ionization rate	0.1%	90%
Electron - neutral collision frequency, ν_e	$1.2\times10^{7}\,s^{-1}$	$3\times10^{9}\,s^{-1}$

Among the other noteworthy findings of the thus produced plasma, are its' stability, repetition, and the persistently rising low frequency electrostatic waves, many of which have become audible through the suitable conversion. The waves may have wave-vector component along the three axis originally, but, as the steady state is established, standing waves are seeking at the radial and cylinder axis direction, and the waves propagate only azimouthally.

The study of these waves has been done theoretically [21,22,24] by using the fluid mechanics equations and its' dispersion relation, the growth rate and damping have been also found. So, three types of dispersion relations and their growth rate are mentioned here; the first dispersion relation is the following,

$$\omega_l \cong l\Omega_i + \frac{l}{\mathbf{2}}(\Omega_R - \Omega_D) - j\frac{\nu_i}{\mathbf{2}} + j|s|C_s\sqrt{\frac{U_R}{U_D} - \mathbf{1}}$$

with the growth rate,

$$\omega_i = |s|C_s\sqrt{\frac{U_R}{U_D} - 1} - \frac{\nu_i}{2} \qquad \text{(2-1)}$$

where, $\Omega_i, \Omega_R, \Omega_D$ are the angular (circular) velocities for ions due to d.c. potential gradient, the rf field and the plasma density gradient, respectively. In addition, there is $C_s^2 \equiv \dfrac{K_B T_e}{m_i}$ and $s \equiv \dfrac{1}{n_0}.\dfrac{dn_0}{d\rho}$.

Afterwards, the second one is,

$$\omega \cong ku_e + j\nu_e\frac{\omega_{pe}^2}{\omega_{pi}^2}.\frac{k^2(u^2 - C_s^2) - \omega_{ci}^2}{\omega_{ce}^2}$$

with the growth rate,

$$\omega_i = \nu_e\frac{\omega_{pe}^2}{\omega_{pi}^2}.\frac{k^2(u^2 - C_s^2) - \omega_{ci}^2}{\omega_{ce}^2} \qquad \text{(2-2)}$$

where, ν_e are the electron-neutral collisions , $\frac{\omega_{pe}^2}{\omega_{pi}^2} = \frac{m_i}{m_e}$, and $u = |u_i - u_e|$.

The third dispersion relation is,

$$\omega \cong k(u_i + C_s) - j\frac{\nu_i}{2} + j\frac{m_e}{m_i}\frac{\nu_e}{2}\frac{u_e - (u_i + C_s)}{C_s}$$

and the growth rate is expressed as,

$$\omega_i = \frac{m_e}{m_i}\frac{\nu_e}{2}\frac{u_e - (u_i + C_s)}{C_s} - \frac{\nu_i}{2} \qquad (2\text{-}3)$$

The first kind of waves is caused by the radial rf –field gradient [21,23], since the second and third kind are identified as electron-neutral and ion-neutral collisional waves, respectively [22].

Figure 2 shows a wave form and the frequency spectra of two electrical plasma waves; each spectrum consists of the fundamental frequency and its' upper harmonics, in full accordance with the dispersion relations (2-1) and (2-2). Figure 2 (a) is the waveform [21], spectrum (b) for the wave caused by the rf-field radial gradient and spectrum (c) for the collisional wave.

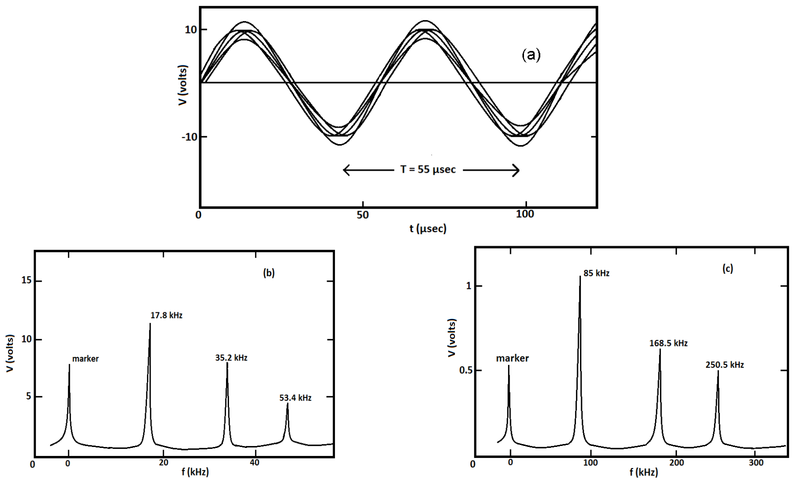


Fig.2. The wave form is shown in (a), whereas the wave spectra are presented in (b) and (c) for rf-drift and collisional wave, respectively.

C. Experimental Data

Although many phenomena appear on the plasma waves, most of which have been presented in the previous publications [21-25], in the present paper only the influence of the gas pressure on the wavy frequency and amplitude is mentioned; this is considered to be enough for the first fitting between an experimental given fact and a suitable model.

The indispensable measurements were taken using an electrical probe placed in the middle of the cylinder radius and the argon was lighted in the following values of the external plasma parameters; magnetic field intensity B=72 mT and microwave power

P=45Watts. The examined wave is the collisional one, which is described by the dispersion relation (2-2), and its' frequency and amplitude was taken from the spectrum on every pressure value. So, Table 2 is completed and the graphic is presented in Fig. 3.

Table 2. The wave Frequency and Amplitude with Pressure values B=72mT, P=45Watts

Gas Pressure (Pa)	Wave Frequency (kHz)	Wave Amplitude (Arbitrary Units
0.001	122	2.9
0.01	102	2.6
0.02	85	2.3
0.03	77	2.0
0.04	65	1.8
0.05	58	1.6
0.06	50	1.4
0.07	46	1.2
0.08	46	1.2
0.09	46	1.2
0.1	46	1.1

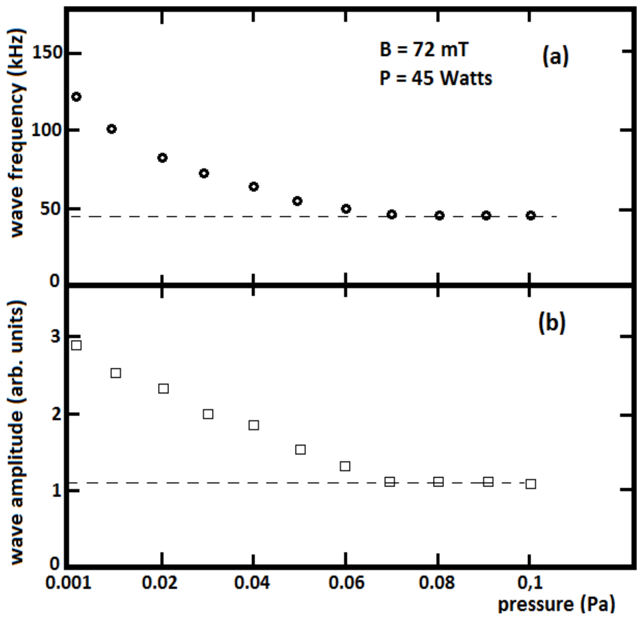


Fig.3. The wave frequency, and the wave amplitude by the gas pressure increase, are presented in (a) and (b) curves, respectively.

3. Physical Quantities with Exponential Changing-Models

Many examples have been taken from other areas of Physics and not only to state the models for the exponentially changed quantities, which is the topic of the present study; the known Radioactive Conversion (Change) Law is taken from the Nuclear Physics and the mortality problem is a clearly statistical subject.

The simple solution of the transitive problems
An easily perceptible example is the solution of the radioactively-law problem. Although the solution of this problem is known since the early university lessons, let us repeat its' solution here, for two basic reasons: i) to give the physical interpretation of every mathematical hypothesis or operation (action) and ii) to study the terms of this simple problem, such as the conversion rate, sub-duplication time, semi-life time e.t.c.

<u>The problem situation:</u>
At the time $t = 0$, the unbroken radioactive nucleus are N_0. How many unbroken nucleus N will still exist after the passing of the time t ?
Starting by the given fact that in the moment of the time t the remaining unbroken nucleuses are N, an infinitesimal increase of the time by dt is considered. A consequence of this is the breaking off dN from the unbroken nucleuses (the infinitesimal increase of the time causes, infinitesimal decrease of the unbroken nucleuses).
The next step is the seeking of the dependences of the dN change of the unbroken nucleuses on the other physical quantities. (the whole physical interest of the issue is concentrated on this point of the solution proceedings). These influences are the following: i) the dN change is proportional to the time increase dt (why?), ii) the dN change is proportional to the available quantity of the unbroken nucleuses N in that moment t. The change dN is proportional to the product of these two factors consequently, and in accordance with the following relation,

$$dN \propto N.dt \qquad (3\text{-}1)$$

If it is considered that there are no other changeable physical quantities that influence the dN, an analogy constant λ (for the quantities units equalization) must be introduced to the above relation (3-1). So, the following differential equation is resulted, which fits the problem,

$$dN = -\lambda.N.dt \qquad (3\text{-}2)$$

The constant λ, is named breaking off constant, depends on the breaking nuclear material, and its' unit is the $\sec^{-1}$. To sign (-) is simply put due to the decrease of the remained unbroken nucleuses.
Although the differential equation (3-2) is solved very easily, at the end of the paper Appendix A gives more details; its' solution is the known relation,

$$N = N_0.e^{-\lambda.t} \qquad (3\text{-}3)$$

The Law's (3-3) study

1. **sub-doubling time:** as sub-duplication time is defined the time $t = t_{1/2}$ at which the remaining unbroken nucleuses are half of the original ones, $N = \frac{N_0}{2}$. With the replacement of the pair of the values $(t_{1/2}, \frac{N_0}{2})$ on the relation (3-3) it is found that,

$$\frac{N_0}{2} = N_0 . e^{-\lambda . t_{1/2}} \quad \textbf{or} \quad 2 = e^{\lambda . t_{1/2}}$$

and finally,
$$t_{1/2} = \frac{\ln 2}{\lambda} \qquad (3\text{-}4)$$

In the same way the time of the sub-quadruplication $t_{1/4}$, for which the remaining unbroken nucleuses are $N = \frac{N_0}{4}$, can be found. With the same mathematical thoughts, the following is resulted,

$$t_{1/4} = \frac{\ln 4}{\lambda} = \frac{2 \ln 2}{\lambda} = 2.t_{1/2} \qquad (3\text{-}5)$$

For the sub-eight time $t_{1/8}$ it is found that,

$$t_{1/8} = \frac{\ln 8}{\lambda} = \frac{3 \ln 2}{\lambda} = 3.t_{1/2} \qquad (3\text{-}6)$$

Thinking that going from $\frac{N_0}{4}$ unbroken nucleuses to $\frac{N_0}{8}$ is actually a sub-doubling, it is valid that,

$$t_{1/8} - t_{1/4} = 3.t_{1/2} - 2.t_{1/2} = t_{1/2} \qquad (3\text{-}7)$$

b) Broken nucleuses

The broken nucleuses N' are: $N' = N_0 - N = N_0 - N_0 . e^{-\lambda . t} = N_0 .(1 - e^{-\lambda t})$ or

$$N' = N_0 .(1 - e^{-\lambda . t}) \qquad (3\text{-}8)$$

The drawing of the relations $N = N(t)$ (3-3) and $N' = N'(t)$ (3-8) is presented in Fig.4.

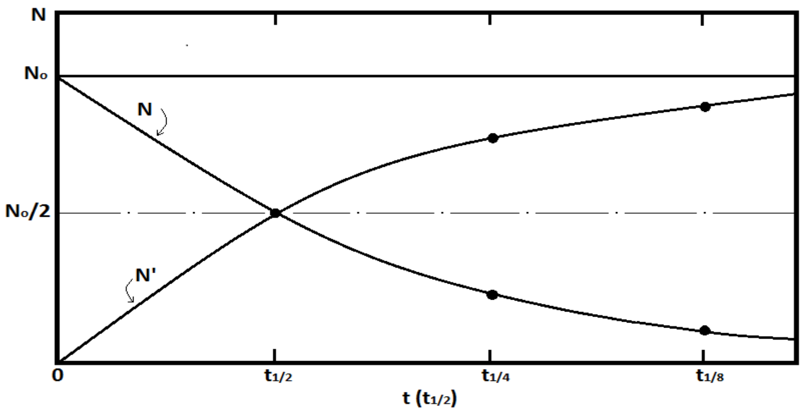


Fig.4. The $N = N(t)$ and $N' = N'(t)$ drawing is presented

c) <u>Conversion rate</u>

The quotient $\dfrac{dN}{dt}$ is defined as conversion rate. Consequently, the derivation of the relation (3-3) gives the conversion rate as following,

$$\frac{dN}{dt} = N_0.(-\lambda).e^{-\lambda.t} = -\lambda.N$$

$$\text{or} \quad \frac{dN}{dt} = -\lambda.N \qquad (3\text{-}9)$$

In Fig.5 the conversion rate versus the time is presented graphically.

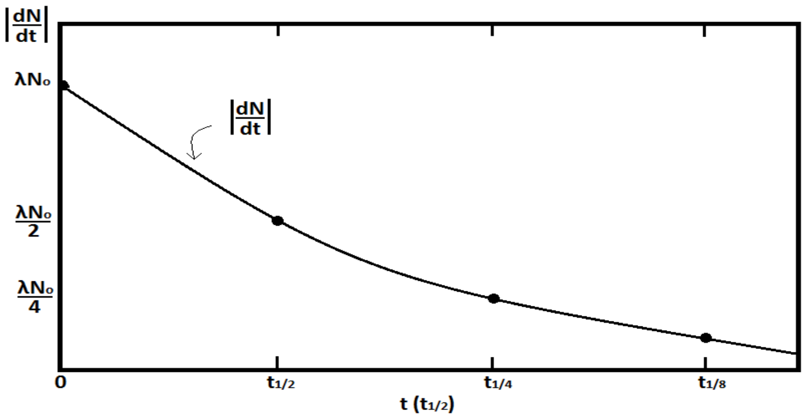


Fig.5. The conversion rate $\dfrac{dN}{dt}$ versus the time t is shown.

Observations-Comments:

1. The sub-doubling time remains constant, apart from the quantity of the unbroken radioactive nucleuses.
2. In accordance with the radioactively law (relation 3-3), when $t = \infty$, the remaining unbroken nucleuses are nullified.
3. The drawings of the remaining nucleuses $N = N_0.e^{-\lambda.t}$ and the already broken ones $N' = N_0(1-e^{-\lambda.t})$ are symmetrical to the straight line $\psi = \frac{N_0}{2}$ (Fig.3).

3. Cases-Models with no constant λ

In most cases the factor λ is not constant, but changeable by the time (quantities changeable by the time), sometimes in a small rate and other times in a big one. Let us consider the radioactively conversion again: two disputes of the results found from the previous solution can be placed here: i) the stability of the sub-life time $t_{1/2}$, apart from the available number of the unbroken nucleuses N , and ii) the total breaking off all the available nucleuses.

The physical perception obtained from the observation of related physical phenomena expects the sub-life time to decrease as the available unbroken nucleuses diminish, while the conversion proceedings have to stop leaving a small quantity of unbroken nucleuses.

Nuclear breaking off with decreased factor λ

I. Case

Let us now consider that the factor λ is not constant, but it has the following influence from the time,

$$\lambda = \lambda_0 - \mu t \qquad (4\text{-}1)$$

where μ is a constant measured in $\sec^{-2}$.

Repeating the formulation of the previous problem, where λ is considered as a constant, and, if at the moment t the remaining unbroken nucleuses are N , then, within the infinitesimal time dt , the change of the unbroken nucleuses dN is given from the following relation,

$$dN = -\lambda.N.dt \quad \text{or} \quad dN = -(\lambda_0 - \mu t).N.dt \quad \text{or}$$

$$\frac{dN}{N} = -(\lambda_0 - \mu t).dt \qquad (4\text{-}2)$$

The integration of the relation (4-2) gives the influence of time for the unbroken nucleuses evolution,

$$N = N_0.e^{-\lambda_0 t + \frac{\mu}{2}.t^2} \qquad (4\text{-}3)$$

The law's (4-3) study

a) Semi-life time: by putting $t = t_{1/2}$ when $N = \frac{N_0}{2}$, the equation $\mu . t_{1/2}^{2} - 2\lambda_0 . t_{1/2} + 2\ln 2 = 0$ is obtained and its' solution gives the semi-life time,

$$t_{1/2} = \frac{\lambda_0 - \sqrt{\lambda_0^2 - 2\mu \ln 2}}{\mu} \qquad (4\text{-}4)$$

If it is put that $t = t_{1/4}$ when $N = \frac{N_0}{4}$, in the same way as above the sub-quadruplication time is obtained,

$$t_{1/4} = \frac{\lambda_0 - \sqrt{\lambda_0^2 - 4\mu \ln 2}}{\mu} \qquad (4\text{-}5)$$

From the last two relations (4-4) and (4-5) and by using the mathematical inducement method, it is easily proved that,

$$t_{1/4} \phi \; 2.t_{1/2} \qquad (4\text{-}6)$$

b) Broken nucleuses: The broken nucleuses $N^{'}$ are calculated from the difference $N^{'} = N_0 - N$ or

$$N^{'} = N_0 (1 - e^{-\lambda_0 . t + \frac{\mu}{2} . t^2}) \qquad (4\text{-}7)$$

The drawing of the relations $N = N(t)$ (4-3) and the $N^{'} = N^{'}(t)$ (4-7) is presented in Fig 6.

c) Conversion rate: The conversion rate $\frac{dN}{dt}$ is defined from the derivative of the relation (4-3). This derivative of the time is,

$$\frac{dN}{dt} = N_0 (-\lambda_0 + \mu . t) . e^{-\lambda_0 . t + \frac{\mu}{2} . t^2} \quad \text{or} \quad \frac{dN}{dt} = -(\lambda_0 - \mu . t) . N \qquad (4\text{-}8)$$

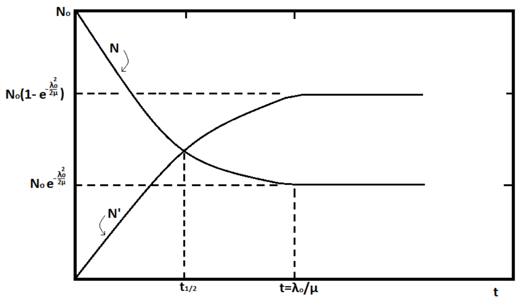


Fig.6. The $N = N(t)$ (relation 4-3) and $N' = N'(t)$ (relation 4-7) drawings are presented.

d) The relation (4-3) study

The derivative of the relation (4-3) gives the conversion rate, which is,

$$\frac{dN}{dt} = N_0(-\lambda_0 + \mu.t).e^{-\lambda_0.t+\frac{\mu}{2}.t^2}$$

If it is put that $\frac{dN}{dt} = 0$, when $t = \lambda_0/\mu$, which is the duration time of the phenomenon, the relation (4-3) has an extremity value as well. The kind of the extremity value is found from the relation $\left(\frac{d^2N}{dt^2}\right)_{t=\lambda_0/\mu}$, and its' value from the relation $N(\lambda_0/\mu)$.

For the second derivative it is concluded that,

$$\frac{d^2N}{dt^2} = N_0\mu.e^{\frac{\mu}{2}t^2-\lambda_0 t} + N_0(\mu.t-\lambda_0)(\mu.t-\lambda_0).e^{\frac{\mu}{2}t^2-\lambda_0 t} \quad \text{or}$$

$$\frac{d^2N}{dt^2} = N_0\left[\mu + (\mu t-\lambda_0)^2\right]e^{\frac{\mu}{2}t^2-\lambda_0 t} \tag{4-9}$$

By setting $t = \lambda_0/\mu$ the relation (4-9) gives,

$$\frac{d^2N}{dt^2}(t = {}^{\lambda_0}\!/\!_{\mu}) = N_0\left[\mu + (\lambda_0 - \lambda_0)^2\right]e^{\frac{\mu}{2}\cdot\frac{\lambda_0^2}{\mu^2} - \lambda_0\cdot\frac{\lambda_0}{\mu}}$$ and, finally,

$$\frac{d^2N}{dt^2}(t = {}^{\lambda_0}\!/\!_{\mu}) = N_0.\mu..e^{-\frac{\lambda_0^2}{2\mu}} \phi\ 0 \qquad (4\text{-}10)$$

It is resulted from the relation (4-10) that the remaining unbroken nucleuses N have a minimum value, which is,

$$N(t = {}^{\lambda_0}\!/\!_{\mu}) = N_0.e^{-\frac{\lambda_0^2}{2\mu}} \qquad (4\text{-}11)$$

In Fig.7 the change by the time of the factor $\lambda(t)$,the unbroken nucleuses $N(t)$ and the conversion rate $\frac{dN}{dt}$ is presented.

e) Comments: By considering the conversion factor λ not constant but changeable by the time, the following advantages arise from the solution of the problem:

1. The sub-doubling time $t_{1/2}$ does not remain constant, but it increases as the unbroken nucleuses diminish.
2. The initially available nucleuses N_0 are not broken in total, but there is a remaining quantity $N_0.e^{-\lambda_0^2/2\mu}$.
3. The solution of the problem and its' results are general and include the results of the solution with $\lambda = cons\tan t$, if it is set on the solution, where $\mu = 0$.
4. The suggested change of the factor λ is linear, which results to the solution being relatively simple, although slightly more complicated from what it is considered to be $\lambda = cons\tan t$.
5. In the problem the change factor μ appears, which is experimentally determinable.

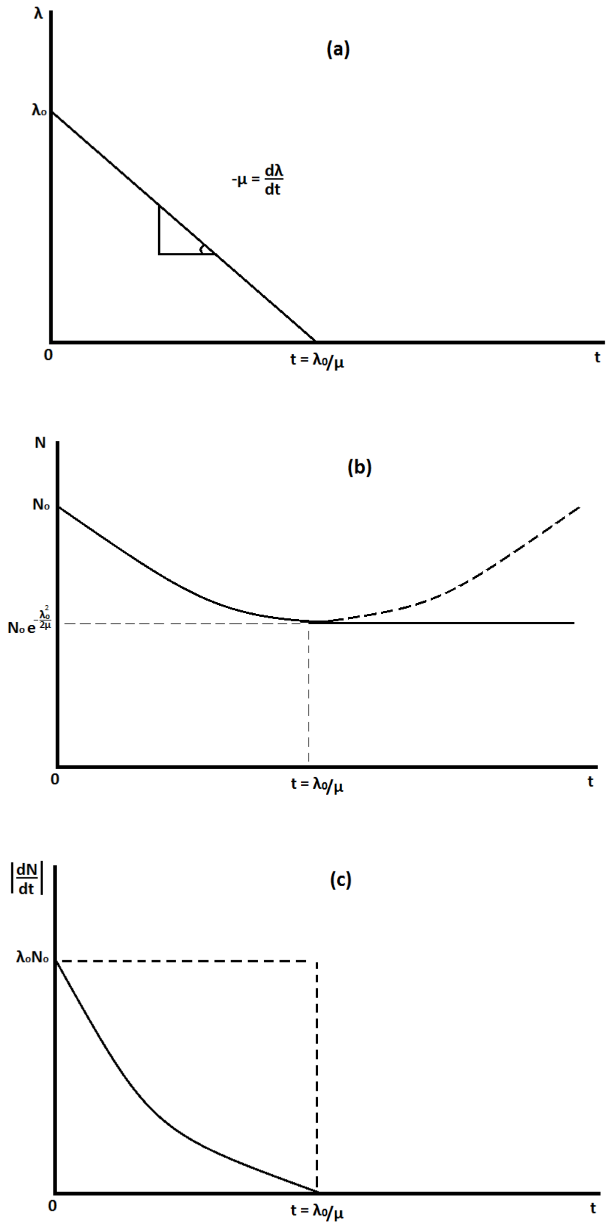


Fig.7. The factor $\lambda(t)$, the unbroken nucleuses $N(t)$ and the conversion rate $\frac{dN}{dt}$ versus the time t is shown.

II. Case

Now, let us consider that the constant λ is influenced by the remaining unbroken nucleuses N (and consequently, indirectly from the time t), in accordance with the relation,

$$\lambda = \lambda_0 + \mu N \qquad (4\text{-}12)$$

Then the differential equation is written as following:

$$dN = -(\lambda_0 + \mu N).N.dt \quad \text{or} \quad \frac{dN}{(\lambda_0 + \mu N).N} = -dt$$

Integrating the last one, it is obtained that,

$$\int \frac{dN}{(\lambda_0 + \mu N).N} = -\int dt + C \qquad (4\text{-}13)$$

The above relation (4-13) has the solution:

$$N = \frac{N_0.\Psi}{\Psi + \mu(1 - e^{-\lambda_0 t})}.e^{-\lambda_0.t} \qquad (4\text{-}14)$$

where is,

$$\Psi = \lambda_0 / \mathrm{N}_0$$

<u>The law's (4-14) study</u>

a) <u>sub-doubling time:</u> By setting into the (4-14) $t = t_{1/2}$ when $N = \dfrac{N_0}{2}$, the next equation is obtained,

$$2 = \frac{\Psi + \mu(1 - e^{-\lambda_0.t_{1/2}})}{\Psi}.e^{\lambda_0.t_{1/2}}$$

the solution of which gives the sub-doubling time,

$$t_{1/2} = \frac{1}{\lambda_0} \ln \frac{2\lambda_0 + \mu N_0}{\lambda_0 + \mu N_0} \qquad (4\text{-}15)$$

If it is set that $t = t_{1/4}$ when $N = \dfrac{N_0}{4}$, in the same way as above the following result is obtained again

$$\boldsymbol{t_{1/4} = \frac{1}{\lambda_0} \ln \frac{4\lambda_0 + \mu N_0}{\lambda_0 + \mu N_0}} \qquad (4\text{-}16)$$

From the last two relations (4-15) and (4-16) and by using the mathematical inducement method it is easily proved that,

$$t_{1/4} \phi 2.t_{1/2}$$

b) Broken nucleuses: The broken nucleuses $N^{'}$ are found from the difference $N^{'} = N_0 - N$ or

$$N^{'} = N_0 (1 - \frac{\Psi}{\Psi + \mu(1 - e^{-\lambda_0 t})} e^{-\lambda_0 . t}) \qquad (4\text{-}17)$$

c) Conversion rate: The conversion rate $\frac{dN}{dt}$ is calculated from the derivative of the relation (4-14). This derivative on the time is,

$$\frac{dN}{dt} = -\lambda_0^2 . \frac{\Psi + \mu}{\left[\Psi - \mu(1 - e^{-\lambda_0 t})\right]^2} . e^{-\lambda_0 . t} \qquad (4\text{-}18)$$

d) The study of the relation (4-14). The derivation on time of the relation (4-14) is the relation (4-18), which is not zero at any moment except the point $t = \infty$. The N(t) does not have extreme values consequently.

4. Interpretation of the results-Conclusions

In Sec.2, Fig.3 represents the plasma wave frequency and wave amplitude decrease by the gas pressure increase; with a first look these two changes have exponential form, since the scrutiny leads to two significant observations; firstly, the required change of the pressure amount for the sub-duplication is not constant, but it increases along with the pressure increase; secondly, the wave frequency and amplitude are not nullified, but remain a sufficient quantity until the plasma is put out. The above results mean that the 'extinguishing factor' λ is not constant, but changeable in some way. The curves of Fig. 3 are similar enough to those of Fig.s 6, 7b.

Although the mechanism of the wave rising is very complicated and in most cases impossible to understand, the difficulty is treated partially by following the thoughts below.

Every wave existence is caused by two antagonism factors. The first one is the cause (motive) for which the wave rises and is expressed by the growth rate. In the low frequency waves for example, the drift waves are caused in different gradients of the plasma quantities (plasma density, plasma temperature, d.c. potential e.t.c.). The second antagonism factor involves the wave damping and expresses the different "resistances", which may interfere with the wave transmission, as the collisions between the plasma particles (collision frequency).

The above mentioned two factors appear together into the imaginary part ω_i of the wave frequency ω in the previous three examples. In the equations (2-1) and (2-3) it is expressed with a sum,

$$\omega_i = |s| C_s \sqrt{\frac{U_R}{U_D} - 1} - \frac{\nu_i}{2} \qquad (2\text{-}1)$$

$$\omega_i = \frac{m_e}{m_i}\frac{\nu_e}{2}\frac{u_e-(u_i+C_s)}{C_s}-\frac{\nu_i}{2} \qquad (2\text{-}3)$$

since in the relation (2-2) it is formed as a product.

$$\omega_i = \nu_e \frac{\omega_{pe}^2}{\omega_{pi}^2}.\frac{k^2(u^2-C_s^2)-\omega_{ci}^2}{\omega_{ce}^2} \qquad (2\text{-}2)$$

The balance of the two factors secures the wave stability and the inclination from the equilibrium gives the growth or the damping, respectively.
The problem rises as the calculated imaginary part of the wave frequency ω_i is not constant but changeable on the time, at least during the wave establishment or extinguishing. The mutual-dependence of the plasma quantities which are involved in the ω_i, is impossible to find and express analytically, so the their modeling becomes necessary.
In the present work such a modeling is set out with the ambition to be completed in the immediate future in a full list of models applicable on any actual experimental data. This approaching fitting between the model and the experimental data must be confirmed by using delay-time methods, as the wave establishment time is in most cases very limited.
With the examples, which are included in the paper and have been taken from the other areas of the Physics (Nuclear Physics) the results are much more satisfactory and acceptable than those believed until now.
In the end the conclusion is that; although the experimental confirmation of the present study's usefulness is feeble now, the effort for the models' development must continue and a list of those models must be composed. This means that the 'Demokritos' team haves to do theoretical future work on the same topic and experimental confirmation of the mathematic models.
In any case, the experimental measurements are very difficult to be carried out; firstly, because of the very little time required for the establishment of the steady state of the plasma waves, and, secondly, due to the great amount of time required for a perceptible physical nuclear decay.

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Appendix A
Solution of the differential equation (3-2)
The equation (3-2) is the simplest form of a differential equation with two changeable quantities (N, t), which can be divided into its’ two parts. So, the following is resulted,

$$\frac{dN}{N} = -\lambda.dt \qquad (A1)$$

The relation (A1) is integrated by parts in two ways: i) by defined integrals, if the changeable quantities' limits are known, or ii) by indefinite integrals, adding the integration constant C . If the second method is prefered, the following is resulted,

$$\int \frac{dN}{N} = -\lambda.\int dt + C$$

or $$\ln N = -\lambda t + C \qquad \text{(A2)}$$

For the finding of the integration constant C , one values pair of the changeable quantities N and t is enough to be known. One known pair of values in this problem is the original conditions, where, for $t = 0$, it is $N = N_0$. The replacement of the quantities t and N on the equation (A2) with the above known values, gives the value of the constant as,

$$C = \ln N_0 \qquad \text{(A3)}$$

By the substitution on the relation (A2), the following relation is resulted,

$$\ln N = -\lambda.t + \ln N_0 \quad \text{or}$$

$$\ln \frac{N}{N_0} = -\lambda.t \qquad \text{(A4)}$$

And, finally, the known law of the radioactivity is obtained,

$$N = N_0.e^{-\lambda.t} \qquad \text{(A5)}$$

Appendix B

Solution of the differential equation (4-13)

By dividing the integral function of the first part of the (4-13) into smaller additives, two factors α and β are seeking for the following equality to be valid,

$$\frac{1}{(\lambda_0 + \mu.N).N} = \frac{\alpha}{N} + \frac{\beta}{\lambda_0 + \mu.N} \qquad \text{(B1)}$$

Finally, the two factors have the values, $\alpha = 1/\lambda_0$ and $\beta = -\mu/\lambda_0$, and the last relation is written,

$$\frac{1}{(\lambda_0 + \mu.N).N} = \frac{1}{\lambda_0 N} - \frac{\mu}{\lambda_0(\lambda_0 + \mu.N)} \qquad \text{(B2)}$$

With the substitution of the relation (B2) into the (B1) one, it is obtained that,

$$\int \frac{dN}{(\lambda_0 + \mu.N).N} = \frac{1}{\lambda_0}.\int \frac{dN}{N} - \frac{\mu}{\lambda_0}.\int \frac{dN}{\lambda_0 + \mu.N} = -\int dt + C$$

or
$$\ln N - \ln(\lambda_0 + \mu.N) = -\lambda_0 t + C' \qquad \text{(B3)}$$

The initial condition ($t = 0, N = N_0$) determines the integration constant C', which takes the value, $C' = \ln N_0 - \ln(\lambda_0 + \mu.N_0)$

With substitution into the relation (B3) and by using suitable mathematical elaboration the following is obtained,

$$N = \frac{N_0.\Psi}{\Psi + \mu(1 - e^{-\lambda_0 t})}.e^{-\lambda_0 .t} \qquad \text{(B4)}$$

where is,

$$\Psi = \lambda_0 / \mathrm{N}_0$$

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Captions

Fig.1 The plasma cavity with probes is presented in (a), whereas a photo of the experimental device is shown in (b).

Fig.2. The wave form is shown in (a), whereas the wave spectra are presented in (b) and (c) for rf-drift and collisional wave, respectively.

Fig.3. The wave frequency, and the wave amplitude by the gas pressure increase, are presented in (a) and (b) curves, respectively.

Figure 4. The $N = N(t)$ and $N' = N'(t)$ drawing is presented

Fig.5. The conversion rate $\frac{dN}{dt}$ versus the time t is shown.

Fig.6. The $N = N(t)$ (relation 4-3) and $N' = N'(t)$ (relation 4-7) drawings are presented.

Fig.7. The factor $\lambda(t)$, the unbroken nucleuses $N(t)$ and the conversion rate $\frac{dN}{dt}$ versus the time t is shown.