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Differential Equations of Ellipsoidal State Estimates for Bilinear-Quadratic Control Systems under Uncertainty

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Abstract. The problem of describing the reachable sets of nonlinear dynamical control systems with combined bilinear and quadratic nonlinearity and with uncertainty in initial states is studied. We assume that the uncertainty is of a set-membership kind when we know only the bounding set for unknown items and any additional statistical information on their behavior is not available. Applying results of the theory of trajectory tubes of control systems and related techniques of differential inclusions theory we present new approaches that allow finding external or internal ellipsoidal estimates of reachable sets. The main result consists in obtaining the differential equations describing the dynamics of centers and matrices of the external ellipsoids estimating the reachable sets of the bilinear-quadratic control system under uncertainty. Examples and numerical simulations related to the proposed techniques and illustrating the theoretical results are also included.
Keywords: Control systems, Nonlinear dynamics, Estimation problem, Set-membership uncertainty, Ellipsoidal calculus, Funnel equations, Trajectory tubes, Simulations for uncertain systems.

1 Introduction

The paper deals with the estimation problems for uncertain systems in the case when a probabilistic description of noise and errors is not available, but only bounds on them are known (Chernousko[4], Schweppe[23], Kurzhanski and Valyi[17], Kurzhanski and Varaiya[18]). Such models may be found in many applied areas ranged from engineering problems from physics to economics as well as to biological and ecological modeling when it occurs that a stochastic nature of the errors is questionable because of limited data or because of complexity of the model. An alternative to a stochastic characterization a so-called bounded-error characterization, also called set-membership approach, has been proposed and intensively developed in the last decades (Bertsekas and Rhodes[1], Chernousko[3], Kurzhanski and Valyi[17], Kurzhanski and Varaiya[18], Milanese *et al.*[21]). The solution of many control and estimation problems under uncertainty involves constructing reachable sets and

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their analogs. For models with linear dynamics under such set-membership uncertainty there are several constructive approaches which allow finding effective estimates of reachable sets (Chernousko[3], Chernousko[4], Filippova[11], Kurzhanski and Valyi[17], Kurzhanski and Varaiya[18], Polyak *et al.*[22]). Certainly however concrete problems are mostly nonlinear in their parameters and the set of feasible system states is usually non-convex or even non-connected. The key issue in nonlinear set-membership estimation is to find suitable techniques, which produce related bounds for the set of unknown system states without being too computationally demanding, some of such approaches may be found e.g. in Brockett[2], Chernousko and Rokityanskii[5], Dontchev and Lempio[6], Filippova[7], Filippova[8], Kurzhanski and Filippova[16], Mazurenko[20], Filippova and Lisin[12].

In this paper the modified state estimation approaches which use the special structure of nonlinearity of studied control system are presented. We assume here that the system nonlinearity is generated by the combination of two types of functions in related differential equations, one of which is bilinear and the other one is quadratic. We find here the set-valued estimates of related reachable sets of such nonlinear uncertain control system. The algorithms of constructing the ellipsoidal estimates for studied nonlinear systems and numerical simulation results related to the proposed techniques are given.

In this paper we continue researches beginning in Filippova and Matviychuk[13], Filippova and *et al.*[14,15]. The paper is organized as follows. Section 2 gives the problem statement and introduces the terminology used throughout the paper. In Section 3 we provide some results on finite difference estimation schemes. Here we present also an algorithm for calculating the external ellipsoidal estimate of reachable sets of the nonlinear system and consider the example. In Section 4 we derive the differential equations describing the dynamics of upper estimates of reachable sets of uncertain control system with bilinear - quadratic nonlinearity. Finally, Section 5 presents conclusions and the last Section contains acknowledgments.

2 Preliminaries and problem formulation

Let us introduce the following basic notation. Let $\mathbb{R}^n$ be the n-dimensional vector space, $\operatorname{comp}\mathbb{R}^n$ be the set of all compact subsets of $\mathbb{R}^n$, $\mathbb{R}^{n\times m}$ stands for the set of all real $n\times m$-matrices, $x'y = (x,y) = \sum_{i=1}^n x_i y_i$ be the usual inner product of $x, y \in \mathbb{R}^n$ with prime as a transpose,

$$\|x\| = \|x\|_2 = (x'x)^{1/2}, \quad \|x\|_\infty = \max_{1\le i\le n} |x_i|$$

be vector norms for $x \in \mathbb{R}^n$, $I \in \mathbb{R}^{n\times n}$ be the identity matrix, $\operatorname{tr}(A)$ be the trace of $n\times n$-matrix A (the sum of its diagonal elements). We denote by $B(a,r) = \{x \in \mathbb{R}^n : \|x-a\| \le r\}$ the ball in $\mathbb{R}^n$ with a center $a \in \mathbb{R}^n$ and a radius $r > 0$ and by

$$E(a,Q) = \{x \in \mathbb{R}^n : \ (Q^{-1}(x-a),(x-a)) \le 1\}$$

the *ellipsoid* in $\mathbb{R}^n$ with a center $a \in \mathbb{R}^n$ and with a symmetric positive definite $n \times n$-matrix Q.

Consider the following system

$$\dot{x} = A(t)x + f(x)d + u(t), \quad x_0 \in \mathcal{X}_0, \quad t \in [t_0, T], \tag{1}$$

where $x, d \in \mathbb{R}^n$, $\|x\| \leq K$ $(K > 0)$, $f(x)$ is the nonlinear function, which is quadratic in x, $f(x) = x'Bx$, with a given symmetric and positive definite $n \times n$-matrix B. Control functions $u(t)$ in (1) are assumed to be Lebesgue measurable on $[t_0, T]$ and satisfying the constraint $u(t) \in \mathcal{U}$ for a.e. $t \in [t_0, T]$ (here $\mathcal{U}$ is a given set, $\mathcal{U} \in \operatorname{comp} \mathbb{R}^n$). The $n \times n$-matrix function $A(t)$ in (1) has the form

$$A(t) = A^0 + A^1(t), \tag{2}$$

where the $n \times n$-matrix A^0 is given and the measurable $n \times n$-matrix $A^1(t)$ is unknown but bounded, $A^1(t) \in \mathcal{A}^1$ $(t \in [t_0, T])$,

$$A(t) \in \mathcal{A} = A^0 + \mathcal{A}^1, \tag{3}$$

$$\mathcal{A}^1 = \big\{A{=}\{a_{ij}\}{\in}\mathbb{R}^{n\times n} : |a_{ij}|{\leq}c_{ij},\ i,j{=}1,\dots n\big\},$$

where $c_{ij} \geq 0$ $(i, j = 1, \dots n)$ are given.

We will assume that $\mathcal{X}_0$ in (1) is an ellipsoid, $\mathcal{X}_0 = E(a_0, Q_0)$, with a symmetric and positive definite matrix $Q_0 \in \mathbb{R}^{n\times n}$ and with a center a_0.

Let the absolutely continuous function $x(t) = x\big(t; u(\cdot), A(\cdot), x_0\big)$ be a solution to dynamical system (1)–(3) with initial state $x_0 \in \mathcal{X}_0$, with admissible control $u(\cdot)$ and with a matrix $A(\cdot)$ satisfying (13)–(3). The *reachable set* $\mathcal{X}(t)$ at time t $(t_0 < t \leq T)$ of system (1)–(3) is defined as the following set

$$\mathcal{X}(t) = \{x \in \mathbb{R}^n : \exists\, x_0{\in}\mathcal{X}_0,\ \exists\, u(\cdot){\in}\mathcal{U},\ \exists\, A(\cdot){\in}\mathcal{A},$$
$$x = x(t) = x\big(t; u(\cdot), A(\cdot), x_0\big)\}.$$

The main problems studied here are as follows.

- **Problem 1**. Find the external ellipsoidal estimate $E(a^+(t), Q^+(t))$ (with respect to the inclusion of sets) of the reachable set $\mathcal{X}(t)$ $(t_0 < t \leq T)$ by using the analysis of a special type of nonlinear control systems with uncertain initial data.
- **Problem 2**. Find differential equations that describe the dynamics of the above ellipsoidal estimates of reachable sets.

3 Finite-difference approximations of trajectory tubes

Consider the general case (1)–(3) of the system dynamics and here we take $\mathcal{X}_0 = E(a_0, Q_0)$ and $\mathcal{U} = E(\hat{a}, \hat{Q})$ where matrices B, $\hat{Q}$ and Q_0 are symmetric and positive definite. The next theorem describes discrete external ellipsoidal estimates of reachable sets $X(t)$ of the uncertain control system (1)–(3), containing both bilinear and quadratic nonlinearities. Denote the maximal eigenvalue of matrix $B^{1/2}Q_0B^{1/2}$ as k^2.

Theorem 1 (Filippova *et al.*[14,15]). *The following external ellipsoidal estimate holds*

$$\mathcal{X}(t_0+\sigma) \subseteq E(a^*(t_0+\sigma), Q^*(t_0+\sigma)) + o(\sigma)B(0,1) \tag{4}$$

where $\sigma^{-1}o(\sigma) \to 0$ *for* $\sigma \to +0$ *and where*

$$a^*(t_0+\sigma) = \tilde{a}(t_0+\sigma) + \sigma(\hat{a} + a_0' B a_0 \cdot d + k^2 d), \tag{5}$$

$$Q^*(t_0+\sigma) = (p^{-1}+1)\tilde{Q}(t_0+\sigma) + (p+1)\sigma^2\hat{Q}, \tag{6}$$

with functions $\tilde{a}(t)$, $\tilde{Q}(t)$ *being solutions of the following differential equations*

$$\dot{\tilde{a}} = \tilde{A}^0\tilde{a}, \;\; \tilde{a}(t_0) = a_0, \quad t_0 \le t \le T, \tag{7}$$

$$\dot{\tilde{Q}} = \tilde{A}^0\tilde{Q} + \tilde{Q}(\tilde{A}^0)' + q\tilde{Q} + q^{-1}G, \;\; \tilde{Q}(t_0) = Q_0, \quad t_0 \le t \le T, \tag{8}$$

where

$$\tilde{A}^0 = A^0 + 2d\cdot a_0' B, \quad q = \big(n^{-1}\,\mathrm{Tr}\,((\tilde{Q})^{-1}G)\,\big)^{1/2}, \tag{9}$$

$$G = \mathrm{diag}\,\Big\{(n-v)\big[\sum_{i=1}^{n} c_{ji}|\tilde{a}_i| + \big(\max_{\sigma=\{\sigma_{ij}\}} \sum_{p,q=1}^{n} \tilde{Q}_{pq}c_{jp}c_{jq}\sigma_{jp}\sigma_{jq}\big)^{1/2}\big]^2\Big\}, \tag{10}$$

the maximum in (10) *is taken over all* $\sigma_{ij} = \pm 1$, $i,j = 1,\dots,n$, *such that* $c_{ij} \neq 0$ *and* v *is a number of such indices* i *for which we have:* $c_{ij} = 0$ *for all* $j = 1,\dots,n$, *and* p *is the unique positive root of the equation*

$$\sum_{i=1}^{n} \frac{1}{p+\alpha_i} = \frac{n}{p(p+1)}$$

with $\alpha_i \ge 0$ $(i = 1,...,n)$ *being the roots of the following equation*

$$|\tilde{Q}(t_0+\sigma) - \alpha\sigma^2\hat{Q}| = 0.$$

The following iterative algorithm is based on Theorem 1.

Algorithm. Subdivide the time segment $[t_0, T]$ into subsegments $[t_i, t_{i+1}]$ where $t_i = t_0 + ih$ $(i = 1,\dots,m)$, $h = (T-t_0)/m$, $t_m = T$.

- Given $\mathcal{X}_0 = E(a_0, Q_0)$, find the smallest $k = k_0 > 0$ such that $E(a_0, Q_0) \subseteq E(a_0, k^2B^{-1})$ (k^2 is the maximal eigenvalue of the matrix $B^{1/2}Q_0B^{1/2}$).
- Take $\sigma = h$ and define by Theorem 1 the external ellipsoid $E(a_1, Q_1)$ such that $\mathcal{X}(t_1) \subseteq E(a_1, Q_1) = E(a^*(t_0+\sigma), Q^*(t_0+\sigma))$.
- Consider the system on the next subsegment $[t_1, t_2]$ with $E(a_1, Q_1)$ as the initial ellipsoid at instant t_1.

Next steps continue iterations 1–3. At the end of the process we will get the external estimate $E(a^*(t), Q^*(t))$ of the tube $\mathcal{X}(t)$ with accuracy tending to zero when $m \to \infty$.

Example. Consider the following control system

$$\begin{cases} \dot{x}_1 = & x_2 + x_1^2 + x_2^2 + u_1, \\ \dot{x}_2 = & c(t)x_1 + u_2. \end{cases} \tag{11}$$

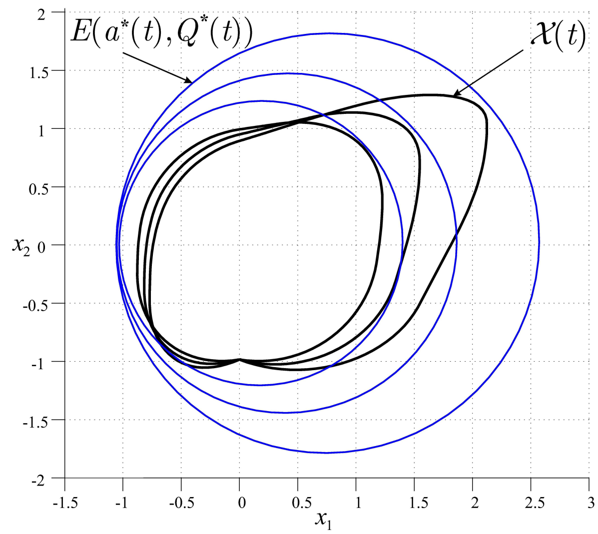

Fig. 1. Reachable sets $\mathcal{X}(t)$ and their external estimates $E(a^*(t), Q^*(t))$ for $t = 0.14; 0.26; 0.4$.

Here we take $x_0 \in \mathcal{X}_0 = B(0, 1)$, $0 \leq t \leq 0.4$ and $\mathcal{U} = B(0, 0.1)$, the uncertain but bounded measurable function $c(t)$ satisfies the inequality

$$|c(t)| \leq 1, \quad 0 \leq t \leq 0.4.$$

The reachable sets $\mathcal{X}(t)$ and their external ellipsoidal estimates $E(a^*(t), Q^*(t))$ calculated by the Algorithm on the base of Theorem 1 are given in Figures 1-2. We see in the figures that the reachable sets $\mathcal{X}(t)$ lose their convexity with increasing time. Nevertheless, their external estimates remain valid. Moreover, in some directions, we see that the sets $\mathcal{X}(t)$ and $E(a^*(t), Q^*(t))$ concern the each other, that is, it is impossible to reduce the estimating ellipsoids $E(a^*(t), Q^*(t))$.

4 Differential equations of external ellipsoidal estimates

Earlier some approaches had been proposed to obtain differential equations describing dynamics of external ellipsoidal estimates for reachable sets of control system under uncertainty, e.g., in Chernousko and Rokityanskii[5] the authors studied estimation problems for systems with uncertain matrices in dynamical equations, but additional nonlinear terms in dynamics were not considered there. In Filippova[10] differential equations of ellipsoidal estimates for reachable sets of a nonlinear dynamical control system were derived for the case when system state velocities contain quadratic forms but in that case the uncertainty in matrix coefficients was not assumed. Here we consider the complex

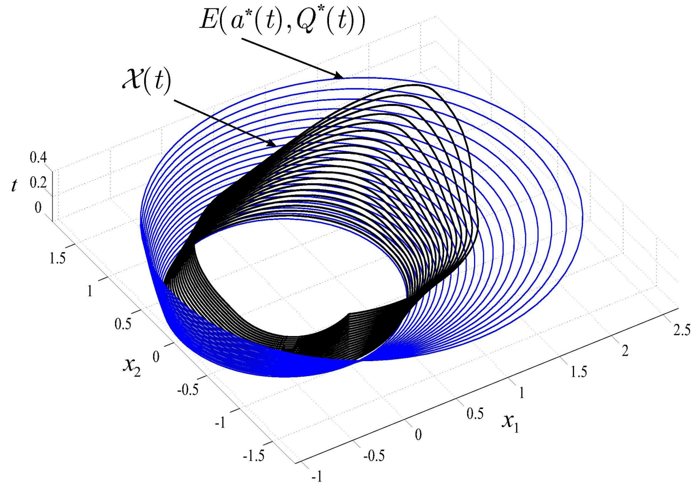

Fig. 2. Trajectory tube $\mathcal{X}(t)$ and its ellipsoidal estimating tube $E(a^*(t), Q^*(t))$ for the bilinear-quadratic control system with uncertain initial states.

situation but in order to slightly simplify related formulas we assume that the system has the following form

$$\dot{x} = A(t)x + f(x)d, \quad x_0 \in \mathcal{X}_0, \quad t \in [t_0, T], \tag{12}$$

where $x, d \in \mathbb{R}^n$, $\|x\| \leq K$ $(K > 0)$, $f(x)$ is the nonlinear function, which is quadratic in x, $f(x) = x'Bx$, with a given symmetric and positive definite $n \times n$-matrix B.

The $n \times n$-matrix function $A(t)$ in (12) has the form

$$A(t) = A^0 + A^1(t), \tag{13}$$

where the $n \times n$-matrix A^0 is given and the measurable $n \times n$-matrix $A^1(t)$ is unknown but bounded, $A^1(t) \in \mathcal{A}^1$ $(t \in [t_0, T])$,

$$A(t) \in \mathcal{A} = A^0 + \mathcal{A}^1, \tag{14}$$
$$\mathcal{A}^1 = \big\{A{=}\{a_{ij}\}{\in}\mathbb{R}^{n \times n} : |a_{ij}|{\leq}c_{ij},\ i,j{=}1, \ldots n\big\},$$

where $c_{ij} \geq 0$ $(i, j = 1, \ldots n)$ are given.

We will assume that $\mathcal{X}_0$ in (12) is an ellipsoid, $\mathcal{X}_0 = E(a_0, Q_0)$, with a symmetric and positive definite matrix $Q_0 \in \mathbb{R}^{n \times n}$ and with a center a_0.

Lemma 1 (Filippova[10]). *The maximal number $k_0^+ > 0$ such that the inclusion holds*

$$E(a_0, Q_0) \subseteq E(a_0, (k_0^+)^2 B^{-1}), \tag{15}$$

is defined by the following equality

$$(k_0^+)^2 = \max_{l \in \{l \in R^n : ||l||=1\}} l' B^{1/2} Q_0 B^{1/2} l. \tag{16}$$

The following theorem provides the differential equations describing the dynamics of centers and matrices of the external ellipsoids estimating the reachable sets $X(t)$ of the bilinear-quadratic control system (12)-(14) under uncertainty in matrix coefficients and initial states and with nonlinear (quadratic) term in state velocities.

Theorem 2. *The following inclusion is true*

$$X(t) \subseteq E(a^+(t), Q^+(t)), \quad t \in [t_0, T], \tag{17}$$

where $Q^+(t) = r^+(t)B^{-1}$ *and functions* $a^+(t)$, $r^+(t)$ *are the solutions of the following nonlinear ordinary differential equations*

$$\frac{da^+(t)}{dt} = A^0 a^+(t) + a^{+\prime}(t) B a^+(t) d + r^+(t) d,$$

$$\frac{dr^+(t)}{dt} = \max_{\|l\|=1} \{ l^{'} (2\tilde{B}_+(t) B^{1/2} \tilde{Q}(t) B^{1/2} + B^{1/2} q_+^{-1}(t) G(t) B^{1/2}) l \} + q_+(t) r^+(t), \tag{18}$$

$$q_+(t) = \{(nr^+(t))^{-1}\, Tr(BG(t))\}^{1/2},$$

$$\tilde{B}_+(t) = B^{1/2}(A^0 + 2d \cdot (a^+(t))' B) B^{-1/2},$$

$$G(t) = \mathrm{diag} \Big\{ (n-v) \Big(\sum_{i=1}^{n} c_{ji} |a_i^+(t)| + \big(\max_{\sigma = \{\sigma_{ij}\}} \sum_{p,q=1}^{n} Q_{pq}^+(t) c_{jp} c_{jq} \sigma_{jp} \sigma_{jq} \big)^{1/2} \Big)^2 \Big\}, \tag{19}$$

where the maximum in (10) *is taken over all* $\sigma_{ij} = \pm 1$, $i, j = 1, \ldots, n$, *such that* $c_{ij} \neq 0$ *in* (3) *and* v *is a number of such indices* i *for which we have:* $c_{ij} = 0$ *for all* $j = 1, \ldots, n$, *with initial conditions*

$$a^+(t_0) = a_0, \quad r^+(t_0) = (k_0^+)^2. \tag{20}$$

Proof. Analyzing results of Theorem 1 and using the general scheme of the proof of Theorem 2 in Filippova[10] (see also techniques in Filippova[9,11]) we obtain the formulas (18) of the Theorem.

Remark 1. Here the differential equations (17)-(18) describing the evolution of upper ellipsoidal estimates of reachable sets $X(t)$ of the studied system are much more complicated then e.g. in Filippova[11] because we assume here simultaneously both the uncertainty in matrix coefficients and also the presence of nonlinear (quadratic) terms in the system dynamics.

5 Conclusion

The paper deals with the problems of state estimation for uncertain dynamical control systems for which we assume that the initial state is unknown but bounded with given constraints and the matrix in the linear part of state velocities is also unknown but bounded.

We study here the case when the system nonlinearity is generated by the presence of bilinear terms and quadratic forms in related differential equations. The problem may be reformulated as the problem of describing the motion of set-valued states in the state space under nonlinear dynamics with state velocities having bilinear-quadratic type.

Basing on results of ellipsoidal calculus developed earlier for some classes of uncertain systems we present the modified state estimation approach which uses the special structure of nonlinearity and uncertainty in the control system and allows constructing the external ellipsoidal estimates of reachable sets.

The differential equations describing the dynamics of the estimating external ellipsoids are derived.

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References

1. D. P. Bertsekas and I. B. Rhodes. Recursive state estimation for a set-membership description of uncertainty. *IEEE Transactions on Automatic Control*, 16, 117–128, 1971.
2. R. W. Brockett. On the reachable set for bilinear systems. *Lecture Notes in Economics and Mathematical Systems*, 111, 54–63, 1975.
3. F. L. Chernousko. *State Estimation for Dynamic Systems.* CRC Press. Boca Raton, 1994.
4. F. L. Chernousko. Ellipsoidal approximation of the reachable sets of linear systems with an indeterminate matrix. *Applied Mathematics and Mechanics*, 60, **6**, 940–950, 1996.
5. F. L. Chernousko, D. Ya. Rokityanskii. Ellipsoidal bounds on reachable sets of dynamical systems with matrices subjected to uncertain perturbations. *Journ. of Optimiz. Theory and Appl.*, 104, **1**, 1-19, 2000.
6. A. L. Dontchev and F. Lempio. Difference methods for differential inclusions: a survey. *SIAM Review*, 34, 263–294, 1992.
7. T. F. Filippova. A note on the evolution property of the assembly of viable solutions to a differential inclusion. *Computers & Mathematics with Applications* , 25, **2**, 115–121, 1993.
8. T. F. Filippova. Estimates of Trajectory Tubes of Uncertain Nonlinear Control Systems. *Lect. Notes in Comput. Sci.*, 5910, 272–279, 2010.
9. T. F. Filippova. Trajectory tubes of nonlinear differential inclusions and state estimation problems. *J. of Concrete and Applicable Mathematics, Eudoxus Press, LLC*, 8, 454–469, 2010.

10. T. F. Filippova. Differential equations of ellipsoidal estimates for reachable sets of a nonlinear dynamical control system. *Proceedings of the Steklov Institute of Mathematics (Supplementary issues)*, 271, suppl.1, S75-S84, 2010.
11. T. F. Filippova. Set-valued dynamics in problems of mathematical theory of control processes. *International Journal of Modern Physics, Series B (IJMPB)*, 26, **25**, 1–8, 2012.
12. T. F. Filippova , D. V. Lisin. On the Estimation of Trajectory Tubes of Differential Inclusions. *Proc. Steclov Inst. Math.: Problems Control Dynam. Systems.* Suppl. Issue 2, S28–S37, 2000.
13. T. F. Filippova and O. G. Matviychuk. Algorithms of Estimating Reachable Sets of Nonlinear Control Systems with Uncertainty. In: *Proceedings of the 7th Chaotic Modeling and Simulation International Conference (Lisbon, Portugal: 7-10 June, 2014). Published by ISAST: International Society for the Advancement of Science and Technology, Christos H Skiadas (Ed.)*, 115–124, Portugal, 2014.
14. T. F. Filippova, O. G. Matviychuk and S. M. Makarov. Reachable Sets for a Class of Nonlinear Control Systems with Uncertain Initial States. In: *Proceedings of the 8th Chaotic Modeling and Simulation International Conference (Henri Poincare Institute (IHP), Paris, France: 26-29 May, 2015). Published by ISAST: International Society for the Advancement of Science and Technology, Christos H Skiadas (Ed.)*, 115–124, France, 2015.
15. T. F. Filippova and O. G. Matviychuk. Algorithms of Estimating Reachable Sets of Nonlinear Control Systems with Uncertainty. *Journal of Chaotic Modelling and Simulation (CMSIM)*, 3, 205–214, 2015.
16. A. B. Kurzhanski and T. F. Filippova. On the theory of trajectory tubes a mathematical formalism for uncertain dynamics, viability and control. In: *Advances in Nonlinear Dynamics and Control: a Report from Russia, Progress in Systems and Control Theory, A. B. Kurzhanski (Ed.)*, 17, 22–188. Birkhauser, Boston, 1993.
17. A. B. Kurzhanski and I. Valyi. *Ellipsoidal Calculus for Estimation and Control*. Birkhauser, Boston, 1997.
18. A. B. Kurzhanski and P. Varaiya. *Dynamics and Control of Trajectory Tubes. Theory and Computation*. Springer-Verlag, New York, 2014.
19. O. G. Matviychuk. Estimation Problem for Impulsive Control Systems under Ellipsoidal State Bounds and with Cone Constraint on the Control. *AIP Conf. Proc.*, 1497, 3–12, 2012.
20. S. S. Mazurenko. A differential equation for the gauge function of the star-shaped attainability set of a differential inclusion. *Doklady Mathematics*, 86, **1**, 476–479, 2012.
21. M. Milanese, J. P. Norton, H. Piet-Lahanier and E. Walter (Eds.). *Bounding Approaches to System Identification*. Plenum Press, New York, 1996.
22. B. T. Polyak, S. A. Nazin S.A., C. Durieu, E. Walter. Ellipsoidal parameter or state estimation under model uncertainty. *Automatica*, 40, 1171-1179, 2004.
23. F. Schweppe. *Uncertain Dynamic Systems*. Prentice-Hall, Englewood Cliffs, New Jersey, 1973.
24. V. M. Veliov. Second order discrete approximations to strongly convex differential inclusion. *Systems and Control Letters*, 13, 263–269, 1989.
25. E. Walter and L. Pronzato. *Identification of parametric models from experimental data*. Springer-Verlag, Heidelberg, 1997.