Phonological Word Proximity in Child Speech Development

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Abstract: In order to establish a universal perspective on phonological word proximity in child speech development, the relationship between phonological word proximity (PWP) and the proportion of consonants correct (PCC) is derived analytically for a whole speech sample, in terms of the proportion of vowels (PV) and the proportion of phonemes deleted minus phonemes added (PPD). PWP depends linearly on the weighted averages of PCC and PPD and non-linearly on the weighted average of PV; the PV dependence is linearized quite accurately for a wide range of PV values. Upper and lower bounds on PWP are obtained for minimum and maximum PPD. Further, PWP changes are obtained relative to PCC and PPD changes, thus, determining which of these measurements better discriminates performance between speech samples. The method and analysis is applied to PWP, PCC and PPD computations from the data of a bilingual child’s speech traced longitudinally from age 2;6 to age 3;9. The results reveal a growth pattern for PCC and PPD and, consequently, for PWP, which is associated with three stages of phonological development. The middle stage is nearly cyclic with a strange attractor and seven months long while the other two stages are progressive of the double-logistic type. The developmental PWP values lie chaotically inside a trapezoid within a triangle bounding general child phonological development in a (PCC, PWP) plane.

Keywords: Child Speech, Development, Phonological Word, Measurement.

1 Introduction

Quantifying progress in child speech has been of interest in the literature since the 1920s. Nice [1] introduced the average length of sentence (ALS) as such an attempt which, in her words, ‘may well be the most important single criterion for judging a child’s progress in the attainment of adult language’. McCarthy [2] set specific rules on how to count words in the sentence and re-named Nice’s measure as mean length of response (MLR). Brown [3] introduced a similar measure, the mean length of utterance (MLU), counting however morphemes in the utterance which, in his words, is ‘an excellent simple index of grammatical development’. In language sample analysis (LSA) which is widely used by speech-language pathologists (see, for example, Kemp & Klee [4]), the mean length of response (MLR) has found yet another name, mean length of utterance.
in words (MLUw), to distinguish it from Brown’s mean length of utterance in morphemes (MLUm). These two measures were compared by Parker & Brorson [5] for 40 language transcripts of 28 typically developing English speaking children between the ages of 3;0 and 3;10. The two measures were found to be perfectly correlated suggesting that, the simpler to calculate, MLUw may be used instead of MLUm. However, correctness of segments or words is totally ignored in all these measures as they are grammatical and not phonological in nature.

Consonant correctness and its measurement has been discussed just about as long (e.g. Wellman et al. [6], Poole [7]) but Shriberg’s work with colleagues has refined the measure by addressing issues such as distortions (Shriberg & Kwiatkowski [8]) and speech profiles (Shriberg et al. [9]). Their proposed proportion of consonants correct (PCC) measures the number of consonants produced correctly in context in proportion to the targeted consonants in the speech sample. As for whole-word correctness, Schmitt et al. [10] suggested that the measure of whole-word accuracy (WWA) would favorably complement other measures such as the proportion of consonants correct (PCC). They based this result on data they collected from children between the ages 3;0 and 3;6.

Whole words, however, do not only vary in their correctness but also in their complexity and intelligibility. Ingram & Ingram [11] and Ingram [12] proposed to measure phonological word complexity in child speech by the phonological mean length of utterance (PMLU). It is a similar measure to Brown’s grammatical mean length of utterance in that it also measures length of utterance even though utterance in PMLU refers to word length while utterance in MLU refers to sentence length. PMLU measures individual segments (consonant or vowel sounds) in the utterance while MLU measures morphemes. But PMLU differs substantially from MLU in that it does not count all the measurable quantities equally, doubling the count of consonant segments produced correctly in the context of intended target, to emphasize the fact that children’s errors more often occur in consonants (e.g. Ingram [13], Stemberger [14]) and do not vary nearly as much as vowel errors between transcribers (e.g. Powell [15]).

Further, Ingram & Ingram [11] and Ingram [12] introduced the phonological whole-word proximity (PWP), an indirect indication of word intelligibility, as a measurement of the phonological proximity between produced and targeted words in child speech. PWP was defined as the ratio of the produced phonological mean length of utterance, PMLU, to the targeted one in which all the consonants are by definition correct in context. For utterances consisting of more than one word, PMLU and PWP were defined as the arithmetic mean of their corresponding single word values.

Following the proposed whole word phonological measures, PMLU and PWP, several studies have used them to assess sample utterances of monolingual or bilingual, normal or phonologically impaired children across languages. Taelman et al. [16] discussed how to use CLAN (MacWhinney [17]) to compute PMLU from children’s data. Other works on the subject published between 2005 and 2009 are described in Bunta et al. [18] and will not be repeated here.
Bunta et al. [18] compared 3-year old Spanish-English bilingual children to their monolingual peers to compute, among other quantities, PWP and the proportion of consonants correct, PCC. They found that while PWP and PCC differ in general, bilinguals only differ on PCC from their monolingual peers in Spanish. They further found that when comparing the Spanish and English of the bilingual participants, PCC was significantly different but PWP was almost the same. Burrows & Goldstein [19] compared PWP and PCC accuracy in Spanish-English bilinguals with speech sound disorders to age-matched monolingual peers. Macleod et al. [20] compared the change in PWP to that in PCC for two samples of twenty children each, both taken at the age of 18 months and at 36 months. One of the samples involved monolingual English children while the other involved bilingual French-English children. For each sample, their results showed that the change in PWP was larger than that in PCC. Saaristo-Helin [21] measured PMLU and PWP for both typically developing children and children with a specific language impairment acquiring Finnish and concluded that the phonologies of the impaired children largely resembled the ones of younger, typically developing children. Goldstein & Bunta [22] compared PWP and PCC for Spanish-English bilingual children, who have parent-reported language use and proficiency measures commensurate with those of their monolingual peers, to the PWP and PCC for their monolingual peers. Bilingual children did not differ from their monolingual peers in Spanish while they outperformed their monolingual English-speaking peers. Last, Freedman & Barlow [23] examined the effect of phonotactic probability and neighborhood density on PWP and compared it between five Spanish-English children and five age-matched monolingual peers. Phonotactic probability refers to the frequency with which sounds occur and co-occur in the language, while neighborhood density is defined as the number of real words that can be created by adding, substituting, or deleting a phoneme in any word position. No differences were found between bilinguals and monolinguals in the respective languages or between languages, even though bilinguals evidenced greater phonological complexity in Spanish than English on words with low phonotactic probability and low neighborhood density.

Besides phonological whole-word proximity and its related phonological mean length of utterance, Ingram & Dubasik [24] proposed six other measures in a multidimensional assessment of phonological similarity (MAPS) for a complete comparison between children’s utterance samples or within a child’s sample. The number of preferred syllable shapes, the proportion of monosyllables, the phonetic inventory articulation score for onsets and codas and the relational articulation score onsets and codas in word initial position and word final position, respectively.

While most studies cited compare speech sample values for the two phonological measures, PWP and PCC, the comparison between them has not been examined in general; Flipsen et al. [25] has compared the two measures by looking into word intelligibility. As a consequence, results though interesting, are not placed in perspective in child speech development, which, if done, would make them more meaningful and useful for practical applications. The
present study achieves this by obtaining analytically the relationship between phonological word proximity (PWP) and proportion of consonants correct (PCC) in terms of the proportion of vowels (PV) and the proportion of phonemes deleted minus added (PPD). Cumulatively, for all the words in a speech sample, PWP is computed as the weighted average of single-word PWPs and not as their arithmetic mean as done in previous studies. This way, the relationship between PWP and PCC for the whole speech sample is expressed analytically and has the same form as for the single word, enabling us to obtain upper and lower PWP bounds in terms of PCC and PV for minimum and maximum PPD, respectively.

The analytical results for general child speech development are applied to a speech sample of a bilingual normal child traced longitudinally between the ages 2;6 and 3;9. The growth patterns of the child’s computed PCC, PPD and PWP values are obtained and are placed in perspective in general child speech development.

2 Method

2.1 The PWP-PCC relationship

In order to place speech sample values of phonological word proximity (PWP) in a universal perspective in child speech development, we obtain the analytical relationship between PWP and PCC. Before considering the whole speech sample, we take a single word. Ingram & Ingram [11] and Ingram [12] defined PWP as the ratio of the produced phonological mean length of utterance (PMLU) to its targeted counterpart, that is, the ratio of the sum of in context correctly produced consonants in context (called from now on correct consonants) and all produced segments (consonants plus vowels) to weighted targeted segments (targeted consonants plus all targeted segments). We re-arrange this formula first by separating the two terms of the sum in the numerator resulting in the sum of two ratios: correct consonants to weighted targeted segments plus all phones to weighted targeted segments. In turn, each ratio is now expressed as the product of two ratios. The former ratio, call it ratio-1, is written as the correct consonants divided by the targeted consonants, called in the literature proportion of consonants correct (PCC) (Shriberg & Kwiatkowski [8], Shriberg et al. [9]), times the targeted consonants divided by the weighted targeted segments. In terms of the proportion of vowels (PV) to segments in the targeted word, this last ratio, call it p, becomes \((1-PV)/(2-PV)\). Ratio-2 is expressed as the product of the produced segments to targeted segments times the ratio of targeted segments to the weighted targeted segments. In terms of the proportion of vowels (PV) to segments in the targeted word, this last ratio, call it \(p\), becomes \((1-PV)/(2-PV)\). Ratio-2 is expressed as the product of the produced segments to targeted segments times the ratio of targeted segments to the weighted targeted segments. This last ratio is clearly equal to 1-p. Since targeted segments may equivalently be written as the produced segments plus the deleted segments minus the epenthetic segments, the former ratio in the product may be expressed as 1-PPD, where PPD stands for the proportion of phonemes (segments) deleted minus phonemes added. Deleted and epenthetic segments are taken into account since it is known (Ingram, [26]; Stoel-Gammon & Dunn [27], Bernhardt &
Stemberger [28]) that during child phonological development it is not unusual to have targeted consonants deleted and vowels added. It is less usual to have epenthetic consonants, even adjacent to a targeted consonant, in normal as well as disordered children (e.g. Ingram [29], Stemberger [14], Babatsouli [30]).

We have, therefore, derived an expression for the phonological word proximity (PWP) of a single word in terms of three phonological parameters PCC, PV and PPD, as

\[
PWP = pPCC + (1 - p)(1 - PPD) \tag{1a}
\]

\[
p = (1 - PV)/(2 - PV) \tag{1b}
\]

Going now from a single word’s PWP to the PWP of a speech sample consisting of N words, we propose taking the weighted average of the single word PWPs instead of their arithmetic mean so that the cumulative PWP becomes the ratio of the arithmetic mean of the produced single word PMLUs to the arithmetic mean of the targeted PMLUs, i.e.,

\[
PWP = \left(\frac{\sum PMLU^{(p)}}{N}\right) / \left(\frac{\sum PMLU^{(t)}}{N}\right) \tag{2}
\]

The choice of weighted average yields a cumulative PWP of exactly the same form as that of the single word given by Eq. (1a, b), with the three phonological parameters PCC, PV and PPD now computed as weighted averages directly from the whole sample. For example, PCC is now the ratio of the correct consonants produced in context to the targeted consonants in the whole sample. Moreover, the choice of weighted average makes it possible to obtain upper and lower bounds on the cumulative PWP by taking the minimum and maximum PWP, respectively.

In general, the three phonological parameters PCC, PV and PPD vary across speech samples within a child or between children. Eq. (1a, b) shows that the phonological word proximity (PWP) depends linearly on PCC and PPD and, nonlinearly, on PV. However, the PV value may be kept constant when comparing samples by appropriately selecting the words in them. At the early age of a few months, an infant’s PCC is negligible giving PWP as \((1-p)(1-PPD)\). At complete acquisition, attained by normal children usually at school age or later, PCC is almost one, PPD is negligible and, therefore, PWP becomes one independent of PV.

### 2.2 The weighting parameter p

The weighting parameter p which represents the proportion of targeted consonants to weighted targeted segments, weights the contribution of PCC to the value of PWP as shown in Eq. (1a). Similarly, \(1-p\) weighs the contribution of 1-PPD. It may be seen that p, as given by Eq. (1b), monotonically decreases with PV. The minimum PV is 0, yielding 0.5 as the maximum p. Thus, \(1-p\) is
greater than \( p \) for all the values of \( PV \). The maximum \( PV \) is 1 giving 0 as the minimum \( p \). PV norms in adult speech are 0.45 for English and Dutch, 0.5 for Italian and Spanish and 0.55 for Japanese (Ramus et al. [31]). For example at \( PV=0.45 \), \( p=0.35 \) and \( 1-p=0.65 \). For children, the proportion of vowels (PV) in targeted word samples in English usually varies in practice between 0.25 and 0.5. When most words contain consonant clusters, the lower value is approached as will be seen in the samples below. For this range of PV values, \( p \) is proposed to approximately depend on \( PV \) in a linear fashion as

\[
p \approx PV^* (1 + PV^* - PV), \quad PV^* = (3 - \sqrt{5})/2
\]  

This is the one-term Taylor series expansion of \( p \) about \( PV^*=0.382 \), the only acceptable PV value less than 1 for which \( p=PV \). For \( PV \) between 0.25 and 0.5, the root mean square error in approximating \( p \) by Eq. (3) is calculated to be 0.0018, while the maximum error is 0.0038 at \( PV=0.25 \). For the range of PV values in discussion, we compared the approximation given by Eq. (3) to that of a linear interpolation of \( p \) between 3/7 (PV=0.25) and 1/3 (PV=0.5). It turns out that the latter approximation’s root means square error is 0.0026, larger than the former’s. Therefore, it is more accurate to use the linear approximation given by Eq. (3).

A consequence of approximating \( p \) linearly on PV values between 0.25 and 0.5 is the linear approximation of PWP of Eq. (1a) on PV. In this range of PV values, the error in approximating PWP, denoted by \( \Delta(PWP) \), is given in terms of the error in \( p \), denoted by \( \Delta p \), as

\[
\Delta(PWP) = -(1 - PCC - PPD) \Delta p
\]

Since the ratio of the produced segments to targeted segments, 1-PPD, is larger than the proportion of consonants correct, as will be discussed in the subsection that follows, the factor multiplying \( \Delta p \) is greater than zero. It is also smaller than one since PCC and PPD are positive. As a result, the error in approximating PWP using, instead of the exact \( p \), that of Eq. (3) is smaller than the error in approximating \( p \), itself.

2.3 Upper and lower PWP bounds

For a given sample of targeted words, the proportion of vowels (PV) is known and, thus, the PWP value depends on the PCC and PPD values which are measured from the produced speech. It is seen from Eq. (1a) that, for the same PCC, PWP is larger for a smaller PPD. Therefore, PPD’s minimum and maximum values yield upper and lower PWP bounds. We take the smallest PPD to be equal to zero when there is no deletion or epenthesis. The largest PPD is equal to \( (1-PCC)(1-PV) \) when there is no epenthesis, no deleted vowels, and no substitutions for targeted consonants; only correct consonant productions and consonant deletions. Substituting these two
extreme values of PPD in Eq. (1a) we obtain, respectively, the upper and lower PWP bounds as:

Upper bound: \[ PWP_{\text{max}} = PCC + (1 - p)(1 - PCC) \]  
Lower bound: \[ PWP_{\text{min}} = PCC + (1 - 2p)(1 - PCC) \]

Clearly, PWP is larger than PCC, except at complete acquisition when PCC equals one and they become equal. For the same p and PCC values, subtracting the two bounds yields the largest possible spread in PWP values between any children as \( p(1-\text{PCC}) \). For \( p_2 < p_1 \) and \( \text{PCC}_2 > \text{PCC}_1 \), which is the case of more consonants produced correctly in context when the proportion of vowels is larger, subtracting the upper bound on \( \text{PCC}_2 \) from the lower bound on \( \text{PCC}_1 \) gives the largest spread of PWP as \( (2p_1-p_2)(1-\text{PCC}_2)+2p_1\Delta(\text{PCC}) \). The PWP bounds may also be used, as follows, to determine sufficient conditions on the changes of p and PCC across samples in order to have an increasing PWP.

For the same targeted sample, PCC generally changes with a child’s age. It also generally changes within a child and between children for two targeted samples of the same p but of distinctly different word constituency, for example, singleton and cluster words. Taking the lower PWP bound for the larger PCC, say \( \text{PCC}_2 \), and the upper PWP bound for the smaller \( \text{PCC}_1 \), we derive from Eqs. (5) and (6) that \( \text{PWP}_2 \) will be for sure larger than \( \text{PWP}_1 \) only when \( \text{PCC}_2 \) is larger than \( (1+\text{PCC}_1)/2 \), meaning that \( \text{PCC}_2 \) will necessarily have to be larger than 0.5. When \( \text{PCC}_2 \) is smaller than \( (1+\text{PCC}_1)/2 \), then \( \text{PWP}_2 \) is either larger or smaller than \( \text{PWP}_1 \) depending on the values of \( \text{PPD}_1 \) and \( \text{PPD}_2 \). This is investigated further in the subsection that follows.

With p also changing across samples, \( \text{PWP}_2 \) is for sure larger than \( \text{PWP}_1 \) only when \( \text{PCC}_2 \) is larger than \( 1-[p_1(1-\text{PCC}_1)/(2p_2)] \), meaning that \( \text{PCC}_2 \) will necessarily have to be larger than \( 1-[p_1/(2p_2)] \). However, since \( \text{PCC}_2 \) was taken larger than \( \text{PCC}_1 \) we must have \( p_2 \) larger than \( p_1/2 \), which from Eq. (1b) yields, \( \text{PV}_2 \) smaller than \( 2/(3-\text{PV}_1) \).

2.4 PWP and PCC changes

Generally, PWP changes between speech samples within a child and between children. It is of interest to determine the magnitude of PWP change in terms of changes in PCC, PV and PPD. In doing so, an answer will be given to the question: which is a better measurement PCC or PWP, in the sense of discriminating performance between speech samples?

Consider two targeted samples with their corresponding single productions or one targeted sample with two different productions. In either case, the parameters in the two sets are distinguished by the subscripts 1 and 2 and the change in their values (2 minus 1) is denoted by the Greek capital letter \( \Delta \). Then, subtracting Eq. (1a) with subscript 1 in the parameters from Eq. (1a) with
subscript 2 in the parameters, results in the following expression for the change of PWP, \( \Delta(PWP) \),

\[
\Delta(PWP) = -(1 - PCC_2 - PPD_2)\Delta p + p_1\Delta(PCC) - (1 - p_1)\Delta(PPD) \tag{7}
\]

The change in \( p \), \( \Delta p \), in terms of the change in \( PV \), \( \Delta(PV) \), is obtained similarly from Eq. (1b) as

\[
\Delta p = -(1 - p_1)(1 - p_2)\Delta(PV) \tag{8}
\]

However, for \( PV \) values between 0.25 and 0.5, \( p \) may be approximated by Eq. (3), as discussed above, which yields a much simpler expression for \( \Delta p \),

\[
\Delta p \approx -(3 - \sqrt{5})/2 \, \Delta(PV) \tag{9}
\]

We see from Eq. (7) that PWP increases in proportion to increases in \( PV \) and \( PCC \) and a decrease in \( PPD \). When the same sample is targeted or when two targeted samples have the same \( PV \), it is concluded from Eq. (7) that PWP increases whenever the change in \( PCC \) is larger than the change in \( PPD \) divided by \( 1-PV \), that is,

\[
\Delta(PWP) \geq 0 : p\Delta(PCC) \geq (1 - p)\Delta(PPD) \tag{10}
\]

We note that positive \( \Delta(PCC) \) and negative \( \Delta(PPD) \) automatically satisfy (10), resulting to a positive change in PWP.

Changes of \( PCC \) and \( PPD \) are, however, bounded since \( 1-PPD-PCC \) is bounded above and below by 1 and 0, respectively, as discussed above. This means that when changes in \( PCC \) and \( PPD \) values have opposite signs, their magnitudes may vary anywhere between 0 and 1. But when the \( PCC \) and \( PPD \) changes have the same sign, they are bounded above by the sum of their magnitudes not exceeding 1. That is, the bounds on \( \Delta(PCC) \) and \( \Delta(PPD) \) are given as

\[
\text{Bounds on } \Delta(PCC), \Delta(PPD) : \quad 
\begin{align*}
&i) \Delta(PCC)\Delta(PPD) < 0, |\Delta(PCC)| + |\Delta(PPD)| \leq 1 \\
&ii) \Delta(PCC)\Delta(PPD) > 0, 0 \leq |\Delta(PCC)|, |\Delta(PPD)| \leq 1
\end{align*} \tag{11}
\]

A practical question that may arise is: which is a better measurement, PWP or \( PCC \), with regard to discriminating performance in two productions? In other words, what is really the difference between the two measurements in practice? We will answer this, generally, by taking the same targeted sample and two different productions and compare the magnitude of the PWP change to that of \( PCC \). Practitioners may want either a small or a large disparity between the two
values of the measurement they use, depending on whether they want to discriminate performance in the two productions. Taking the absolute value of $\Delta(PWP)$ of Eq. (7) with $\Delta p=0$ and comparing it with the absolute value of $\Delta(PCC)$, we find that it is larger only when $\Delta(PCC)$ $\Delta(PPD)$ satisfy the following conditions:

$$\left|\Delta(PWP)\right| \geq \left|\Delta(PCC)\right|;$$

\[
i) \Delta(PCC)\Delta(PPD) < 0, \quad \left|\Delta(PCC)\right| \leq \left|\Delta(PPD)\right| \\
ii) \Delta(PCC)\Delta(PPD) > 0, \quad (1 + p)\left|\Delta(PCC)\right| \leq (1 - p)\left|\Delta(PPD)\right|
\] (12)

in which $\Delta(PCC)$ and $\Delta(PPD)$ are bounded according to (11). If conditions (12) are violated, the magnitude of $\Delta(PCC)$ is larger than the magnitude of $\Delta(PWP)$.

Considering (10) and (12) simultaneously, since $(1-p)/p$ is larger than $(1-p)/(1+p)$ we conclude that only when $\Delta(PPD)$ is negative, $\Delta(PWP)$ may be positive and, at the same time, larger than $\Delta(PCC)$. Even then, this will be true only when

$$\left|\Delta(PCC)\right| \leq \left|\Delta(PPD)\right|, \quad (1 + p)\left|\Delta(PCC)\right| \leq (1 - p)\left|\Delta(PPD)\right|$$ (13)

with $\Delta(PCC)$ and $\Delta(PPD)$ bounded according to (11).

When comparing two productions between two stages in general, the largest possible $\Delta(PWP)/\Delta(PCC)$ as obtained in the preceding subsection is $2p+p(1-PCC_2)/\Delta(PCC)$, where $\Delta(PCC)$ is positive. This will approach $2p$ from above as stage 2 reaches complete acquisition, where $PCC_2$ is almost 1. Therefore, $\Delta(PWP)/\Delta(PCC)$ will generally be larger than 1 when necessarily $(1-2p)\Delta(PCC)<p(1-PCC_2)$ in accordance with the first case of (12), as $\Delta(PPD)$ is $-(1-PCC_1)(1-PV)$. Last, when two speech samples are compared in general, and $\Delta(PCC)$ is positive while $\Delta(PPD)$ is negative, we are in the first case of (12) and, therefore, the ratio $\Delta(PWP)/\Delta(PCC)$ will be smaller than 1 only when the magnitude of $\Delta(PCC)$ is larger than the magnitude of $\Delta(PPD)$.

These observations have also implications on the age dependence of the PWP change relative to the PPC change between two speech samples. We take sample-2 to be much easier to produce correctly than sample-1. For example, take the words in sample-2 to contain only singletons and all the words in sample-1 to contain consonant clusters. Then, near complete acquisition, the first term of Eq. (7) is negligible, $\Delta(PPD)$ is expected to be negative approaching zero before $\Delta(PCC)$ which is expected to be positive and, thus, $\Delta(PWP)/\Delta(PCC)$ becomes smaller than one, approaching $p_1$. As we get away from complete acquisition, the first term of Eq. (7) is positive since $p_2$ is smaller than $p_1$, and we expect that in general $\Delta(PWP)/\Delta(PCC)$ will increase with decreasing age with its largest value, obtained in the preceding subsection, being equal to $2p_1+(2p_1-p_2)(1-PCC_2)/\Delta(PCC)$ as $\Delta(PPD)$ is $-(1-PCC_1)(1-PV_1)$. 


Therefore, \( \Delta(PWP)/\Delta(PCC) \) will generally be larger than 1 when necessarily \((1-2p_1)\Delta(PCC) < (2p_1-p_2)(1-PCC_2)\). Since \( \Delta(PPD) \) is negative and \( \Delta(PCC) \) is positive for the case at hand, the necessary and sufficient condition for \( \Delta(PWP) \) to be larger than \( \Delta(PCC) \), on use of Eq. (7), becomes

\[
\Delta(PCC) \leq |\Delta(PPD)| + |\Delta p|(1 - PCC_2 - PPD_2)/(1 - p_1)
\]

Comparing (14) with (12), we see that when the proportion of vowels is not the same between targeted samples, the range of \( \Delta(PCC) \) values for which \( \Delta(PWP) \) is greater than \( \Delta(PCC) \) is larger. This means that, when \( \Delta(PCC) \) is smaller than \( \Delta(PPD) \), condition (14) is automatically satisfied and \( \Delta(PWP) \) is for sure larger than \( \Delta(PCC) \).

If one wants to measure only consonants in a speech production and ignores vowels altogether, the phonological word proximity (PWP) becomes what we will call ‘phonological word consonants proximity’ (PWCP). It is interesting to compare directly the two measures, PCC and PWCP, as the first measure counts consonants only when they are produced correctly in context while the second measure counts consonants when they are produced correctly, independent of context, even though the correct consonants in context are counted twice. In this case, the magnitude of \( \Delta(PWCP) \) is larger than the magnitude of \( \Delta(PCC) \) only when the changes of the proportion of consonants deleted minus added, \( \Delta(PCD) \), and of \( \Delta(PCC) \) satisfy the inequalities given by (12) with PWCP in place of PWP, PCD in place of PPD, and \( p=0.5 \) since \( PV=0 \).

3 The speech data

The data is taken from a Greek/English bilingual female child’s speech in English from age 2 years and 6 months to age 3 years and 9 months. Her spontaneous speech in English during thirty-minute daily routine interactions with the first author was recorded and, subsequently, time aligned and phonetically transcribed by the first author in a CLAN (MacWhinney [17]) database, using the International Phonetic Association (IPA) symbols. The purpose here is to compute the child’s phonological word proximity (PWP) and trace its monthly change with age together with its components PCC and PPD, placing them in perspective within general child speech development, as examined above. For this reason, the same sample of targeted words was considered at each age. The sample taken consists of 25 words which were selected in order to satisfy two main criteria: first, that the same 25 words could be found in the child’s speech at least once a month between the ages of 2;6 and 3;9 and, second, that they are a mixture of different complexities in terms of consonant place and manner of articulation, consonant position in the word, singleton consonants and consonant clusters, and number of syllables. As expected, the child’s natural utterances contained a varying number of words, so that the 25 word types in the sample were extracted from different utterances, on a different day or week of the month, in general. However, the first production
of each word in the month was included in the sample, so that the child’s age increased by about a month between word productions. The targeted words in the child’s speech sample in alphabetical order are: again, also, and, another, any, bag, blanket, close, clothes, come, don’t, English, finished, give, go, hold, inside, make, play, ready, the, together, took, why, wolf.

It is expected that when the speech sample considered changes substantially, PWP will in general also change as its components PV, PCC and PPD vary between samples. In the present study, this is exemplified by selecting a second sample comprising of all the word types in the child’s speech at the age of 3;0 that contain at least one consonant. In order to have a large sample of word types, we selected the words within ten days after the child’s third birthday upon first production. As a result, the following 158 targeted word types are included in the sample: accident, again, airplane, already, also, and, animals, another, any, back, bag, balance, beach, because, bed, birdie, bit, blanket, block, boots, box, bread, breakfast, bridge, bring, brush, bunnies, bunny, called, case, cat, chicken, chicken, chocolate, clean, clock, close, clothes, colors, come, cotton, counter, crunchy, cucumber, destroying, dirty, dog, dolphin, don’t, donkeys, door, downhill, downstairs, dream, English, every, excellent, falling, farm, finished, fish, five, floor, food, found, full, garden, give, glasses, go, grab, grandpa’s, hair, have, head, help, here, hide, hold, inside, juice, kettle, kiss, later, leave, left, lick, licking, look, loose, lost, make, meatballs, middle, milk, moon, more, morning, myself, nice, no, nose, now, once, open, outside, panty, pieces, piglet, plain, play, polite, potatoes, pull, pushing, put, puzzles, rain, ready, red, remember, restaurant, scatter, seeds, shopping, shoulder, shower, slide, small, someone, space, spaghetti, stopped, street, stroller, sunscreen, table, teacher, the, there, things, throw, toast, today, together, took, train, trash, trouble, umbrella, upset, washed, what, where, why, wolf, working, yes. The changes in the phonological parameters PWP, PCC, and PPD between the two samples will be calculated and viewed in relation to the method and analysis presented above.

4 Numerical results

4.1 Child data in general

The method and analysis for general child speech development, as far as phonological word proximity (PWP) and its components are concerned, were examined above. Here, numerical results will be presented graphically. In terms of its components, PWP is given by Eq. (1a, b). In a three-dimensional (PCC, PV, PWP) rectangular coordinate system, all children’s PWP values lie inside a body which is bounded above and below by the surfaces given by Eq. (5) and Eq. (6), respectively. These bounds on PWP are calculated for PCC values ranging from 0 to 1 and PV from 1/3 to 3/4 and are plotted in a (PCC, (2-PV)/(1-PV), PWP) space where they are easy to view. Note that (2-PV)/(1-PV)=1/p ranges from 2.5 to 5. The results are shown in Fig. 1.
The black surface in Fig. 1 is the upper bound while the red surface is the lower bound. These surfaces are shaped as hyperbolic paraboloids and they form the wings of a phonological word proximity (PWP) glider. When we have the same sample of targeted words, PV does not change and children’s PWP values lie inside the glider’s section which, as seen in the figure, is triangular with its base equal to $p$ at zero PCC. As PV increases, $1/p$ also increases and the base of the bounding triangle becomes smaller. The largest triangular section base in the figure is equal to 0.4 at $1/p=2.5$ (the left end of the figure) and the smallest is equal to 0.2 at $1/p=5$ (the right end of the figure). At complete acquisition, PCC is one, PPD is zero and PWP becomes one independent of PV. This defines the glider’s ceiling shown in the figure along $1/p$.

When PV is the same between speech samples, it was shown in the method and analysis above by conditions (10) - (13) that what matters in the change of PWP is the change of PCC relative to the change of PPD. This is shown schematically in Fig. 2. Regions of positive and negative $\Delta$(PWP) are bounded by the blue-green and blue-red lines respectively in a $\Delta$(PCC), $\Delta$(PPD) plane. On the blue line which represents the equation in (10), PWP remains unchanged between the two speech samples. In the figure, the irregular hexagon bounding $\Delta$(PCC) and $\Delta$(PPD) values represents the equations in (11).

Discriminating measurements between two productions of the same targeted speech sample is of interest to practitioners. To this end, a comparison of the magnitude of $\Delta$(PWP) to that of $\Delta$(PCC) was made in the method above and was given by (12). In Fig. 3, the regions where (12) is satisfied are drawn in dashed lines in a $\Delta$(PCC), $\Delta$(PPD) plane. That is, the magnitude of $\Delta$(PWP) is
larger than that of $\Delta(PCC)$ in the dashed regions and smaller in the rest. As in Fig. 2, changes in PCC and PPD are bounded by the irregular hexagon shown also in this figure.

Fig. 2. PWP changes ($>0$, $<0$) relative to PPD, PCC changes ($\Delta$) in child speech development.

Fig. 3. The shaded region of the $\Delta(PPD)$, $\Delta(PCC)$ plane where the magnitude of $\Delta(PWP)$ is larger than that of $\Delta(PCC)$ in child speech development.

4.2 The phonological data of this study’s child

For the bilingual child’s speech sample described above, which was taken monthly between the ages 2;6 and 3;9, we calculate PCC, PPD and subsequently
PWP. Cumulatively for the 25 words in the speech sample, PCC and PPD are computed as two ratios, respectively. The first is the ratio of the number of produced correct consonants divided by 63, the total number of consonants in the speech sample, while the second ratio is the number of deleted consonants and vowels minus the added ones divided by the total number of segments in the speech sample, which is 109. Thus, the proportion of vowels, PV, in the targeted sample is 46/109 or 0.42. The developmental PCC and PPD values were subsequently computed monthly. In turn, PWP was computed using Eq. (1a, b).

The numerical results are depicted in solid lines in Fig. 4. We see that three distinct stages of phonological development may be identified, associated with the growth patterns of the phonological parameters PCC, PPD and PWP. In each stage their growth pattern may be fitted by a straight line shown by the dashed line in Fig. 4. As a result, the overall developmental pattern is tri-linear.

The first stage lasts for three months from age 2;6 to age 2;9 and is progressive in PWP as, according to (10), PCC increases and PPD decreases. The increase in PWP is from 0.32 to 0.70 and is of the double-logistic type. The second stage lasts for seven months and is nearly cyclic as PCC, PPD and also PWP fluctuate, even though non-uniformly, about the same level. In fact, at this stage, PCC has a strange attractor of the value 0.44 and PCC has a strange attractor of the value 0.16 resulting in a strange attractor for PWP of the value 0.69. The third stage is again progressive with PCC increasing, PPD decreasing and PWP increasing from 0.70 to 0.90 in a double-logistic fashion with a nearly cyclic regime separating the two logistic-like sub-stages within this stage.
The existence of a plateau stage during speech development has been reported in the literature on a qualitative basis (e.g. Ingram [13]). Moreover, the plateau is the well known middle stage of the U-shaped learning pattern in developmental psychology (e.g. Werker et al. [32]). Here, on a quantitative basis, we see that this stage exists and is, in fact, nearly cyclic.

Now, in view of the analysis and discussion above, it will be interesting to compare the ratio $\Delta(\text{PWP})/\Delta(\text{PCC})$ in the child’s speech performance between different ages. Calculating this ratio between the first and last speech samples in stage-1 and in stage-3 we obtain 0.73 and 0.50 respectively. In both stages, the ratio is smaller than 1 since $\Delta(\text{PCC})$ is 0.118 and 0.372 in the two stages respectively, while $\Delta(\text{PPD})$ is negative and its magnitude is smaller at 0.068 and 0.148 and, therefore, the first of (12) is violated in both stages. The targeted sample is the same all along with $p=0.37$. Then, according to the method and analysis above, the ratio will approach 0.37 when two speech productions are compared near this child’s or any child’s complete phonological acquisition. In fact, the ratio between the last two months (3;8 and 3;9) in stage-3 is 0.46, that is, even closer to 0.37 than the average ratio 0.50 over the whole stage-3.

The bilingual child’s developmental PWP and PCC values are placed in perspective in general child speech development by comparing them to the upper and lower bounds on phonological word proximity (PWP) given by Eqs. (5) and (6), respectively. This comparison is depicted graphically in Fig. 5 where PWP is plotted versus PCC.

The upper bound on PWP given by Eq. (5) for all children is represented by the black dashed line in Fig. 5, while the lower bound given by Eq. (6) is the red dashed line. These two lines meet at the point PCC=1, PWP =1 forming a triangular bounding region in which all children’s values lie during speech development. The vertices of the triangle’s base are given by the points (0,1-p=0.63) and (0, 1-2p=0.26). This triangular region is the section of the PWP glider of Fig. 1 at 1/p=2.7. However in practice, as it is also the case here, a child’s speech samples are taken following increased production of intelligible words. Thus, it is expected that the smallest computed PCC will be larger than zero and the largest computed PPD will be smaller than all children’s maximum value which is given by $(1-PV)(1-PCC)$, resulting in a smaller bounding region for PWP. In Fig. 5, this region for the bilingual child is defined inside the black solid lines because the child’s minimum PCC is 0.33 and maximum PPD is 0.23 at this PCC. Inside this trapezoid, the child’s actual developmental (PCC, PWP) values are shown by dots. Their correlation is rather chaotic in the nearly cyclic regime of their developmental paths where PCC has a strange attractor of the value 0.44 and PPD has a strange attractor of the value 0.16.
Now, it will be of interest to compare the bilingual child’s PWP, PCC and PPD values between productions of the targeted sample considered above and the larger targeted sample at age 3;0 described in the methodology. The 158 words in the larger sample have $PV=0.37$ ($p=0.39$). The child’s production resulted in PCC equal to 0.53 and PPD equal to 0.127. Then, on use of Eq. (1), we get 0.74 for PWP. The corresponding values for the smaller targeted sample at age 3;0 are $PWP=0.713$, $PCC=0.47$ and $PPD=0.145$. This shows that the smaller targeted sample traced along development is more difficult for the bilingual child to produce correctly than the larger sample. Therefore, the PCC and PWP growth patterns of Fig. 1 are conservative. Even though the two targeted samples differ significantly in size, we see that the disparity in their PWP, PCC and PPD values is relatively small with the largest disparity being that of PCC. Calling sample-2 the larger targeted sample, we have $\Delta(PCC)=0.06$, $\Delta(PPD)=-0.018$ and $\Delta(PWP)=0.027$. The ratio $\Delta(PWP)/\Delta(PCC)$ is 0.45 smaller than 1 since $\Delta(PPD)$ is negative and $\Delta(PCC)$, $\Delta(PPD)$ are such that (14) is violated.

It is also of interest to compare the child’s performance between words that contain singleton consonants and words that contain consonant clusters (at least two consonants next to each other). We call here sample-1 the 89 cluster words included in the 158 words sample whose weighted proportion of vowels is $PV_1=0.43$. We call sample-2 the 69 singleton words with $PV_2=0.34$. The child’s corresponding productions give $(PCC_1, PPD_1, PWP_1)$ and $(PCC_2, PPD_2, PWP_2)$, respectively, as: $(0.52, 0.18, 0.70)$ and $(0.66, 0.03, 0.86)$. We see that the child
produces singleton words better than cluster words. The disparity in the values of PCC, PPD and PWP is expressed in terms of the ratio \( \Delta(PWP)/\Delta(PCC) \) which becomes 1.14. It is larger than 1 since \( \Delta(PPD) \) is negative and its magnitude is larger than \( \Delta(PCC) \), so that (14) is automatically satisfied. As discussed in the methodology following Eq (7), the ratio will overall decrease with age and it will approach \( p_1 = 0.36 \) near complete phonological acquisition, where PCC\(_2\) is nearly 1 and PPD is negligible for both singleton and cluster words.

### 5 Conclusions

Measurements of phonological word proximity (PWP) and proportion of consonants correct (PCC) in child speech development are placed in perspective having obtained analytically their relationship as well as upper and lower bounds in terms of the proportion of phonemes deleted minus added (PPD) and the proportion of vowels (PV). Child data reveal the existence of a nearly cyclic stage with strange attractors for PCC and PPD and, consequently for PWP, before the final progressive double-logistic stage in phonological development, and the relative advantages of using PWP instead of PCC in discriminating performance between speech samples within a child and between children, of the same or different age.

### References


