Parhelic-like Circle and Chaotic Light Scattering

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Abstract. The propagation of light in foams creates patterns which are explained by the Geometrical Theory of Diffraction, and some of these patterns are known as Parhelic-like circle or Parlaseric circle. We also present the analogy between the atmospheric phenomena known as Parhelic circle, Sun dogs and Sun pillars and the patterns obtained from light scattering in foams.

Keywords: Parlaseric circle, Parhelic circle, Plateau borders, Geometrical Theory of Diffraction.

1 Introduction

Scattering problems are at very heart of Physics, from Celestial to Quantum Mechanics, with particles or waves, we are always looking for a target. In our experiment, we are observing the light scattering in foams, and the results can be applied in Acoustics, Optics and Spectroscopy. Light through foams presents a complex behavior, for example, though the laws of ray reflection and refraction are simple, the boundary conditions for the light scattering are very difficult to be determined precisely due to many awkward technical aspects, such as nonlinearities, or if the thickness of the liquid films is sufficiently close to the wave length of visible light, there is light interference and thus produce the iridescent colors of soap bubbles.

In this paper we discuss the chaotic scattering, diffusion and some aspects of the interface between wave and geometric optics. We have observed that the light scattering dynamics in foams can present two main process: a diffusive one related to Gaussian process and another one related to chaotic dynamics, similar to those observed in chaotic saddles, with some rays of light bouncing back and forth for a certain time, and leaving it through one of several exits. In addition to those behaviors, between geometrics and wave optics, we also have observed the phenomena of the theory of geometrical refraction, with the parlaseric circle.

2 The experimental apparatus

This experiment involves light scattering in foam. The foam is obtained by shaking a liquid inside a transparent box consisting of two parallel Plexiglas
plates separated by a gap (19.0 x 19.0 x 2.0 cm\(^3\)). We can inspect the profiles of light scattered inside the foam and we have used a photographic camera to detect the resulting light patterns. The box contains air and an amount of dishwashing liquid diluted in water (V = 115 cm\(^3\)). The surfactant is Lyner Alkylbenzene Sulfonate (LAS), with surface tension around 25 dyne/cm and refractive indices equal to 1.333 for the liquid and 1.0 for the air. The light sources used when photographing the light scattered by the foam were laser diodes with colors red (635 nm), green (532 nm), and blue (405 nm).

Fig. 1. In (a) is shown the cross section of a Plateau border. The example of ray tracing in a Plateau border (b). The diagram of the experiment is shown in (c).

Fig. 2. Image of the laser beam in a Plateau border.
We use foams composed of bubbles in a network of surfactant solution and some single structures of foams known as Plateau borders and vertex. The Plateau border is an edge of foam in a junction of three soap films, while the vertex is a place where four Plateau borders meet.

### 3 Diffusion and Chaotic scattering

Using experiments involving the transport of light in foams, we have observed two main processes of light transport \[1\]: a diffusive one related to a Gaussian function, and another one related to a chaotic dynamics, similar to those observed in chaotic saddles, in which a ray of light bounces back and forth for a certain time in the scattering region, and leaves it through one of several exits, as it is shown in Figure 3 which represents a slice of the profile of light scattering. In this figure, we present the result obtained from the experiment involving light scattering in foams, in which a laser beam is injected in liquid foam inside a box.

![Light intensity plot](image.png)

**Fig. 3.** Light intensity plot of the light scattering in foam.

The scattering process spreads the light and limits the depth of light penetration, creating a center glow located just above the interface of the box and the foam. We can represent the overall behavior of the light scattering \(g(x,y)\) in foams with the following model:
in which the first term represents the diffusive process present in all places, while the effects of the chaotic dynamics \( f(x,y) \) are present mainly at the center of the foam, around the laser beam direction.

\[
g(x, y) = \frac{1}{2\pi\sigma^2} e^{\frac{x^2+y^2}{2\sigma^2}} + f(x, y),
\]

4 The Parlaseric Circle

Besides these phenomena, we also observed the formation of some caustics. One of these caustics is the light pattern involving the parlaseric circle [2], explained by the Theory of Geometrical Diffraction [3].

The parlaseric circle is a luminous ring generated by light scattering in foams or soap bubbles. In analogy to the atmospheric phenomena known as parhelic circle, sun dogs, and sun pillars, we have named the features of the patterns observed as parlaseric circle, laser dogs, and laser pillars. The triangular symmetry of the Plateau borders is analogous to the hexagonal symmetry of ice crystals which produce these atmospheric phenomena. Working with one Plateau border at a time, we have observed wave optics phenomena that are not used in the explanation of the atmospheric phenomena, such as diffraction and interference.
Fig. 5. The parlaseric circle, laser dogs and laser pillars obtained with green light.

Fig. 6. The parlaseric circle, two laser dogs and laser pillars obtained with blue light.
The main features of these patterns are the following: the laser spot and the laser dogs are always inscribed at the circumference of the parlaseric circle, and they are explained by the laws of geometrical optics. The lines crossing the laser spot are explained by the wave optics, and they represent the typical Fraunhofer diffraction of a triangle. The parlaseric circle is explained by the Theory of Geometrical Diffraction [3].

According to the Geometrical Theory of Diffraction suggested by Keller [3], when a light beam hits a straight edge obliquely, there is a cone of diffracted rays $u_c$ and the cross section of this cone is a circle given by:

$$u_c = D u_i e^{i k r},$$

(2)

where $D$ is a diffraction coefficient, $u_i$ is the incident field, $r$ is the distance between the edge and the screen, and $k=2\pi/\lambda$ is the wave number of the incident field with wavelength $\lambda$. This is the case when a laser light hits some structures of foams known as Plateau border, as previously observed by Tufaile and Tufaile [4].

Fig. 7. According to the Geometrical Theory of Diffraction, when a light beam hits a straight edge obliquely, there is a cone of diffracted rays. $\phi$ is the semi angle of the diffraction cone that is equal to the incident angle measured from the laser beam direction.
The diagram of Figure 8 shows some important aspects of the parlaseric circle pattern, in which LS represent the laser spot, LD1 and LD2 are the laser dogs and the laser pillars are represented by the lines of diffraction and interference $\eta_1$, $\eta_2$, and $\eta_3$. Depending on the angle of incidence of light we can observe two or four laser dogs.

**Laser Pillars and The Parlaseric Circle**

The Plateau border has three sharp edges, and therefore the resulting point spread function produces a six pointed star.

$$U(\xi, \eta) \approx \iint f(r,s) e^{i \frac{2\pi (r^2 + s^2)}{d^2}} dr ds \Rightarrow \eta_1(\xi) = \xi \sqrt{3}, \eta_2(\xi) = -\xi \sqrt{3}, \eta_3(\xi) = 0$$

Fig. 8. Laser pillars and the parhelic circle pattern diagram. LS is the laser spot, LDs are the laser dogs, $\eta_n$ are the laser pillars.

### 5 Vertex Diffraction

In addition to the previous case, we have inspected the case known as vertex diffracted ray, when light hits a vertex formed by a junction of four Plateau borders, in which the corresponding diffracted waves are spherical with vertex at their center. Keller suggested that the field on the diffracted ray for this case is given by:

$$u = C \phi \left( \frac{e^{ikr}}{r} \right).$$

The images in Figure 9 and Figure 10 obtained in our experiment show concentric fringes, could be the vertex diffraction, and in this case the amplitude varies as $r$, since the cross sectional area of a tube of diffracted rays is proportional to $r^2$, according to Keller.
However, when we have moved the screen where the images were obtained, the interference pattern remained the same and the width of the fringes change, indicating that this is spherical wave of diffraction, resembling the interference patterns observed in the case of Newton rings or some patterns obtained in the Michelson interferometer. Increasing the angle of laser beam incidence and the axis of the Plateau border, the size of the fringes decrease.

Fig. 11. Another image of the circular fringes at the center of the parlaseric circle.
By inspection of the patterns like those in Figures 9, 10 and 11, we are exploring the aspects of thin-film interference which occurs in a curved thin liquid film and, trying to associate with the case of Pohl interferometer, that is a device based in the amplitude splitting interference [5].

Conclusions

We are proposing the combination of two main processes of the light transport in foams: a diffusive one related to Gaussian function, and another one related to chaotic dynamics. Just considering the aspects of the geometrical optics, the curvature of soap film structures cause incident light to be scattered in different directions, because the direction of the rays leaving the soap film boundaries can vary greatly for the same incident angle with a small positional offset. Besides this, we have obtained some interesting patterns of light diffraction and compared them with the theory of geometrical diffraction suggested by Keller.

The classification of the optical phenomena somehow involves the scale of the scatterer, and the Plateau border is not just a regular prism. The pattern of the parlaseric circle present some features of the geometrical optics, wave optics and the theory of geometrical diffraction. All these phenomena are related to the nonlinear effects of light transport in foams, with applications in image processing, construction of new optical elements to generate halos or spherical waves, and with the possibility of improvements in the studies of atmospheric optics. Because all these phenomena can be observed with a tabletop experiment, they can be performed in order to get a better understanding of some concepts of optics, such as the Huygens principle.

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