Chaotic Modeling and Simulation (CMSIM) 3: 281–289, 2016

A New Dynamical Control Scheme to Control of Abnormal Synthetic ECG Signals

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Abstract. Spurious coupling between pacemaker components may turn normal ECG signals into chaotic ones. Present study introduces a new chaos control approach known as dynamical control to retain normal signals. To this end, phase space diagram method is used for comparing between before and after of control. The obtained results confirm that the proposed method is effective in enforcing the heart to reassume a limit cycle.
Keywords: Chaos, Dynamical Feedback Control, Electrocardiogram (ECG).

1 Introduction

The study of cardiac system dynamics within the framework of Chaos Theory has found significant progress in developing new methods to overcome the real-world challenges of heart failure [1–5]. The interest of the approach lies in the fact that the electrical behavior of the heart may be chaotic duo to the abnormal functioning of cardiac pacemakers [3,6]. On the other hand, the regularity of cardiac signals as a result of normal functioning of the cardiac pacemakers [3,7] demands new approaches to enforce the heart to reassume a stable limit cycle.
The stabilization of unstable desired orbits can be performed by various methods such as discrete OGY method [8], time-delayed feedback approach (TDF) [9] and extended time-delayed feedback (ETDF) control technique [10]. However, observer dependence is one of the main challenges of methods mentioned in feasible implementations. So, present study introduces *dynamical control* as a new control scheme for stabilization of cardiac signals [11].
Here, a system of three coupled modified delayed van der Pol oscillators [14] is used as a mathematical model to describe heart rhythms dynamics. The dynamical structure of the model is investigated through phase space diagrams and then based on dynamical control approach a controller is proposed for controlling chaos in the system.
The rest of the paper was organized as follows. In Sect. 2 the mathematical model used in this study is described. The proposed dynamical control

Received: 19 April 2015 / Accepted: 14 February 2016

ISSN 2241-0503

approach and one-parameter families of chaotic maps which are cornerstone of the proposed dynamical control method are explained in Sect. 3. Results are discussed in Sect. 4. Finally, summary and outline are presented in Sect. 5. Furthermore, Sect. A includes a brief introduction on the heart and its electrical activity.

2 Model Description

The model used here for simulating ECG signals is an extending of the model proposed in Ref. [13]. First, each of the natural pacemakers of the cardiac system (AV node, His-Purkinje fibers and SA node) is modeled by a unique modified delayed van der Pol oscillator. Then, by suitable coupling of them, dynamical behavior of an electrocardiogram signal is simulated. Electrocardiogram is a procedure for quantifying the electrical potential and so the electrical activity of the heart and ECG recording is one of the simple clinical approaches for investigation of the heart health and its proper functioning [15].
The proposed model is as follow [14]

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -a_{SA}x_2(x_1 - w_{SA_1})(x_1 - w_{SA_2}) - x_1(x_1 + d_{SA})(x_1 + e_{SA}) \\ \quad + k_{SA-AV}(x_1 - x_3^{\tau_{SA-AV}}) + k_{SA-HP}(x_1 - x_5^{\tau_{SA-HP}}), \\ \dot{x}_3 = x_4, \\ \dot{x}_4 = -a_{AV}x_4(x_3 - w_{AV_1})(x_3 - w_{AV_2}) - x_3(x_3 + d_{AV})(x_3 + e_{AV}) \\ \quad + k_{AV-SA}(x_3 - x_1^{\tau_{AV-SA}}) + k_{AV-HP}(x_3 - x_5^{\tau_{AV-HP}}), \\ \dot{x}_5 = x_6, \\ \dot{x}_6 = -a_{HP}x_6(x_5 - w_{HP_1})(x_5 - w_{HP_2}) - x_5(x_5 + d_{HP})(x_5 + e_{HP}) \\ \quad + k_{HP-SA}(x_5 - x_1^{\tau_{HP-SA}}) + k_{HP-AV}(x_5 - x_3^{\tau_{HP-AV}}). \end{cases} \tag{1}$$

where $x_i^\tau = x_i(t-\tau)$, $i = 1, ..., 6$, τ represents time delay and $k_\circ$ symbolizes coupling terms. Then, the ECG signal is built from the composition of these signals as follows:

$$ECG = \alpha_0 + \alpha_1 x_1 + \alpha_3 x_3 + \alpha_5 x_5. \tag{2}$$

In present study, k_{SA-AV} was taken as a control parameter. The remaining parameters were fixed at the values suggested by the original paper [14] as $a_{SA} = 3$, $a_{AV} = 3$, $a_{HP} = 5$, $w_{SA_1} = 0.2$, $w_{SA_2} = -1.9$, $w_{AV_1} = 0.1$, $w_{AV_2} = -0.1$, $w_{HP_1} = 1$, $w_{HP_2} = -1$, $d_{SA} = 3$, $d_{AV} = 3$, $d_{HP} = 3$, $e_{SA} = 4.9$, $e_{AV} = 3$, $e_{HP} = 7$, $k_{SA-AV} = 5$, $k_{AV-HP} = 20$, $\alpha_0 = 1$, $\alpha_1 = 0.1$, $\alpha_3 = 0.05$, $\alpha_5 = 0.4$, $\tau_{SA-AV} = 0.8$, $\tau_{AV-HP} = 0.1$ and all other parameters vanish.

3 Control Scheme

Our idea for control is based on the fact that the control parameter can be a variable in time through a chaotic map. In this section, first we try to explain mathematical description of the chaotic map which we used in this paper. Then, we expand our idea of control.

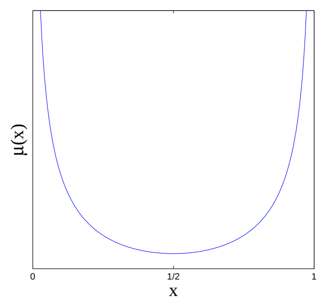

Fig. 1. Invariant measure of Logistic map.

3.1 One-parameter families of chaotic maps

The Logistic map is one of the most familiar maps in unit interval which serves as a prototype of chaos in classical nonlinear maps. One of the exciting features of the Logistic map is that it has an invariant measure which provides frequency of visits to any given interval in $[0, 1]$.

$$\mu(x) = \frac{1}{\pi\sqrt{x(1-x)}}. \tag{3}$$

This density function is graphed in Fig. 1 and ensures the ergodicity of the Logistic map.
In previous work [16] we generalized the Logistic map to a Hierarchy of one parameter families of maps with some special properties in unit interval $[0, 1]$:

- They map the interval $[0, 1]$ into itself,
- They have $(N - 1)$ critical points,
- They have $(N - 1)$ real roots,
- They have at most $(N + 1)$ attracting periodic orbits [18].

The mathematical form of the proposed Hierarchy one parameter families of maps is as follows

$$\Phi_N(k, \alpha) = \frac{\alpha^2(T_N(\sqrt{k}\,))^2}{1 + (\alpha^2 - 1)(T_N(\sqrt{k}\,)^2)}. \tag{4}$$

where $N > 1$ is an integer and T_Ns are Chebyshev polynomials of type 1 [17]. Invariant measure of the $\Phi_N(k, \alpha)$ is defined as follows

$$\mu(k, \beta) = \frac{1}{\pi}\frac{\sqrt{\beta}}{\sqrt{k(1-k)(\beta + (1-\beta)k)}}. \tag{5}$$

provided that $\beta > 0$ and

$$\begin{cases} \alpha = \dfrac{\sum_{k=0}^{[\frac{(N-1)}{2}]} C_{2k+1}^{N}\beta^{-k}}{\sum_{k=0}^{[\frac{N}{2}]} C_{2k}^{N}\beta^{-k}} & \text{N: odd,} \\ \\ \alpha = \dfrac{\beta\sum_{k=0}^{[\frac{(N)}{2}]} C_{2k}^{N}\beta^{-k}}{\sum_{k=0}^{[\frac{(N-1)}{2}]} C_{2k+1}^{N}\beta^{-k}} & \text{N: even.} \end{cases} \tag{6}$$

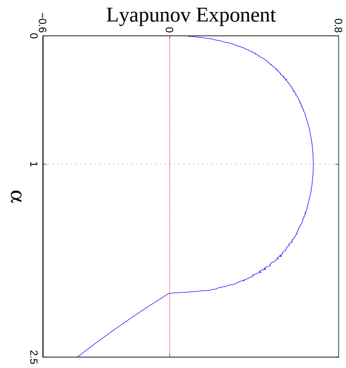

Fig. 2. Illustration of the effect of the α variation on the Lyapunov exponent of the Eq. 10. The positive value of the Lyapunov exponent proves the chaotic nature of the generalized Logistic map (Eq. 10). Furthermore, the maximum value of the Lyapunov exponent is occurred at $\alpha = 1$ which was used to generate dynamical map (Eq. 11) for control of chaos.

As an example

$$\Phi_2(k,\alpha) = \frac{\alpha^2(2k-1)^2}{4k(1-k)+\alpha^2(2k-1)^2}, \qquad \alpha = \frac{2\beta}{(1+\beta)}. \tag{7}$$

By the aid of the invertible map $h(k) = \frac{1-k}{k}$ which maps $[0,1]$ into $[0,\infty)$ one can transform $\Phi_N(k,\alpha)$ into $\Psi_N(k,\alpha)$ as

$$\Psi_N(k,\alpha) = h \circ \Phi_N(k,\alpha) \circ h^{(-1)} = \frac{1}{\alpha^2}\tan^2\left(N \arctan\sqrt{k_m}\right) \tag{8}$$

which in terms of k_{m+1} can be written as

$$k_{m+1} \equiv \Psi_2(k,\alpha) = \frac{1}{\alpha^2}\tan^2\left(2\arctan\sqrt{k_m}\right) = \frac{4}{\alpha^2}\frac{\tan^2\left(\arctan\sqrt{k_m}\right)}{(1-\tan^2\left(\arctan\sqrt{k_m}\right))^2}. \tag{9}$$

Finally, it can be simplified as

$$k_{m+1} = \frac{4k_m}{\alpha^2(1-k_m)^2}. \tag{10}$$

The Lyapunov exponent diagram for this map is shown in Fig. 2. Obviously, maximum value has been reached at $\alpha = 1$. So, in the following we set $\alpha = 1$. $\Phi_2(k)$ and $\Psi_2(k)$ for $\alpha = 1$ are shown in Fig. 3 and Fig. 4, respectively.

3.2 Controlling Procedure

The observer dependence of previous methods [8–10] for control of chaos is a high risk for heart health. Our previous work [11] allows one to overcome

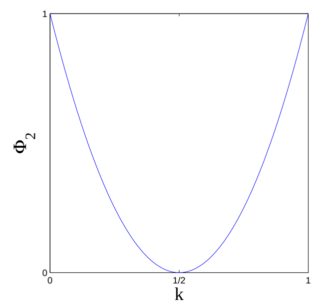

Fig. 3. Illustration of the $\Phi_2(k)$ for $\alpha = 1$.

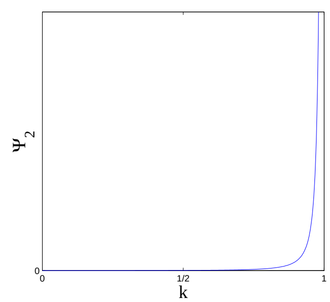

Fig. 4. Illustration of the $\Psi_2(k)$ for $\alpha = 1$.

the risk. Flexibility and observer independence are the main features of the method. The proposed method is based on the fact that the control parameter as a variable in time is changeable by another chaotic map. We improve the method by considering the hierarchy of one parameter families of ergodic solvable chaotic maps with invariant measure [16]. So, the behavior of original system may be replaced by

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}, k_m), \\ \\ k_{m+1} = \frac{4k_m}{(1-k_m)^2}. \end{cases} \tag{11}$$

where $\mathbf{x} \in \mathrm{R}^n$, $k \in \mathrm{R}^1$ denotes k_{HP-SA} and $\mathbf{F}$ is the dynamical model (Eq. 1).

4 Results and Discussion

4.1 Introducing the dynamics of the master ECG

Fig. 5 depicts the phase space of the system under different situations. $k_{SA-AV} = 6.42$, $k_{SA-AV} = 7.57$, $k_{SA-AV} = 7.64$ and $k_{SA-AV} = 10$ have been chosen as samples to reveal diverse configurations. As is evident, the system includes a wide range of behaviors. Due to the variation of k_{SA-AV} its response may

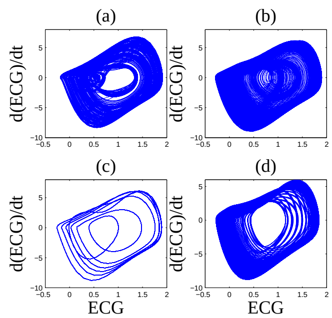

Fig. 5. Phase space of ECG without applying the control process. (a) $k_{SA-AV} = 6.42$, (b) $k_{SA-AV} = 7.57$, (c) $k_{SA-AV} = 7.64$, (d) $k_{SA-AV} = 10$. (a), (b) and (d) demonstrate non-periodic and unstable responses, and (c) represents periodic and stable response.

be periodic and stable (see Fig. 5(c)) or non-periodic and unstable (see Fig. 5(a)-(b)-(d)).

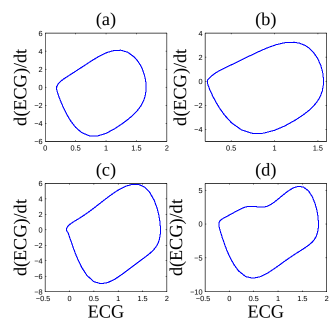

Fig. 6. Phase space of ECG after applying the control process. (a) $k_{SA-AV} = 6.42$, (b) $k_{SA-AV} = 7.57$, (c) $k_{SA-AV} = 7.64$, (d) $k_{SA-AV} = 10$. In comparison with Fig. 5 it is seen that all of the responses have suppressed to 2-period orbits.

4.2 Applying the chaos control method

The results of control method have been shown in Fig. 6. In order to reveal the control method efficiency, $k_{SA-AV} = 6.42$, $k_{SA-AV} = 7.57$, $k_{SA-AV} = 7.64$ and $k_{SA-AV} = 10$ were chosen as samples to be subjected to the control method. Pertinent phase spaces have been plotted in Fig. 6. Obviously, the chaotic motion has suppressed to a 2-period orbit. The results have confirmed the efficiency of proposed control method.

5 Summary

The development of control methods that accurately modulate the undesirable behavior of cardiac system is a fast growing research in interdisciplinary sciences. In present study based on phase space diagrams, nonlinear behavior and the unstable signals suppression problem were studied in an electrocardiogram model. Here, based on one-parameter families of chaotic maps a new controller was introduced for controlling chaos. Moreover, it was shown that the proposed technique can modulate underlying dynamics.

A Heart and Electrical Activity

The heart is a four-chambered organ which pumping blood for circulation is its basic function. The heart is divided into right and left parts, each part with its own atrium and ventricle. Receiving deoxygenated blood from the rest of the body and propelling oxygenated received blood from lungs to other organs of the body are fulfilled through the coordinate contractions of the heart organs. For contractions to be occurred, the conducting cells of heart must be excited by impulses initiated at a network of pacemaker cells. There are three types of pacemaking cells.

- The sinoatrial (SA) node which contains main pacemaking cells.
- The atrioventricular (AV) node which serves as a pacemaker should the SA node fail.
- The bundle of His-Purkinje (HP) fibers, responsible for contracting the ventricles, which may initiate impulses at low rates compared to the SA and AV nodes [15].

In resting state cardiac cells are polarized electrically, i.e., the outside of the cell membrane has a positive charge and the inside of the cell has a negative charge instead. Depolarization is the fundamental electrical event of the heart within it positive ions flow across the cell membrane into the cell and negative ions to the outside of the cell membrane. Through a process namely repolarization, polarity returns and the relaxation or resting state occurs. The waves of depolarization and repolarization represent electrical activity of the heart known as ECG [15]. A schematic illustration of the heart organ has been shown in Fig. 7.

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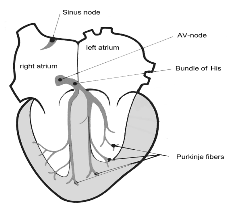

Fig. 7. Schematic picture of the heart. The heart is a four-chambered organ which is divided into right and left parts, each part with its own atrium and ventricle. For contractions to be occurred, the conducting cells of heart must be excited by impulses initiated at a network of pacemaker cells. There are three types of pacemaking cells. The sinoatrial (SA) node which contains main pacemaking cells, the atrioventricular (AV) node which serves as a pacemaker when the SA node fails and the bundle of His-Purkinje (HP) fibers (responsible for contracting the ventricles), may initiate impulses at low rates compared to the SA and AV nodes [15].

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