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HYPERCHAOS SET BY FRACTAL PROCESSES SYSTEM

Salah NASR[1], Kais BOUALLEGUE[2], and Hassen MEKKI[3]

[1] CEM Laboratory ENIS, sfax, National Engineering School of Sousse,University of Sousse , Tunisia
(E-mail: `nasrsalah-tun@hotmail.fr`)
[2] Department of Electrical Engineering. Higher Institute of Applied Sciences and Technology of Sousse, Tunisia
[3] National Engineering School of Sousse, Tunisia

Abstract. This paper presents a new class of hyper chaotic attractor. This hyperchaos is a set of chaotic attractors with different numbers of scrolls. It has different behavior at forms either separated or not with or without other nested chaotic attractors. This class of systems is constructed by using fractal processes system (FPS). For each parameter value, which is treated by process that is presented in the FPS, generates a new behavior and increases the number of scrolls. Therefore,creating a multi chaotic attractors with nested ones is a theoretically very attractive and yet technically a quite challenging task.It is obviously significant to create more complicated multi chaotic attractor and multi hyper-chaotic attractor, in both theory and engineering application. Simulation demonstrates the validity and feasibility of the proposed method.
Keywords: Multi-chaotic attractor, hyper chaotic attractor, fractal processes.

1 Introduction

Chaotic system has become a popular research area around the world after the first three-dimensional chaotic system was discovered by Lorenz, and many new chaotic systems have been proposed (i.e., Chen system, L system, Liu system)[3]

Recently, exploiting chaotic dynamics in high-tech and industrial engineering applications have attracted much interest, where in more attention has been focused on creating chaos effectively[4].

Compared with the simple chaotic attractors, multi chaotic attractors can provide more complex dynamic behaviors, more adjustability and more encryption parameters. These properties indicate that multi- chaotic attractors have a general potential applications to communications, cryptography and many other fields.

Many methods have been used to construct a hyperchaotic system and several hyper chaotic have been discovered in high dimensional dynamics such as

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Hyperchaotic Rössler system [6], hyperchaotic Chua's circuit [7] and hyperchaotic Lorenz system [8]. Our approach is regarded as a new class of hyper chaos.

The rest of the paper is organized as follows: In section 2, We describe generation of multi chaotic attractors separated and non separated with the same behavior. In section 3 introduce another generation of multi chaotic attractors with different form of behavior.Finally ,in section 4 we conclude this paper by providing a summary of the above finding.

2 Chaos with the same form of behavior

We recall the structure of fractal processes system described in paper [10]. Here, we present a structure of a system of fractal processes by associating multiple fractal processes in a cascading manner. This structure starts with a set of initial conditions, a number of fractal processes, and a set of transformations.

Let $\mathcal{E}$ be the complete metric unit, Φ a system of fractal processes in $\mathcal{E}$ such as:

$$\begin{gathered} \mathcal{E} \to \mathcal{E} \\ \Phi\!:\!(x_i, y_i) \to (x_m, y_m) \end{gathered}$$

The fractal processes system Φ is represented by:

$$\Phi \begin{cases} (x_0, y_0) \\ (x_{i+1}, y_{i+1}) = P_1(\alpha x_i + \gamma, \beta y_i + \lambda) \\ (x_{i+2}, y_{i+2}) = P_2(x_{i+1}, y_{k+1}) \\ (x_{i+3}, y_{i+3}) = T_1(x_{i+2}, y_{i+2}) \\ (x_{i+4}, y_{i+4}) = P_3(x_{i+3}, y_{i+3}) \\ \vdots \\ (x_{j+1}, y_{j+1}) = T_k(x_{j-1}, y_{j-1}) \\ \vdots \\ (x_m, y_m) = P_{m-k}(x_{m-1}, y_{m-1}) \end{cases} \tag{1}$$

The dynamics of the system of fractal processes is controlled by the assignment of x_{i+1} to x_i and of y_{i+1} to y_i.

$$\begin{cases} x_i \leftarrow x_{i+1} \\ y_i \leftarrow y_{i+1} \end{cases} \tag{2}$$

The system (1) is a combination of different transformations and different processes. It consists of m equations and k transformations, so $m-k$ processes for n iterations where the first iteration is (x_0, y_0).

We give different examples using FPS to show and validate our approach.

2.1 Chaos with separated chaotic attractors

In this paper, we use two classical chaotic attractors: in the first one we take Lorenz attractor in the second we choose Chua attractor. We recall the structure of the two chaotic attractors.

The Lorenz system [2]has become one of paradigms in the research of chaos, and is described by where x_1, x_2 and x_3 are system states and σ, ρ and β are system parameters

$$M\begin{cases} \dot{z}_1 = \sigma(z_2 - z_1) \\ \dot{z}_2 = \rho z_1 - z_2 - z_1 z_3 \\ \dot{z}_3 = (z_1 z_2 - \beta z_3) \end{cases} \tag{3}$$

And the second is Chua's circuits[1], which were introduced by Leon Ong Chua in 1983, are simplest electric circuits operating in the mode of chaotic oscillations. Different dynamic systems were inspired by Chua circuit such as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = a(-x_1 - x_2 - x_3 + f(x_1)) \end{cases} \tag{4}$$

where $\dot{y}_1$,$\dot{y}_2$ and $\dot{y}_3$ are the first time derivatives and a is a real parameter. Where $f(y_1)$ is a statured function as follows :

$$f(x) = \begin{cases} k, if x > 1 \\ kx, if |x| < 1 \\ -k, if x < -1 \end{cases} \tag{5}$$

2.2 Chaos with same behavior of separated chaotic attractors

Consider the following Φ a fractal processes system :

Let $\mathcal{E}$ be the complete metric unit, Φ a fractal processes system of $\mathcal{E}$ in $\mathcal{E}$ such as:

$$\mathcal{E} \to \mathcal{E}$$
$$\Phi{:}(f_1, f_2) \to (X_G, Y_G)$$

The fractal processes system Φ is represented by:

$$\Phi\begin{cases} (u_1, v_1) = P_J(\dot{z}_3 + \arctan(\dot{x}_1), \dot{x}_1 + \arctan(\dot{z}_3)) \\ (u_2, v_2) = P_J(\dot{z}_2 + \beta_1, \dot{z}_2) \\ (X_G, Y_G) = P_J(u_1 - \alpha u_2, v_1 - \alpha v_2) \end{cases} \tag{6}$$

Figure 1 shows eight chaotic attractors with the same behavior. Each chaotic attractor contains two scrolls one from Lorenz attractor and the other from Chua attractor.

(a) (b)

Fig. 1. Chaotic attractors separated with the same form of behavior

2.3 Chaos with same behavior of non separated chaotic attractors

In this subsection, we take Rössler system which was introduced in the 1970s as prototype equation with the minimum ingredients for continuous time chaos. This system is minimal for continuous chaos for at least three reasons: Its phase space has the minimal dimension three, its nonlinearity is minimal because there is a single quadratic term, and it generates a chaotic attractor with a single scroll, in contrast to the Lorenz attractor which has two scrolls. Rössler system is described as follows:

$$M\begin{cases} \dot{y}_1 = -(y_2 + y_3) \\ \dot{y}_2 = y_1 + \alpha y_2 \\ \dot{y}_3 = (y_1 y_3 - \beta y_3 + \gamma y_1) \end{cases} \tag{7}$$

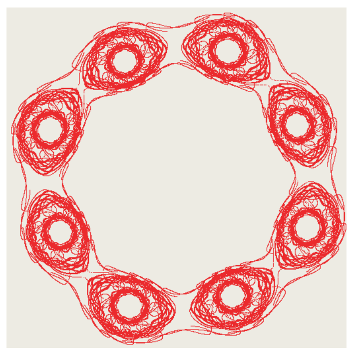
Let $\mathcal{E}$ be the complete metric unit, Φ a fractal processes system of $\mathcal{E}$ in $\mathcal{E}$ such as:

$$\begin{gathered} \mathcal{E} \to \mathcal{E} \\ \Phi{:}(f_1, f_2) \to (X_G, Y_G) \end{gathered}$$

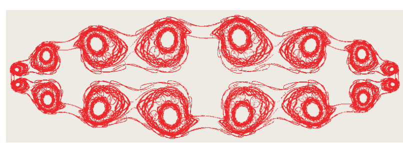
The fractal processes system Φ is represented by:

$$\Phi\begin{cases} (u_1, v_1) = P_J(\dot{y}_1 + \beta_1, \dot{y}_2 + \beta_2) \\ (p_1, q_1) = P_J(\dot{y}_1 - \beta_1, \dot{y}_2 - \beta_2) \\ (m_1, n_1) = P_J(u_1 - \alpha p_1, v_1 - \alpha q_1) \\ (m_2, n_2) = P_J(m_1, n_1) \\ (m_3, n_3) = T(m_2, n_2) \\ (m_4, n_4) = P_J(m_3 + 4, n_3) \\ (m_5, n_5) = P_J(m_3 - 4, n_3 + 1) \\ (X_G, Y_G) = P_J(m_4 - \arctan(m_5/3) + 2, n_4 + \arctan(1.5 n_5)) \end{cases} \tag{8}$$

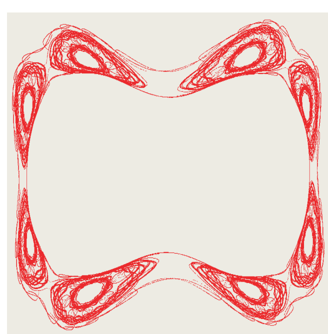
Figure 3 shows result of implementation. These behavior of chaotic attractors contain multi levels of scales.



(a) 8 chaotic attractors generated by FPS using Rössler attractors

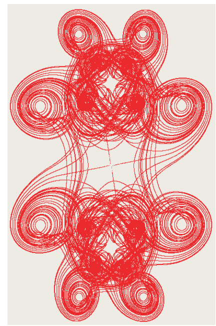


(b) 16 chaotic attractors generated by FPS using Rössler attractors

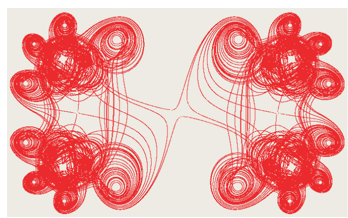


(c) Eight Chaotic attractors generated by FPS using Lorenz attractor

Fig. 2. HyperChaos Contains Eight linked chaotic attractors with same behavior



(a) Eight Chaotic attractors generated by FPS using Lorenz attractor



(b) multi chaotic attractors generated by FPS using Chua attractor

Fig. 3. HyperChaos Contains Eight linked chaotic attractors with same behavior

3 Chaos with the different form of behavior

3.1 Chaos with two behavior of chaotic attractors

A new chaotic attractors with two forms of behavior is established by the following Fractal Processes System:

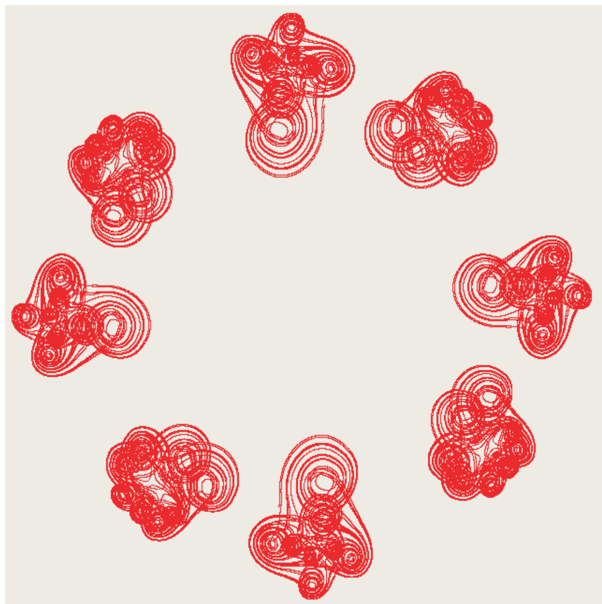
Let $\mathcal{E}$ be the complete metric unit, Φ a fractal processes system of $\mathcal{E}$ in $\mathcal{E}$ such as:

$$\begin{gathered}\mathcal{E} \to \mathcal{E}\\ \Phi{:}(f_1, f_2) \to (X_G, Y_G)\end{gathered}$$

The fractal processes system Φ is represented by:

$$\Phi \begin{cases} (u_1, v_1) = P_J(\dot{x}_1 + \arctan(\dot{x}_1), \dot{x}_2 + \arctan(\dot{x}_2)) \\ (u_2, v_2) = (1 - u_1, 1 - v_1) \\ (p_1, q_1) = P_J(\dot{z}_2 + \beta, \dot{z}_3 - \beta) \\ (p_2, q_2) = (\alpha_1 p_1(1 - p_1), \alpha_2 q_1(1 - q_1)) \\ (m_1, n_1) = P_J(u_1 - p_1, v_1 - q_1) \\ (r_1, s_1) = P_J(\dot{x}_1 - \beta_1, \dot{x}_2 - \beta_2) \\ (X_G, Y_G) = P_J(r_1 - \rho m_1 + \lambda, s_1 - \rho n_1) \end{cases} \tag{9}$$

Figure 4 shows eight chaotic attractors alternated with two forms of behavior.



(a) 8 chaotic attractors separated

(b) First behavior chaotic attractors

(c) Second behavior chaotic attractors

Fig. 4. Chaos with two behavior of chaotic attractors

3.2 Chaos with three behavior of chaotic attractors with four times

Let $\mathcal{E}$ be the complete metric unit, Φ a fractal processes system of $\mathcal{E}$ in $\mathcal{E}$ such as:

$$\mathcal{E} \to \mathcal{E}$$

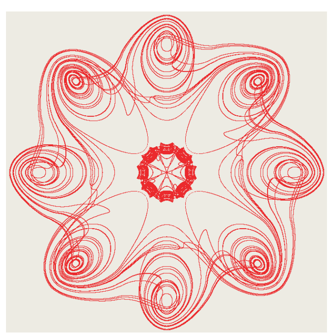
$$\Phi{:}(f_1, f_2) \to (X_G, Y_G)$$

The fractal processes system Φ is represented by:

$$\Phi \begin{cases} (u_1, v_1) = P_J(\dot{x}_1 + \arctan(\dot{x}_1), \dot{x}_2 + \arctan(\dot{x}_2)) \\ (u_2, v_2) = P_J(\dot{z}_2 + \beta_1, \dot{z}_3) \\ (u_3, v_3) = P_J(u_1 - 2u_2 + \beta_2, v_1 - v_2) \\ (p_1, q_1) = P_J(\dot{z}_2 + \beta_3, \dot{z}_3) \\ (p_2, q_2) = P_J(u_1 - 2p_1, v_1 - q_1) \\ (X_G, Y_G) = P_J(u_3 - 2p_2, v_3 - 2q_2) \end{cases} \tag{10}$$

Fractal processes system contains two different chaotic attractors the first one we choose Chua attractor noted with x the other Lorenz attractor noted with z. Implementation of fractal processes system shows result in figure

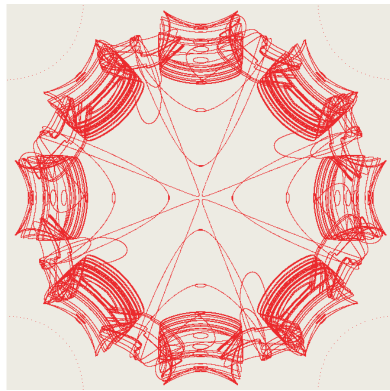
Figure 5 shows the result of implementation, It contains nested chaotic attractors with eight scrolls.



(a)

Fig. 5. hyper Chaotic attractors with nested chaotic attractor

Figure6 illustrates a new forms of scrolls in nested chaotic attractors .



(a)

Fig. 6. Zooming of nested chaotic attractor

3.3 Hyper Chaos with separated chaotic attractors

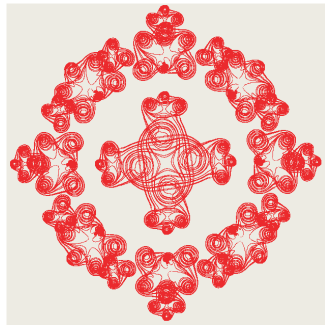
In this subsection , we give an hyper chaotic attractor with nine chaotic attractors,that has three forms of behavors,for generate this forms of hyper chaotic attractor using fractal processes system.

Let $\mathcal{E}$ be the complete metric unit, Φ a fractal processes system of $\mathcal{E}$ in $\mathcal{E}$ such as:

$$\mathcal{E} \to \mathcal{E}$$
$$\Phi{:}(f_1, f_2) \to (X_G, Y_G)$$

The fractal processes system Φ is represented by:

$$\Phi \begin{cases} (u_1, v_1) = P_J(\dot{x}_1 + \arctan(\dot{x}_1), \dot{x}_2 + \arctan(\dot{x}_2)) \\ (u_2, v_2) = P_J(\dot{z}_2 + \beta_1, \dot{z}_3 - \beta) \\ (u_3, v_3) = P_J(u_1 - \alpha u_2 + \beta_2, v_1 - v_2) \\ (p_1, q_1) = P_J(\dot{z}_2 + \beta_3, \dot{z}_3) \\ (p_2, q_2) = P_J(u_1 - \alpha p_1, v_1 - q_1) \\ (X_G, Y_G) = P_J(u_3 - 2p_2, v_3 - 2q_2) \end{cases} \tag{11}$$



(a)

Fig. 7. Chaotic attractors separated within nested chaotic attractors

3.4 Chaos with four behavior of chaotic attractors

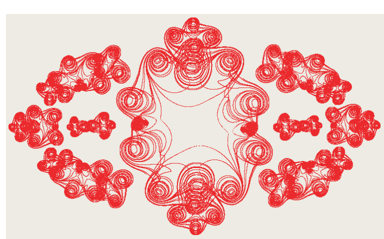
Let $\mathcal{E}$ be the complete metric unit, Φ a fractal processes system of $\mathcal{E}$ in $\mathcal{E}$ such as:

$$\begin{gathered}\mathcal{E} \rightarrow \mathcal{E} \\ \Phi\!:\!(f_1, f_2) \rightarrow (X_G, Y_G)\end{gathered}$$

The fractal processes system Φ is represented by:

$$\Phi \begin{cases} (u_1, v_1) = P_J(\dot{x}_1 + \arctan(\dot{x}_1), \dot{x}_2 + \arctan(\dot{x}_2)) \\ (u_2, v_2) = P_J(\dot{z}_2 + \beta_1, \dot{z}_3) \\ (u_3, v_3) = P_J(u_1 - \alpha u_2 + \beta_2, v_1 - v_2) \\ (p_1, q_1) = P_J(\dot{z}_2 + \beta_3, \dot{z}_3) \\ (p_2, q_2) = P_J(u_1 - 2p_1, v_1 - q_1) \\ (X_G, Y_G) = P_J(u_3 - 2p_2 + \beta_4, v_3 - 2q_2) \end{cases} \tag{12}$$

Figure 8shows result of implementation.



(a)

Fig. 8. Chaotic attractors with four behavior forms

4 Conclusion

In this paper, different techniques of generating a new classes of chaos attractors by Chua attractor with fractal and multi fractal behavior. Many of them are new and interesting in both theory and engineering application. It seems this approach of generation multi separated chaotic attractor can be used to drive and control complex systems for example autonomous robots. Moreover, many of them have some novel properties, therefore deserve further investigation in the future. Some numerical simulation results are provided to show the effectiveness of the method proposed in this work.

References

1. Chua, L. O., Komuro, M. & Matsumoto, T. [1986]"The double scroll family," *IEEE Trans. Circuits Syst* **33**, pp. 1072–1118.
2. E.N. Lorenz, Deterministic nonperiodic flow, J. Atmos. Sci. 20(1963)130 − 141.
3. Xianming.Wu, Yigang.He,Wenxin.Yu,Baiqiang.Yin, A New Chaotic attractor and Its synchronization implementation. Circuit Syst. Sig. Process. DOI 10.1007/s00034-014-9946-7
4. Guanghui.Sun,MaoWang.Lilian Huang.Liqun Shen, Generating Multi-Scroll Chaotic Attractors via Switched Fractional Systems.Circuits Syst.Sig.Process 30, 1183 − 1195(2011)
5. E.Ott Chaos in dynamical system Combridge University Press, Combridge 1993.
6. E.O Rössler, Phys.Lett A 71(1979) 155.
7. Cafagna G. Grassi Int J Biffurcation Chaos 13 (2003) 2889.
8. EE. Mohmoud Dynamics and synchronization of a new hyperchaotic complex Lorenz system, Math Computation Model 55(2012) 1951-1962.
9. K.Bouallegue, A. Chaari, A. Toumi, Multi-scroll and multi-wing chaotic attractor generated with Julia process fractal,Chaos, Solitons & Fractals 2011; 44 : 79 − 85
10. K.Bouallegue, Gallery of Chaotic Attractors Generated by Frcatal Network, Int J Biffurcation Chaos (25)1(2015)1530002. DOI 10.1142/S0218127415300025.
11. K.Bouallegue, Chaotic Attractors with separated scrolls, Int J AIP, Chaos 25, 073108(2015). DOI 10.1063/1.4923302.