Jerky Systems Derived From Sprott D and Sprott C Systems

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Abstract:

The Sprott C and D systems are among the three variable simplest chaotic systems with resonant normal form characteristics and serve as possible candidates for demonstrating Hopf Bifurcation. Investigation of Sprott systems for deriving jerky Dynamics has also become of interest. The jerky dynamics has been analyzed with approximating discontinuities with a continuous function.

In this work, we will use slightly generalized forms of these two Sprott systems, derive jerky systems compatible with them, analyze their transition to chaotic behavior and their attractors using our two parametrizations for approximating discontinuous functions with continuous ones, one of which is the same as that used in the literature(Hacınlıyan and Kandıran[5]).

Chaotic invariants such as Lyapunov exponents and fractal dimensions have been calculated. Comparisons of these invariants between the normal and jerky systems have been analyzed.

Keywords: Chaotic modeling, Chaotic systems, Sprott systems, Fractal dimension, Lyapunov exponents, Simulation, Chaotic simulation, Hopf bifurcation.

Introduction

The characterization of the nature of dynamical systems is a widely addressed issue. There are many different techniques to define the nature of these systems, especially in the case of three-dimensional systems. In 1994 Sprott studied a wide range of three-dimensional autonomous dynamical systems. The models that he studied are three-dimensional ordinary differential equations with five or six terms and one or two quadratic nonlinearities. With one exception, they possess one or two equilibrium points.

Although the knowledge about fixed points of the system gives important information about the nature of the system, it is not sufficient to characterize the whole structure of the chaotic attractors. The relationship between non-linear dynamical systems and the topology of the solution depends on the algebra, namely linear stability analysis. Although Poincare sections or phase portrait

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with analytical properties of the governing equations gives important information about dynamics of the system, when we have a system which have two or more asymptotically stable equilibria and other attracting sets of point as the system parameters change, another type of complexity may occur. The trajectories of the kinds of system selectively converges on either of the attracting sets depending on the initial state of the system.

In Sprott's study mentioned above, he found 19 distinct chaotic models total of only five or six terms with one or two quadratic nonlinearities. A further simplification to Sprott models can be achieved by transformation to explicit third-order ordinary differential equations, namely jerky systems of the form:

$$\ddot{x} = J(x, \dot{x}, \ddot{x});$$

where dots represent the time derivative. By defining $\dot{x} = y$ and $\ddot{x} = z$, the jerky dynamics can be written as:

$$\dot{x} = y, \, \dot{y} = z, \, \dot{z} = J(x, y, z).$$

First two terms of the given representation are common in jerky system but the last terms is generally called as nonlinear jerk function J(x, y, z). In the last term, nonlinearities can lead to complexity, for example in the numerical simulation or estimation of Lyapunov exponents which gives information about chaotic properties of the system.

The jerky terms in jerky systems may include some discontinuous functions and they may create a problem when the chaotic characteristic of the system is investigated. To overcome this problem, we can replace this discontinuous parts with continuous ones. Some of the discontinuous functions and their continuous alternatives are given below.

Discontinuous Functions	Continuous Alternatives
sgn(x)	$\frac{2}{\pi} \tan^{-1} Nx \text{ or } \tanh(Nx)$
x	xsgn(x)
$\min(a, b)$	$\frac{a+b}{2} - \left \frac{a-b}{2}\right $
$\max(a, b)$	$\frac{a+b}{2} + \left \frac{a-b}{2}\right $

In this paper we have worked on the Sprott C and Sprott D systems and we tried to investigate chaotic and dynamical properties of the system. We have

calculated Lyapunov exponents, Hurst exponents, fractal dimension of these system and performed their numerical simulations.

Sprott C System

Generalized form of the Sprott C system can be written as follows:

 $\dot{x} = a(y - x)$ $\dot{y} = -cy - xz$ $\dot{z} = y^2 - b$

where a, b, c are constants.

The fixed points of the system are $(\sqrt{b}, \sqrt{b}, -c)$ and $(-\sqrt{b}, -\sqrt{b}, -c)$, if b>0 and $a\neq 0$. For this fixed point the system has the following eigenvalues: $\lambda_1 = -a$ and $\lambda_2 = \lambda_3 = \pm \sqrt{2b}$. Therefore, these last two fixed points have local 1-dimensional stable manifolds tangent to the eigenvector associated to the eigenvalue λ_1 and local 2-dimensional center manifolds tangent to the plane generated by the eigenvectors associated to the complex eigenvalues λ_2 and λ_3 . Furthermore we expect resonant normal forms in all odd orders.

As it can be seen easily there is no-equilibrium when b<0. In addition to this when b=1, three is only one equilibrium point (0,0, z), z is real number. If c=0, the Jacobian matrix has a pair of purely imaginary eigenvalues and a nonzero real eigenvalue. That's why, the calculated fixed points are both not hyperbolic but weak repelling focus.

Jerky Sprott D model

Sprott D Model consists of the following set of equations:

 $\dot{x} = -y$ $\dot{y} = x + z$ $\dot{z} = xz + \alpha y^{2}$ If we use the following set of transformations to convert Sprott D system: $\dot{x} = -y$ $\ddot{x} = -x - z$ We obtain the following jerky system: $\ddot{x} = x\ddot{x} - \dot{x} - \alpha \dot{x}^{2} + x^{2}$ We can get its piecewise linear version by introducing signum function

$$\dot{x} = -y$$
$$\dot{y} = x + z$$
$$\dot{z} = sgn(x)z + ay$$

As we mentioned above, it is difficult to calculate Lyapunov exponents for systems that involve discontinuous functions such as the signum. We use a simple method in which sgn(x) is smoothed by $\frac{2}{\pi} \tan^{-1} Nx \ or \tanh(Nx)$

approximation with sufficiently large N. The final set of differential equations are:

$$\dot{y} = x + z \dot{z} = tanh(Nx)z + \alpha y^2$$

The given system has a fixed point at (0,0,0) with eigenvalues $\lambda_1 = -0.9529$ and $\lambda_{2,3} = -1.0236 \pm 0.0396i$. So the fixed point is stable.

Numerical Simulations and Analysis of Sprott C and Sprott D Systems Sprott C Model

We have made numerical simulations for both Sprott C and D systems. Firstly, we have studied the Sprott C system. We tried to investigate possible chaotic regions. We have made a numerical simulation for Sprott C model with following initial conditions and parameter values: $(x_0, y_0, z_0) = (0.26, 0.19, -0.14)$ and a=10, b=100 and c=0.4.

Figure 1:Simulation for Sprott C system



As it is seen from the graphs of the simulation, there are two fixed points. Dynamic of Lyapunov exponents are presented in Figure 2. This two points are stable with the following eigenvalues $\lambda_1 = -10.1355$ and $\lambda_{2.3} = -0.1321 \pm 14.0465i$.

Figure 2:Lyapunov Exponents of the Sprott C system.



Then, we have calculated Lyapunov exponent of the system: (1.2670, -0.0193, -11.6476). That's why, the system has a chaotic attractor for these initial conditions. We have calculated fractal dimension for the system using three different methods. The Kaplan-Yorke dimension for the system is 2.107. We also used the Higuchi and Katz methods to calculate the fractal dimension of the system. According the Higuchi method, the fractal dimension is 2,012 and for the Katz method, it is equal to 1.96.

We also applied time series analysis methods to investigate the chaotic behavior of the system. We have used mutual information analysis method to estimate the delay time of the system. According to this analysis, we have found that delay time of the system is 3.

Figure 3: Mutual Information Analysis



Figure 4: False Nearest Neighborhood Analysis



After determining the delay time, we can determine the embedding dimension of the system using the False Nearest Neighborhood Analysis. We have found that the embedding dimension of the system which is 2.

According to Detrended Fluctuation Analysis, we have found the fluctuation exponent as 2.104, which means that there is a correlation other than power law.



Figure 5: Detrented Fluctuation Analysis of the system

Then we tried to see that whether the dynamics of the system changes according to initial conditions, In Table 1, we give the Lyapunov exponents of the system for each initial conditions, indicating a possible bifurcation.

Initial Conditions LE₁ LE₂ LE₃ (11.2, 4.81, -0.2)1.2804 0.0001 -11.7305 (11.2, 4.86, -0.2)-0.1268 -0.1296 -10.1553 (11, 4.85, 1.2708 -0.0002-11.6678 -0.2) $(1\overline{1, 4.88, -0.2})$ -0.1228 -0.1308 -10.1482

Table 1: Lyapunov Exponents for Different Initial Conditions

As it can be seen from the Table 1, a small change in the initial condition of the system leads totally different dynamic behavior. In summary, for different initial conditions, trajectory of system with only stable equilibria can converges to two types of attractors (chaos or stable equilibrium).

Sprottt D Model

The simulation plots of linearized jerky Sprott D system is given in Figure 6,7 and 8.



Figure 8:Simulation of jerky Sprott D System

The initial condition is (x,y,z)=(0.05,0.05,0.05) and $\alpha = 3$ with time step 0.01 from t=0 to t=1000.

According to this simulation values we have found delay time of the system as 4.



Figure 9: Mutual Information Analysis

After finding the delay time, we have determined the embedding dimension of the system as 2 by using false nearest neighborhood analysis. In addition to this, we have found largest Lyapunov Exponent as 0.2525.



Figure 10:False Nearest Neighbour Analysis



Figure 11:False Nearest Neighborhood Analysis

Conclusion

We have tried to investigate the possible chaotic behavior of the Sprott C system. We have made a numerical simulation and calculated Lyapunov exponents. Lyapunov exponents and numerical simulation shows us that the Sprott C systems display chaotic behavior for investigated initial conditions. We have also seen that small changes in initial conditions will lead change in the dynamical behavior of the system. For the jerky Sprott D system, we have determined largest Lyapunov exponent, embedding dimension and delay time of the system.

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