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Construction of compound chaotic multiattractors containing the same type of local attractors with different parameters

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Abstract: Presents a method for constructing a composite chaotic multiattractor that allows you to set the required difference between local chaotic attractors that are part of multiattractor
Keywords: Heterogeneous multiattractor, Compound multiattractor, Chaotic attractor, Replication operator, Dynamic system.

1 Introduction

Currently known methods for constructing uniform (homogeneous) composite chaotic multiattractors, that consist from identical chaotic attractors – elements of multiattractor (see, e.g., [1-8]).
It is obvious that homogeneous multiattractors are only part of a larger family of similar objects, which should also include multiattractors, containing dissimilar elements. It is therefore of interest to find ways of introducing differences between the elements of compound chaotic multiattractors.
Let us consider a method for constructing dynamical systems, having nonuniform composite chaotic multiattractor, local chaotic attractors which belong to the same dynamic system, but correspond to the different values of the constants, included in the equations of motion.
The system of ordinary differential equations with constant coefficients describing the motion in a homogeneous composite chaotic multiattractor has the form [1]:

$$\dot{x} = \alpha H(x) + f[x, \eta], \tag{1}$$

where

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$$\dot{x}=\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dots\\ \dot{x}_n\end{bmatrix};\quad \dot{x}_j=\frac{dx_j}{dt},\quad j=1,2,\dots n;\quad H(x)=\begin{bmatrix}H_1(x_1)\\ H_2(x_2)\\ \dots\\ H_n(x_n)\end{bmatrix};\quad \alpha=\begin{bmatrix}\alpha_{11} & \alpha_{12} & \dots & \alpha_{1n}\\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2n}\\ \dots & \dots & \dots & \dots\\ \alpha_{n1} & \alpha_{n2} & \dots & \alpha_{nn}\end{bmatrix};$$

$$f(x,\eta)=\begin{bmatrix}f_1\left[H_1(x_1),\ H_2(x_2),\dots\ H_n(x_n),\eta_{11},\eta_{12},..\eta_{1m_1}\right]\\ f_2\left[H_1(x_1),\ H_2(x_2),\dots\ H_n(x_n),\eta_{21},\eta_{22},..\eta_{2m_2}\right]\\ \dots\\ f_n\left[H_1(x_1),\ H_2(x_2),\dots\ H_n(x_n),\eta_{n1},\eta_{n2},..\eta_{nm_n}\right]\end{bmatrix}.$$

Here $x=x_1, x_2, \dots x_n$ – variables of replication, each of which represents either the independent variable or linear combination of independent variables of the original dynamic system; α – constants of the linear part of the equations; $fj(\xi)$ – nonlinear terms; η_i – constants included in the equations of non-linear members; $H_j(x_j)$ – replicate operators, providing creating copies of the chaotic attractor of the original dynamical system and combining them into a single multiattractor.

Replicate (reduplication, replicator) operators (functions) are nonlinear functions of replication variables, consisting of line segments of unit slope, interconnected intermediate portions (which may be linear or non-linear segments of opposite slope, discontinuities of the first kind or the area of the hysteresis).

From 0 to $n-1$ replicate functions in equations (1) can consist of a single segment with unit slope, i.e. to coincide with your argument. If the system (1) contains such, "singular", operators, replication will be performed only on part of replication variables.

The action of m $(m\leq n)$ nondegenerate replicate operators on the dynamic system can be represented as a formation in the phase space of the system (1) m-dimensional array of phase cells, within each of which is a fragment of the phase space is identical to the fragment of the phase space of the original dynamic system containing its attractor. The inner area of the phase cells correspond to the segments with unit slope. The intermediate segments replicate functions correspond to the layers of the phase space, dividing phase of the cell.

Continuous replicate functions can be represented, for example, the following expression:

$$\begin{aligned}H_j(x_j)=x_j+(d_j+1)\Bigg\{&P\left(x_j+s_j+h_j+\frac{h_j}{d_j}\right)+P\left(x_j+s_j-h_j-\frac{h_j}{d_j}\right)-\\ &-\sum_{m=0}^{M_j}\left[P\left(x_j+s_j-(2m-1)\left(h_j+\frac{h_j}{d_j}\right)\right)+\frac{h_j}{d_j}\right]-\sum_{n=0}^{N_j}\left[P\left(x_j+s_j+(2n-1)\left(h_j+\frac{h_j}{d_j}\right)\right)-\frac{h_j}{d_j}\right]\Bigg\};\end{aligned}\qquad(2)$$

where $P(x_j)=\frac{1}{2}\left(\left|x_j+\frac{h_j}{d_j}\right|-\left|x_j-\frac{h_j}{d_j}\right|\right)$, h_j – is half of the length of the phase cell, that contains the chaotic attractor of the original dynamical system, up to j-th replication variable; M_j – number of local chaotic attractors, up to j-th replication variable from the original chaotic attractor; N_j is the number of local chaotic attractors down on the j-th replication variable from the original chaotic attractor; d_j – module of the slope of the intermediate segments of replication function for the j-th replication variable; s_j is a constant that takes into account the asymmetry of the attractor of the original dynamical system for the j-th replication variable.

To local attractors differ, obviously, must vary the corresponding equations of motion. In the present case, this requires that the values of the constants in equations (1) had individual values in each phase bin.

2 Establishment Differences between of the Elements of Multiattractor

For this purpose it is necessary to multiply all the constants on a special weight ("individualizing") functions, the values of which remain constant within each phase cell, but vary from cell to cell. In this case the arrays of constants and nonlinear terms in equations (1) is converted to the following form:

$$\alpha_\gamma(x)=\begin{bmatrix} \gamma_{\alpha 11}(x)\alpha_{11} & \gamma_{\alpha 12}(x)\alpha_{12} & \dots & \gamma_{\alpha 1n}(x)\alpha_{1n} \\ \gamma_{\alpha 21}(x)\alpha_{21} & \gamma_{\alpha 22}(x)\alpha_{22} & \dots & \gamma_{\alpha 2n}(x)\alpha_{2n} \\ \dots & \dots & \dots & \dots \\ \gamma_{\alpha n1}(x)\alpha_{n1} & \gamma_{\alpha 11}(x)\alpha_{n2} & \dots & \gamma_{\alpha nn}(x)\alpha_{nn} \end{bmatrix};$$

$$f_\gamma[x,\eta]=\begin{bmatrix} f_1\left[H_1(x_1),\ H_2(x_2),\dots\ H_n(x_n),\ \gamma_{\eta 11}(x)\eta_{11}, \gamma_{\eta 12}(x)\eta_{12}, \dots\ \gamma_{\eta 1m_1}(x)\eta_{1m_1}\right] \\ f_2\left[H_1(x_1),\ H_2(x_2),\dots\ H_n(x_n),\ \gamma_{\eta 21}(x)\eta_{21}, \gamma_{\eta 22}(x)\eta_{22}, \dots\ \gamma_{\eta 2m_2}(x)\eta_{2m_2}\right] \\ \dots \\ f_n\left[H_1(x_1),\ H_2(x_2),\dots\ H_n(x_n),\ \gamma_{\eta n1}(x)\eta_{n1}, \gamma_{\eta n2}(x)\eta_{n2}, \dots\ \gamma_{\eta nm_n}(x)\eta_{nm_n}\right] \end{bmatrix},$$

where $\gamma(x)$ – "individualizing" function; $x = x_1, x_2, \dots x_n$.

Due to changes in the constants in the equations of motion is to change parameters of the attractor, including its configuration in the phase space of the system. In particular, changing the size and relative position of the phase cells containing local attractors.

3 Creating Conditions for Merging of Different Chaotic Attractors in a Single Multiattractor

Be combined with each other such cells without gaps or overlaps in the general case impossible. Therefore, after introducing into the equations the weighting functions it is necessary to change the size and position (relative to the centers of the local coordinate systems) of all local attractors, so that the phase cells, containing them, again took the same still (same as in a homogeneous multiattractor) ordered arrangement. (The sizes of all local attractors and their position within their phase cells can convert, for example, to the size and position of a chaotic attractor of the original dynamic system).
The normalization of the size of the local attractors requires to perform a local scale transformation of the phase space within each phase of the cell. That is, within the *k*-th cell have to do a change of variables of the form:

$$x^*_j=\phi_{k,j}x_j, \quad j=1,2,...n, \qquad (3)$$

where $\phi_{k,i}$ – is the conversion factor scale in the *k*-th cell to *j*–th variable, which will change the size of the attractor by a new variable in $\phi_{k,j}$ times.
For conversion of the local attractors to the same position relative to the centers of the local coordinate system the expression (3) must be reduced to the form:

$$x^*_j=\phi_{k,j}(x_j+\theta_{k,j}), \quad j=1,2,...n,$$

where $\theta_{k,i}$ – is the interval displacement of the *k*-th local attractor for the *j*-th replication variable.
In order for the transform coefficients of the scale and balancing the coefficients have individual values in each phase cell, the equations of motion need to enter the appropriate scaling and balancing functions, providing different values for scale and balancing coefficients within different phase cells. This can be done by conversion replicate operators to the following form:

$$H^*_j(x_j)=\phi_j(x)H(x_j+\theta_j(x)), \quad j=1,2,...n,$$

where $\phi_i(x)$ and $\theta_i(x)$ – respectively the scaling and balancing functions on the *j*-th replication variable. The values of these functions, as well as the values of the weighting functions $\gamma(x)$, remain constant within each phase of the cell but vary from cell to cell.
After that, the equations describing the dynamics of a system with chaotic multiattractor composite, consisting of different local attractors will take final look:

$$\dot{x} = \begin{bmatrix} \dfrac{\dot{x}_1^*}{\phi_1(x)} \\ \dfrac{\dot{x}_2^*}{\phi_2(x)} \\ \ldots \\ \dfrac{\dot{x}_n^*}{\phi_n(x)} \end{bmatrix}; \tag{4}$$

where $\dot{x}_j = \dfrac{dx_j}{dt}$, $j=1,2,...n$; $\dot{x}^* = \alpha_\gamma(x) H^*(x) + f_\gamma(x,\eta)$;

$$H^*(x) = \begin{bmatrix} \phi_1(x) H_1[x_1 + \theta_1(x)] \\ \phi_2(x) H_2[x_2 + \theta_2(x)] \\ \ldots \\ \phi_n(x) H_n[x_n + \theta_n(x)] \end{bmatrix};$$

$$\alpha_\gamma(x) = \begin{bmatrix} \gamma_{\alpha 11}(x)\alpha_{11} & \gamma_{\alpha 12}(x)\alpha_{12} & \ldots & \gamma_{\alpha 1n}(x)\alpha_{1n} \\ \gamma_{\alpha 21}(x)\alpha_{21} & \gamma_{\alpha 22}(x)\alpha_{22} & \ldots & \gamma_{\alpha 2n}(x)\alpha_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ \gamma_{\alpha n1}(x)\alpha_{n1} & \gamma_{\alpha 11}(x)\alpha_{n2} & \ldots & \gamma_{\alpha nn}(x)\alpha_{nn} \end{bmatrix};$$

$$f_\gamma[x,\eta] = \begin{bmatrix} f_1\left[H^*{}_1(x_1),\ H^*{}_2(x_2),\ldots\ H^*{}_n(x_n),\ \gamma_{\eta 11}(x)\eta_{11}, \gamma_{\eta 12}(x)\eta_{12}, \ldots\ \gamma_{\eta 1m_1}(x)\eta_{1m_1}\right] \\ f_2\left[H^*{}_1(x_1),\ H^*{}_2(x_2),\ldots\ H^*{}_n(x_n),\ \gamma_{\eta 21}(x)\eta_{21}, \gamma_{\eta 22}(x)\eta_{22}, \ldots\ \gamma_{\eta 2m_2}(x)\eta_{2m_2}\right] \\ \ldots \\ f_n\left[H^*{}_1(x_1),\ H^*{}_2(x_2),\ldots\ H^*{}_n(x_n),\ \gamma_{\eta n1}(x)\eta_{n1}, \gamma_{\eta n2}(x)\eta_{n2}, \ldots\ \gamma_{\eta nm_n}(x)\eta_{nm_n}\right] \end{bmatrix}.$$

4 Example of Heterogeneous Composite Chaotic Multiattractor, Consisting of Lorentz Attractors

Let us consider an example. We introduce differences between the local attractors in "two-dimensional" composite multiattractor, consisting of Lorentz attractors, existing in a dynamic system following [10]:

$$\begin{cases} \dfrac{dx}{d\tau} = A\left[H_2(y+\mu x) - (1+\mu)H_1(x)\right]; \\ \dfrac{dy}{d\tau} = H_1(x)\left[B - z + \mu\right] - H_2(y+\mu x); \\ \dfrac{dz}{d\tau} = \left[H_2(y+\mu x) - \mu H_1(x)\right]H_1(x) - Cz\ , \end{cases} \tag{5}$$

Replication of the original Lorenz attractor in this system is performed in two replication variables: x and $w=y+\mu x$ (μ – constant) by operators $H_1(x), H_2(w)$.

Before the introduction of inhomogeneities in multiattractor of system (5) it must be transformed to the form (1), that is, to write the equations of motion with respect to the replication variables:

$$\begin{cases} \dfrac{dx}{d\tau} = A\left[H_2(w) - (1+\mu)H_1(x)\right]; \\ \dfrac{dw}{d\tau} = H_1(x)\left[B - z + \mu\right] - H_2(w) + \mu A\left[H_2(w) - (1+\mu)H_1(x)\right]; \\ \dfrac{dz}{d\tau} = \left[H_2(w) - \mu H_1(x)\right]H_1(x) - Cz\ , \end{cases} \tag{6}$$

(Due to the fact that in this system replication on z is not made, the operator $H_3(z)$ is degenerate: $H_3(z)=z$).
To introduce differences between the local attractors, assign the constants A, B, C distinct values in each phase bin. To do this, transform equation (6) to the form (4):

$$\begin{cases} \dfrac{dx}{d\tau} = \dfrac{\gamma_A(x,w)A0\left[H_2^*(w) - (1+\mu)H_1^*(x)\right]}{\phi_1(x,w)}; \\ \dfrac{dw}{d\tau} = \dfrac{H_1^*(x)\left[\gamma_B(x,w)B0 - \phi_3(x,w)(z+\theta_3(x,w)) + \mu\right] - H_2^*(w) + \mu\gamma_A(x,w)A\left[H_2^*(w) - (1+\mu)H_1^*(x)\right]}{\phi_2(x,w)}; \\ \dfrac{dz}{d\tau} = \dfrac{\left[H_2^*(w) - \mu H_1^*(x)\right]H_1^*(x) - \gamma_C(x,w)C0\phi_3(x,w)(z+\theta_3(x,w))}{\phi_3(x,w)}, \end{cases} \tag{7}$$

where A0, B0, C0 – constant values, corresponding to the attractor of initial dynamic system.
Since the Lorenz attractor is symmetric on variables x and w the corresponding balancing function is equal to zero: $\theta_1(x,w)=\theta_2(x,w)=0$, therefore in equations (7)

$$H_1^*(x)=\phi_1(x,w)H_1[x+\theta_1(x,w)]=\phi_1(x,w)H_1(x),$$
$$H_2^*(w)=\phi_2(x,w)H_2[w+\theta_2(x,w)]=\phi_2(x,w)H_2(w).$$

Replicates functions are defined by equations (2). Individualizing, scaling and balancing function, in the case of two-dimensional composite multiattractor can be represented by the following expression:

$$\gamma_A(x,w)=S(x,w,\gamma A);\ \gamma_B(x,w)=S(x,w,\gamma B);\ \gamma_C(x,w)=S(x,w,\gamma C);$$
$$\phi_1(x,w)=S(x,w,\phi 1);\ \phi_2(x,w)=S(x,w,\phi 2);\ \phi_3(x,w)=S(x,w,\phi 3);$$
$$\theta_1(x,w)=S(x,w,\theta 1);\ \theta_2(x,w)=S(x,w,\theta 2);\ \theta_3(x,w)=S(x,w,\theta 3);$$

where γA, γB, γC – arrays of values of weight coefficients specifying individual values of the constants *A, B, C*, respectively, in each phase cell; $\gamma\phi 1$, $\gamma\phi 2$, $\gamma\phi 3$ – arrays of values of scaling factors that ensure the standardization of the sizes of all local attractors in terms of the variables *x, w, z*, respectively; $\gamma\theta 1$, $\gamma\theta 2$, $\gamma\theta 3$ – arrays of values of the balancing factors that ensure the balancing of local attractors in terms of the variables *x, w, z*, respectively.
$S(x,w,\Psi)$ – is a function specifying the distribution of coefficients over cells in phase space; it is a sum of components, each of which has a specified value within a single cell and equals zero in all other cells. The equation defining this function, are shown in table.I.

Table.I

$S(x,w,\Psi)=S1(x,w,\Psi)+S2(x,w,\Psi)+S3(x,w,\Psi)$
$S1(t,q,\Psi)=\sum_{j_2=1-N_2}^{M_2-1}\sum_{j_1=1-N_1}^{M_1-1}\Psi_{j_2+O_2,j_1+O_1}Pa[t-j_1c(1),q-j_2c(2)]+$ $\Psi_{j_2+O_2,M_1+O_1}Pb2[t-(M_1-1)c(1),q-j_2c(2)]+$ $\Psi_{j_2+O_2,O_1-N_1}Pb4[t+(N_1-1)c(1),q-j_2c(2)];$
$S2(t,q,\Psi)=\sum_{j_1=1-N_1}^{M_1-1}\Psi_{M_2+O_2,j_1+O_1}Pb1[t-j_1c(1),q-(M_2-1)c(2)]+$ $\Psi_{M_2+O_2,M_1+O_1}Pc1[t-(M_1-1)c(1),q-(M_2-1)c(2)]+$ $\Psi_{M_2+O_2,O_1-N_1}Pc3[t+(N_1-1)c(1),q-(M_2-1)c(2)];$
$S3(t,q,\Psi)=\sum_{j_1=1-N_1}^{M_1-1}\Psi_{O_2-N_2,j_1+O_1}Pb3[t-j_1c(1),q-(1-N_2)c(2)]+$ $\Psi_{O_2-N_2,M_1+O_1}Pc2[t-(M_1-1)c(1),q-(1-N_2)c(2)]+$ $\Psi_{O_2-N_2,O_1-N_1}Pc4[t-(1-N_1)c(1),q-(1-N_2)c(2)];$
$Pa(t,q)=P3(t,1)P3(q,2);\ Pb1(t,q)=P3(t,1)P4(q,2);$ $Pb2(t,q)=P4(t,1)P3(q,2);$

$Pb3(t,q)=P3(t,1)P5(q,2)$; $Pb4(t,q)=P5(t,1)P3(q,2)$; $Pc1(t,q)=P4(t,1)P4(q,2)$; $Pc2(t,q)=P4(t,1)P5(q,2)$; $Pc3(t,q)=P5(t,1)P4(q,2)$; $Pc4(t,q)=P5(t,1)P5(q,2)$;
$P3(\xi,\lambda)=P1(\xi,\lambda)-P2(\xi,\lambda)$; $P4(\xi,\lambda)=\frac{1}{2}+P2(\xi,\lambda)$; $P5(\xi,\lambda)=\frac{1}{2}-P1(\xi,\lambda)$;
$P1(\xi,\lambda)=\frac{d_\lambda}{4h_\lambda}\left[\left\|\xi+h_\lambda+\frac{2h_\lambda}{d_\lambda}\right\|-\left\|\xi+h_\lambda\right\|\right]$; $P2(\xi,\lambda)=\frac{d_\lambda}{4h_\lambda}\left[\left\|\xi-h_\lambda\right\|-\left\|\xi-h_\lambda-\frac{2h_\lambda}{d_\lambda}\right\|\right]$; $c(\lambda)=2h_\lambda\left(1+\frac{1}{d_\lambda}\right)$.
M1 and *N1* – is the the number of local attractors respectively up and down on the variable x relatively of the attractor of the original dynamic system; *M2* and *N2* – is the number of local attractors respectively up and down on of the variable w relatively of the attractor of the original dynamic system; *h1* and *d1* – is half the length of the phase of the cell that contains the chaotic attractor of the original dynamical system, and the module of the slope of the intermediate segments of the replicate function, respectively, on the of the variable x; *h2* and *d2* – is half the length of the phase cell, that contains the chaotic attractor of the original dynamical system, and the module of the slope of the intermediate segments of the replicate function, respectively, on of the variable w.

In Fig.1 shows an example of nonuniform chaotic multiattractor which is observed in the system (8) with the following constants: $\mu=-0.45$; $h_1=17.1$; $h_2=17.7$; $d_1=d_2=100$; $s_1=s_2=0$; $s_3=24.65$; $A0=10.5$; $B0=28$; $C0=8/3$; $M=2$; $N=1$; $M_1=2$; $M_2=1$; $N_1=N_2=1$; $O_1=O_2=2$.

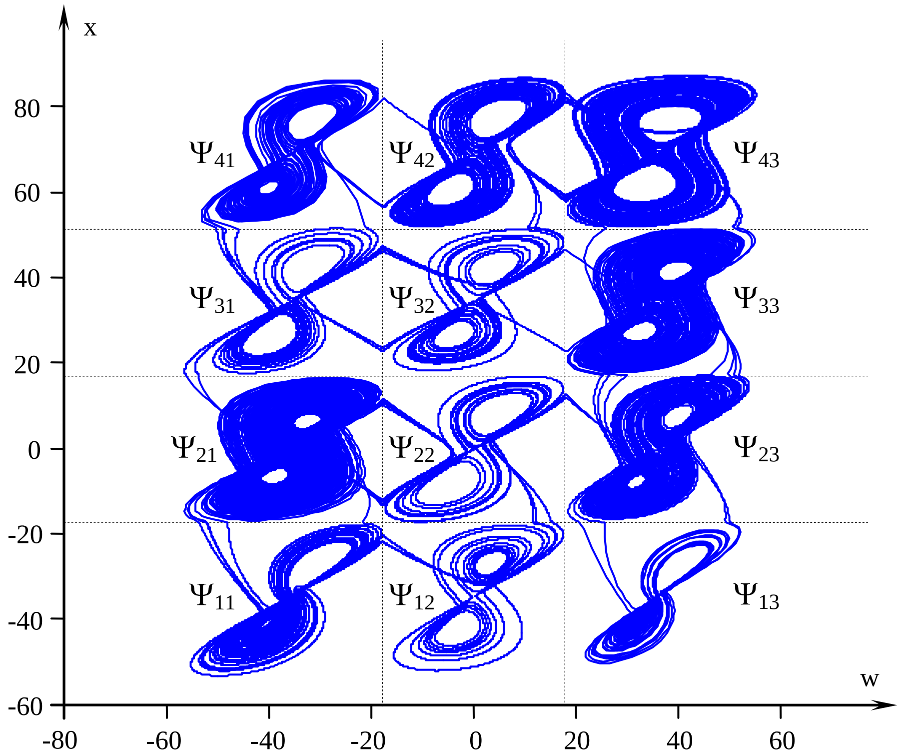

Fig.1. Nonuniform compound chaotic multiattractor whith Lorenz local attractors

$$\gamma A=\begin{bmatrix} 1.429 & 4.762 & 0.286 & 31.714 \\ 0.19 & 1 & 0.381 & 1.429 \\ 28.571 & 2.762 & 5.714 & 0.286 \end{bmatrix};\ \gamma B=\begin{bmatrix} 0.893 & 14.286 & 0.536 & 35.714 \\ 0.714 & 1 & 0.714 & 1.786 \\ 14.286 & 3.571 & 14.286 & 2.679 \end{bmatrix};$$

$$\gamma C=\begin{bmatrix} 0.375 & 3.75 & 0.188 & 30 \\ 0.113 & 1 & 0.375 & 1.5 \\ 7.5 & 2.25 & 5.625 & 0.263 \end{bmatrix};\ \phi 1=\begin{bmatrix} 0.77 & 9 & 0.33 & 35 \\ 0.31 & 1 & 0.55 & 1.6 \\ 14.9 & 3.1 & 10 & 0.9 \end{bmatrix};$$

$$\phi 2=\begin{bmatrix} 0.56 & 11 & 0.35 & 35 \\ 0.4 & 0.9 & 0.6 & 1.6 \\ 11.9 & 3.2 & 12.3 & 1.6 \end{bmatrix};\ \phi 3=\begin{bmatrix} 0.75 & 11.8 & 0.37 & 35 \\ 0.4 & 1 & 0.63 & 1.65 \\ 13.3 & 3.2 & 13.3 & 1.7 \end{bmatrix};\theta 3=\begin{bmatrix} 5.5 & 9 & 11 & 2 \\ 21 & 0 & 2.5 & 3 \\ 4 & 5 & 4 & 17 \end{bmatrix}.$$

Moreover, these constants have the following values:

$$A=\begin{bmatrix} 15 & 50 & 3 & 333 \\ 2 & 10.5 & 4 & 15 \\ 300 & 29 & 60 & 3 \end{bmatrix};\ B=\begin{bmatrix} 25 & 400 & 15 & 1000 \\ 20 & 28 & 20 & 50 \\ 400 & 100 & 400 & 75 \end{bmatrix};\ C=\begin{bmatrix} 1 & 10 & 0.5 & 80 \\ 0.3 & 8/3 & 1 & 4 \\ 20 & 6 & 15 & 0.7 \end{bmatrix}.$$

All local attractors in Fig.1 are given the same dimensions close to the dimensions of the attractor of the original dynamical system (it is located in the phase cell, containing the origin).
Figure 2 illustrates the difference between local chaotic attractors that are part of multiattractor shown in Fig.1.

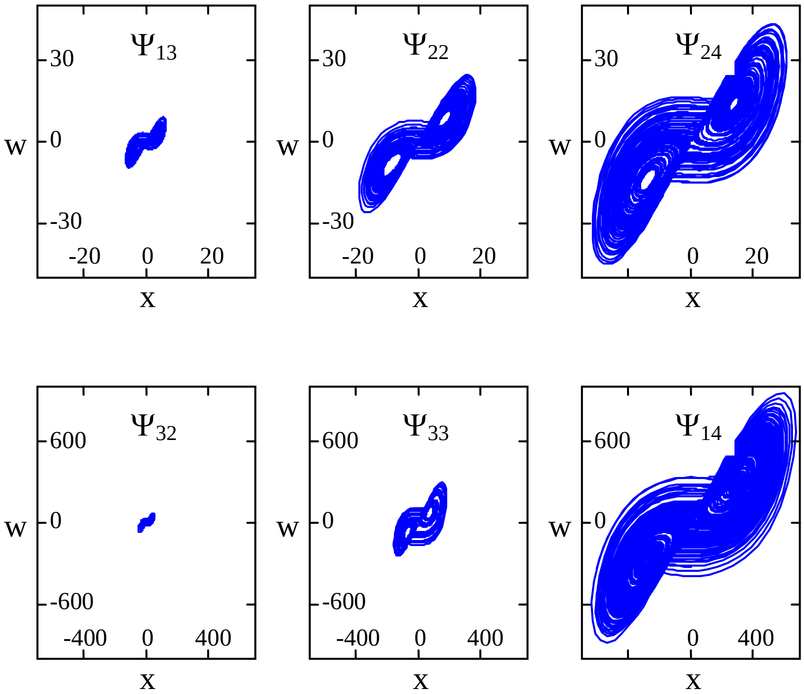

Fig.2. Illustration of differences between local chaotic attractors that are part of multiattractor shown in Fig.1.

In Fig.3 is example of time dependences of variable *x*.

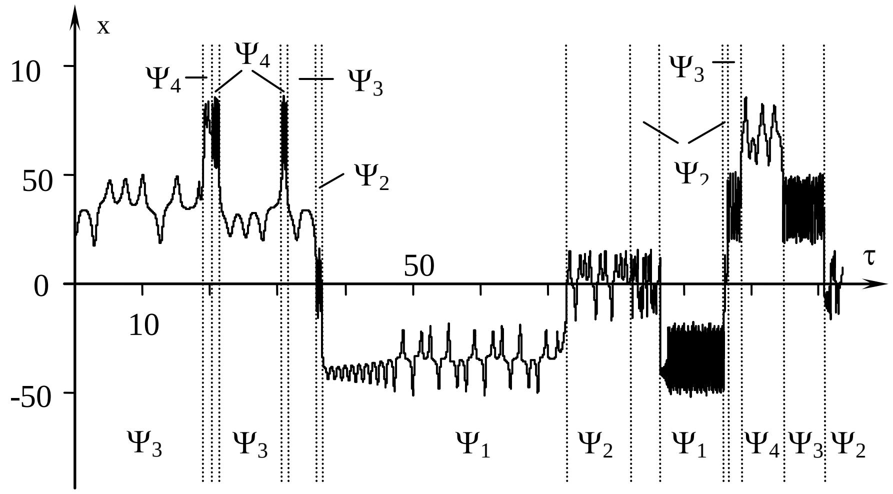

Fig.3. Example of time dependence of variable x.

5 Conclusions

Thus, there is the possibility of building dynamic systems that have composite chaotic multiattractors, in which it is possible a priori assignment of the differences between local chaotic attractors. The necessary modification of the original dynamic system with compound chaotic multiattractor, includes the introduction in the equations of motion of the three groups of additional nonlinear functions, one of which actually was responsible for the introduction of differences between the local attractors, and the other two for regulation of the size and configuration of the phase cells containing local chaotic attractors.

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