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Chaos in a coupled shaker – oscillator model

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Abstract: The purpose of this paper is to study the interaction of the oscillating system and the shaker, which is driven by the amplifier. The amplifier is considered as self-exciting system which has a power source of limited power. The current produced by it is converted by the shaker into mechanical force, which leads to vibrations of the shaker base. A mechanical oscillator is mounted on the shaker base. The influence of oscillator vibrations on the formation of the driving force leads to a number of specific effects, in particular, to the Sommerfeld's effect. New nonlinear effects in the coupled shaker–oscillator system is studied in details. Steady-state regimes of the constructed model are investigated by methods of the theory of dynamical systems. Regular periodic and chaotic regimes are found and studied.
Keywords: shaker-oscillator system, Sommerfeld-Kononenko's effect, chaotic steady-state regimes.

1. Introduction

The coupling effect between an excitation machine and vibrational loads was found by Sommerfeld [1-3], is a universal phenomenon and a manifestation of the law of conservation of energy. A rather complete study of the Sommerfeld effect has been given in the works of Kononenko [4], so that we call these phenomena as Sommerfeld-Kononenko's effect [5-8]. As shown by Kononenko for a linear oscillator with limited excitation the characteristics of a nonlinear oscillator arise, such as the occurrence of instability regions. In view of this, in the present study, the existence of new possible characteristics is investigated for an oscillator with damping and an electrodynamic shaker. Presence of both direct and feedback interactions between the oscillator and the shaker are main goal of our modelling and study in present paper. The mutual influence between an oscillating system and the mechanism of its excitation, when the later has limited power, gives rise to a number of unusual phenomena in their behaviour [9-12]. The effects of the interaction of an electrodynamic shaker powered by a vacuum-tube amplifier of limited power, and a linear

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oscillator which affects the amplitude and frequency of the driving force, are studied in this paper.

2. The mathematical model with strong interaction

Let us consider an oscillator with damping, mounted on the base of a shaker which undergoes displacements $w(t)$ (Fig. 1). The equation of vibrations of the oscillator of mass m with the vibrational resistance coefficient β_0 has the following form

$$m\ddot{x} + \beta\dot{x} + cx = -m\ddot{w} \tag{1}$$

The base of the shaker has a displacement $w(t)$ as a result of the action of the force $H_0 i_0$ [5, 7] applied to the coil L_1, which is rigidly attached to the base. The quantity H_0 is a constant characterizing the electromagnetic field of the vibrator; i_0 is the current of the shaker circuit. The law of motion of the centre of mass of the coil with the base (their mass is m_1) and the oscillating system may be written in the form

$$m_1\ddot{w} + m(\ddot{w} + \ddot{x}) = H_0 i_0 \tag{2}$$

The current of the shaker is related to the amplifier current $(i_2 + i_3)$ and the displacement $w(t)$ by the differential relationship [5, 7]

$$(L_0 + L_1)\frac{di_0}{dt} - M\frac{d(i_2 + i_3)}{dt} + H_0\frac{dw}{dt} = 0 \tag{3}$$

Suppose that the tube operates under conditions when the anodic current equals [5]

$$i_a = a_0 + a_1(e_g + De_a) - \varepsilon a_3(e_g + De_a)^3 \tag{4}$$

where e_g is the tube grid voltage; e_a is the anodic voltage; D is the penetration factor of the tube; and ε is a small positive parameter.
Applying the method of contour currents, we can write the following Kirchhoff's equations for each branch of the generator current:

$$i_a = i_1 + i_2 + i_3,$$

$$e_a = E_a - R_a i_1,$$

$$e_a = -L_k \frac{di_2}{dt} - R_k i_2,$$

$$L_k \frac{di_2}{dt} + R_k i_2 = \frac{1}{C_k}\int i_3 dt,$$

$$e_c = -E_c + M_k \frac{di_2}{dt}.$$

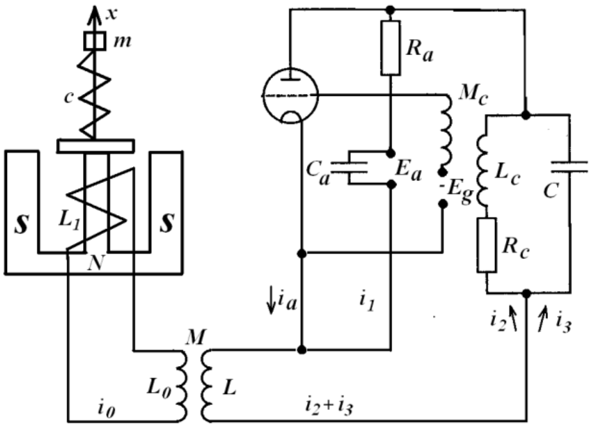


Fig. 1. Schema of a shaker with an amplifier interacting with an oscillator .

After setting up these Kirchhoff's equation for each branch of the amplifier current, let us reduce them to one equation with respect to a new variable $u(t) = \int (e_g - E_g)dt$ ($-E_g$ is the constant component of the voltage e_g). We retain only terms of the first order of smallness. Here we assume that $\left(L - \left[M^2/\left(L_0 + L_1\right)\right]\right) = \varepsilon\alpha_1,\ D = \varepsilon\alpha_2,\ H_0 = \varepsilon\alpha_3$.

Selecting the slope of the tube characteristic in (4), a_1 in accordance with the equation of amplitude balance, we assume it to be equal to

$$a_1 = \frac{R_c R_a C_a + L_c}{R_a (M_c - DL_c)} + \varepsilon a \quad (\varepsilon a > 0).$$

With this value of a_1 we obtain the following nonlinear equation for the function $u(t)$:

$$\frac{d^2u}{dt^2}+\omega^2u=\left(-\frac{E_aM_c}{L_cC_cR_a}+a_0\frac{M_c}{L_cC_c}\right)+\varepsilon a\frac{M_c}{L_cC_c}\frac{du}{dt}$$

$$+\varepsilon a_1\left(\frac{R_c\omega^2}{R_aL_c}u+\frac{R_c}{R_a{}^2L_cC_c}\frac{du}{dt}\right)-\varepsilon a_3\frac{M_c}{L_cC_c}\left(\frac{du}{dt}-E_g\right)^3+\varepsilon\gamma\dot{x} \qquad (5)$$

Here

$$\omega^2=\frac{R_a+R_c}{L_cC_cR_a};\ \varepsilon\gamma=\varepsilon\alpha_3\frac{MM_cm}{L_cC_cR_a(L_0+L_1)(m_1+m)}$$

The tube obtains energy from the energy sources E_a and $-E_g$, which are rectifiers of the supply voltage. We assume the rectifiers to be nonideal sources of energy [4], since the output voltage E of the rectifier depends on the current i flowing through the load (of the tube oscillator, in this case), according to the external characteristic |5], which is given approximately by $\mathrm{E}=\mathrm{E}_{\mathrm{oc}}-\varepsilon\mathrm{ri}$ (E_{oc} is the open-circuit voltage; $\varepsilon\mathrm{r}$ is a quantity equivalent to the rectifier resistance). Neglecting the grid current, we assume $\mathrm{E}_{\mathrm{g}}=\mathrm{E}_{\mathrm{oc}}$. By considering the equality of rectifier output voltage on a shunt of high capacitance C_{a} (assuming $\mathrm{C}_{\mathrm{a}}\approx 1/\varepsilon$), we obtain the following relationship for the voltage E_a:

$$E_a=\frac{R_e}{R_e+\varepsilon r}E_{oc}-\varepsilon r_1u(t)-\varepsilon r_2\frac{du}{dt} \qquad (6)$$

where R_e is the equivalent resistance (the sum of R_a, the tube resistance, and εr); εr_1 and εr_2 are constants determined by the parameters of the tube oscillator and rectifier.

Therefore, E_a is not a constant but depends on the variable function $\mathrm{u}(\mathrm{t})$. This fact clearly must be reflected in the formulation of $\mathrm{u}(\mathrm{t})$. After substituting (6) into (5), the components $(\mathrm{M}_c/\mathrm{R}_{\mathrm{a}}\mathrm{L}_{\mathrm{c}}\mathrm{C}_{\mathrm{c}})[\varepsilon\mathrm{r}_1\mathrm{u}+\varepsilon\mathrm{r}_2(\mathrm{du}/\mathrm{dt})]$ reflecting the nonideal character of the energy source of the excitation mechanism, appear on the right side. Terms on the right side of equation (5) may be regarded as ‘internal forces’ and as the effect of interaction with the oscillator. Therefore, we write

$$\frac{d^2u}{dt^2}+\Omega_0^2u=\varepsilon L\left(u,\frac{du}{dt}\right)-\varepsilon K\left(u,\frac{du}{dt}\right)+\varepsilon\gamma\frac{dx}{dt} \qquad (7)$$

Here $\varepsilon L(\mathrm{u},\ \mathrm{du}/\mathrm{dt})$ is the sum of the internal forces causing energy influx; $\varepsilon K(\mathrm{u},\ \mathrm{du}/\mathrm{dt})$ is the sum of the internal resistance forces; Ω_0 is the frequency of the self-oscillation conditions of the unloaded excitation mechanism; i.e., Ω_0 and amplitude ξ_0 are determined for the function $u=\xi_0\cos\Omega_0 t$ from the serf-oscillation equation

$$\frac{d^2u}{dt^2}+\Omega_0^2u=\varepsilon L\left(u,\frac{du}{dt}\right)-\varepsilon K\left(u,\frac{du}{dt}\right)=0$$

We call the function $\varepsilon L(\mathrm{u},\ \mathrm{du}/\mathrm{dt})$ the static characteristic of the energy source, since under stationary conditions $\varepsilon L(\mathrm{u},\ \mathrm{du}/\mathrm{dt})$ opposes the energy loss $\varepsilon K(\mathrm{u},\ \mathrm{du}/\mathrm{dt})$. These functions have the following form:

$$\varepsilon L(u,\frac{du}{dt})=\varepsilon\left\{[(a+\frac{r_2}{R_a})\frac{M_c}{L_cC_c}+\alpha_1\frac{R_c}{R_a^2L_cC_c}]\frac{du}{dt}\right\}$$

$$+3a_3\frac{M_c}{L_cC_c}E_g(\frac{du}{dt})^2;$$

$$\varepsilon K(\mathrm{u},\frac{du}{dt})=\varepsilon a_3\frac{M_c}{L_cC_c}\left\{3E_g^2\frac{du}{dt}+(\frac{du}{dt})^3\right\}; \qquad (8)$$

And the frequency Ω could be determined from

$$\Omega_0^2=\omega^2-\frac{\varepsilon r_1M_c}{R_aL_cC_c}\mathrm{u}-\frac{\varepsilon\alpha_1R_c\omega^2}{R_aL_c}\mathrm{u}.$$

We should note that the nonideal model of the shaker with amplifier (7) has principal difference from the model constructed and used in the papers [9-11], where it has unlimited energy source of variable current. So it is impossible to influence on the frequency what is crucial for stability of the process of interaction [13].

Transforming equations. (1), (2), and (3) and expressing the current (i_2+i_3) by $\mathrm{u}(\mathrm{t})$ enable us to define

$$\frac{d^2x}{dt^2}+\Omega_1{}^2x=\varepsilon\lambda u-\frac{\varepsilon\mu}{\Omega_0}\frac{du}{dt}-\varepsilon\beta\frac{dx}{dt}, \tag{9}$$

where

$$\Omega_1{}^2=\frac{c(m_1+m)}{mm_1};\ \varepsilon\lambda=\varepsilon\alpha_3\frac{HMR_c}{m_1M_cR_a(L_0+L_1)};$$

$$\frac{\varepsilon\mu}{\Omega_0}=\varepsilon\lambda R_aC_c;\ \varepsilon\beta=\frac{\beta_0(m_1+m)}{mm_1}.$$

Concluding, the system of equations (7) and (9) represents of the coupled shaker- oscillator model with nonideal amplifier.

3. Numerical simulations results

Introducing the following dimensionless variables

$$\xi=\frac{u\omega}{E_g},\ \dot{\xi}=\frac{d\xi}{d\tau}=\zeta,\ x_1=\frac{x}{w},\ \dot{x}_1=\frac{dx_1}{d\tau},\ \tau=\Omega_0t,$$

the system of equations (7) and (9) can be written in the form:

$$\begin{cases}\dot{\xi}=\zeta\\ \dot{\zeta}=-\xi+\gamma_1\zeta+\gamma_2\zeta^2-\gamma_3\zeta^3+\gamma_4p\\ \dot{x}_1=p\\ \dot{p}=\gamma_5\xi+\gamma_6\zeta-\gamma_0x_1-\gamma_7p.\end{cases} \tag{10}$$

Where the coefficients are

$$\gamma_1=\frac{\varepsilon}{\Omega_0}\left\{(a+\frac{r_2}{R_a})\frac{M_c}{L_cC_c}+\alpha_1\frac{R_c}{R_a^2L_cC_c}-3a_3E_g^3\frac{M_c}{L_cC_c}\right\};$$

$$\gamma_2=3a_3\frac{M_c}{L_cC_c}E_g;\gamma_3=\frac{M_ca_3\Omega_0}{L_cC_c};\ \gamma_4=\frac{\varepsilon\gamma}{\Omega_0};\ \gamma_5=\frac{\varepsilon\lambda}{\Omega_0^2};$$

$$\gamma_6=-\frac{\varepsilon\mu}{\Omega_0^3};\ \gamma_7=\frac{\varepsilon\beta}{\Omega_0};\ \gamma_0=\frac{\Omega_1{}^2}{\Omega_0^2}.$$

The system (10) is nonlinear, so we may study it numerically. The following values of variables and constants are used in our numerical simulations [7]:

$E_g = 700V;\ E_a = 2000V;\ a = 6.5\times10^{-5}\ \mathrm{A/V};\ R_a = 160\Omega;\ R_c = 10\Omega;$

$a_3 = 5.184\times X\times10^{-9}\ \mathrm{A/V^3};\ D = 0.015;\ L_c = 0.094H;\ L = 100H;$

$M = 1H;\ M_c = 0.275H;\ C_c = 1.0465mF.$

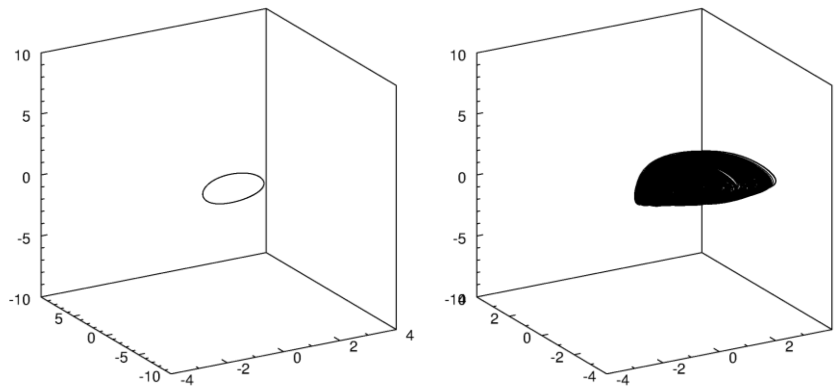
Using these variables one may obtain the following coefficients for the system (10):

$\gamma_0 = 0.995,\ \ \gamma_1 = 0.0535,\ \ \gamma_2 = 0.63X,\ \ \gamma_3 = 0.21X,\ \ \gamma_4 = 0.5$

$\gamma_5 = -0.0604,\ \ \gamma_6 = -0.12,\ \ \gamma_7 = 0.01,$ X is the bifurcation parameter.

The phase portraits of steady state solutions for the initial conditions $\xi = 0.3, \zeta = 0.2,\ x = p = 0.1$ are shown in Figure 2. The limit cycle graph is shown in Figure 2 a) and corresponds to regular regimes of oscillations [14] with periodically changing variables ξ and ζ. Of course, the variables *x* and *p* are also regular and periodic in time. The phase portrait for chaotic regimes of interaction are presented in Figure 2 b).

The spectrum in Figure 3 a) has discrete peaks. So that, this graph indicates that there is regular regimes in the system at *X*=1.0. With increasing value of *X* the transition to chaos occurs. Thus, at *X*=2.0 chaos is realized in the system, when the spectrum in Figure 3 b) is continuous [14] and the projection of the phase portrait occupies some area in the phase space (Fig.2 b)).



a b

Fig. 2. Graphs of projection of the phase portrait at X=1 a), X=2 b)

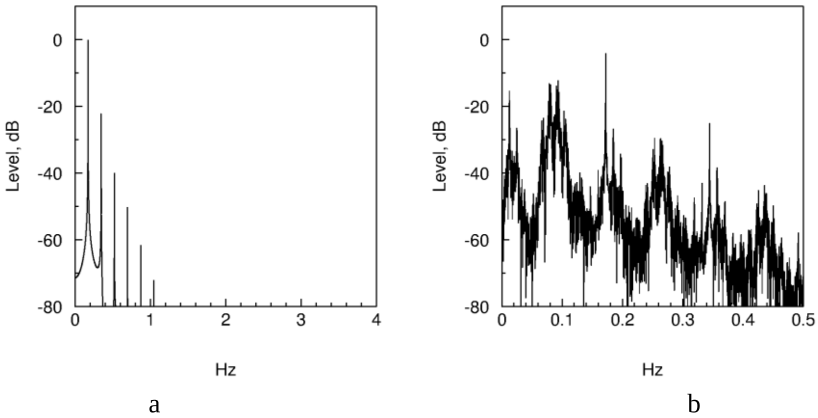


Fig. 3. Graphs of the power spectra at X=1 a), X=2 b)

4. Conclusions

The coupled shaker-oscillator model, which takes into account both direct and reverse influence of subsystems is worked out. The methods of modern theory of the dynamical systems are used to study laws of the steady-state regimes of the complex model with strong interaction. The chaotic regimes were found out. The dynamics of the oscillator system is in good correspondence with experimental information of a limited power shaker behavior [10, 11, 12]. Found irregularities of phase trajectories of the complex model depend on intensity of the amplifier tube.

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