Chaotic Modeling and Simulation (CMSIM) **3: 191-196, 2021**

The Effect of Different Hopping Amplitudes on Discrete Rogue Waves

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Abstract. We study discrete rogue waves in Hermitian and non- Hermitian waveguides. We show that the hopping amplitudes, forward and backward direction, indicate the site where rogue waves occur. In non-Hermitian waveguides, discrete rogue waves tend to accumulate to one side.
Keywords: Rogue waves, non-Hermitian system.

1 Introduction

Rogue waves are generally known as abruptly large amplitude waves that rarely appear in the open sea (Haver, 2004). Their living time is concise, and they are at least two times higher than the rest of the sea. Their interesting feature appears without warning and disappears like it never happened (Dysthe et al., 2008; Guo, Boling; Tian, 2017; C. Kharif et al.., 2008; C. Kharif & Pelinovsky, 2003; Wen-Rong Sun et al., 2021). These waves can occur in open water and can be excessively dangerous for several reasons (M. Onorato et al., 2001). An important one is an unpredictability. The other one is their large amplitude.
Rogue waves have attracted the great attention of researchers because these waves have found not only in hydrodynamics but also in other physical fields such as nonlinear optics (Kibler et al., 2010), Bose-Einstein condensates (Wen et al., 2011), superfluids (Chabchoub et al., 2012), plasmas (Moslem, 2011), even in finance (Yan, 2011; Zhen-Ya, 2010).
For centuries, giant waves were thought to be part of marine folklore, but their existence has been proved by taking sensitive measurements (Christou & Ewans, 2014; C. Kharif et al.., 2008). Seafarers describe freak waves as "walls of water" because the resulting amplitude may reach the height of 20–30 m or even higher. Freak waves can be observed in groups of waves, as well as a single one. In literature, two small waves accompanying to central peak, recalling "three sisters," are often mentioned (C. Kharif et al.., 2008).

Received: 24 February 2021 / Accepted: 16 July 2021

ISSN 2241-0503

The first scientific record was taken in the North Sea at Draupner platform on 1 January 1995 (Haver, 2004). The maximum wave height was 25.6 m. It is vital since it brings light to how rogue waves occur experimentally.

Since rogue waves are infrequent, measurements of this event troublesome, underlying mechanisms of these giant waves are still in debate (C. D. Pelwan et al., 2020). A variety of mathematical explanations of freak waves in the ocean are even linear or nonlinear (Dysthe et al., 2008). Although it is too early to state the most common causes, main linear and nonlinear physical factors can be said as geometrical focusing, temporal focusing, modulational instability, nonlinear wave interaction, wind, and strong currents. There is a consensus that the nonlinear Schrödinger equation (NLSE) is a convenient and widely-used way to investigate rogue waves (Guo, Boling; Tian, 2017; M. Onorato et al.., 2016).

Bludov, Konotop, and Akhmediev considered the discrete nonlinear Schrödinger equation (DNLS) to model an array of nonlinear waveguides. They constructed a controlled formation of a discrete rogue wave (Bludov et al., 2009).

DNLS equation can be defined as follows (Bludov et al., 2009),

$$i\frac{d\Psi_n}{dt}+Q_n(\Psi_{n+1}+a\Psi_{n-1})+g\,|\Psi_n|^2\Psi_n=0 \qquad (1)$$

where n is the complex field amplitude at the n-th waveguide, t is the propagation direction, Q_n The coupling coefficient between the n-th waveguide and neighboring waveguides and a is a dimensionless constant which $0 < a$ and g is nonlinear interaction constant. We study with the initial condition that closes to the exact solution of NLS. References (Bludov et al., 2009; Efe & Yuce, 2015) already show that the initial state has significant importance in observing discrete rogue waves. We take the initial condition as,

$$\Psi(n,0) = \varepsilon e^{-i\varepsilon^2 L}\left(1-4\frac{1-2i\varepsilon^2 L}{1+2\varepsilon^2 n^2+4\varepsilon^4 L^2}\right) \qquad (2)$$

in references (Bludov et al., 2009; Efe & Yuce, 2015). Here ε is a very small parameter (ε<<1) and background amplitude. L is the length of each waveguide in the array.

We work on a numerical solution, and we suppose that equation 1 is subject to periodic boundary conditions and initial condition given in eq.2. We begin with choosing a=1 in eq.1. This means that forward difference length and backward difference are equal to each other, and all waveguides are disseverance same. We can define this desired system as a Hermitian waveguide.

Consider ε=0.2, L=100 and g=1. We take the coupling coefficient of the DNLS equation Q_n=1.2,

$$i\frac{d\Psi_n}{dt}+1.2\,\Psi_{n+1}+1.2\,\Psi_{n-1}+|\Psi_n|^2\Psi_n=0 \qquad (3)$$

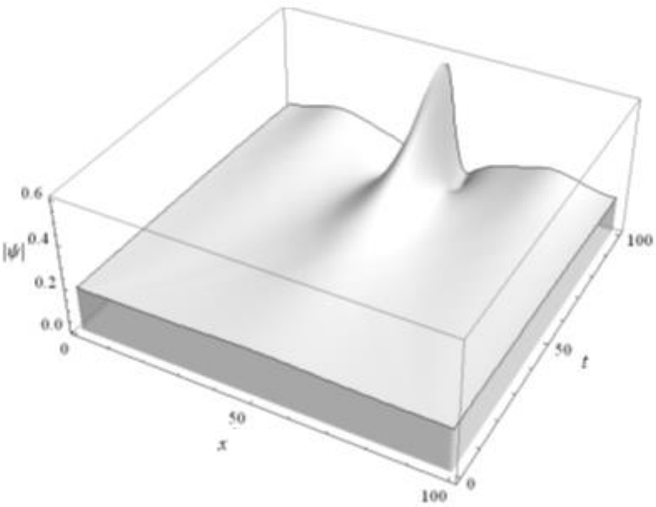

Figure 1: The absolute of field amplitude for ε=0.2 and L=100. Eq. 3 is solved numerically with the initial form of field given in eq. 2. Peregrine-like soliton occurred.

Figure 1 shows that when forward and backward differences equal to each other and proper initial condition is chosen, Peregrine soliton has occurred.
Reference (Efe & Yuce, 2015) shows that a small degree of disorder affects rogue wave formation deeply. However, it is not shown that if the length of forwarding and backward difference waveguides are not equal to each other, this situation causes the rogue waves to gather in one region. In this study, we show rogue waves' behavior when the lengths of waveguides are not equal. This means the system is non-Hermitian.
Now, let us change the coupling constant and dimensionless constant of the DNLS equation, Q_n=1.3, $a \approx 0{,}923$,

$$i\frac{d\Psi_n}{dt}+1.3\ \Psi_{n+1}+1.2\ \Psi_{n-1}+|\Psi_n|^2\Psi_n=0$$

note that forward difference and backward difference lengths are not equal to each other now. Forward difference length is a bit bigger than backward difference length.

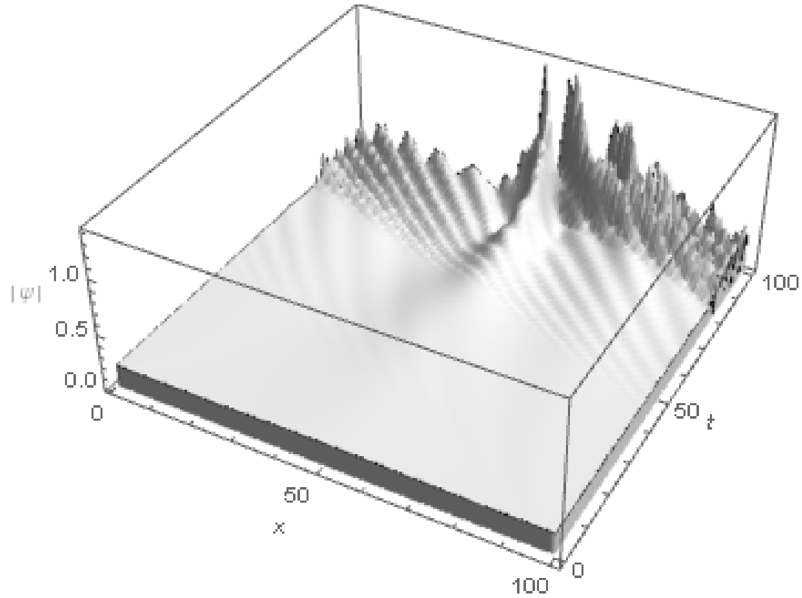

Figure 2: The absolute of field amplitude for ε=0.2, L=100, and g=1. Eq. 3 is solved numerically with the initial form of field given in eq. 2. rogue waves accumulate right side.

As shown in Figure 2, although some chaotic fluctuations happen, rogue waves accumulate right side.

Does discrete rogue waves evaluation really depend on irregular grids? There is the question of what happens if backward difference length is a bit bigger than forwarding ones. We will change the coupling constant and dimensionless constant of DNLS equation as, Q_n=1.2, $a = 1.08\overline{3}$,

$$i\frac{d\Psi_n}{dt}+1.2\ \Psi_{n+1}+1.3\ \Psi_{n-1}+|\Psi_n|^2\Psi_n=0$$

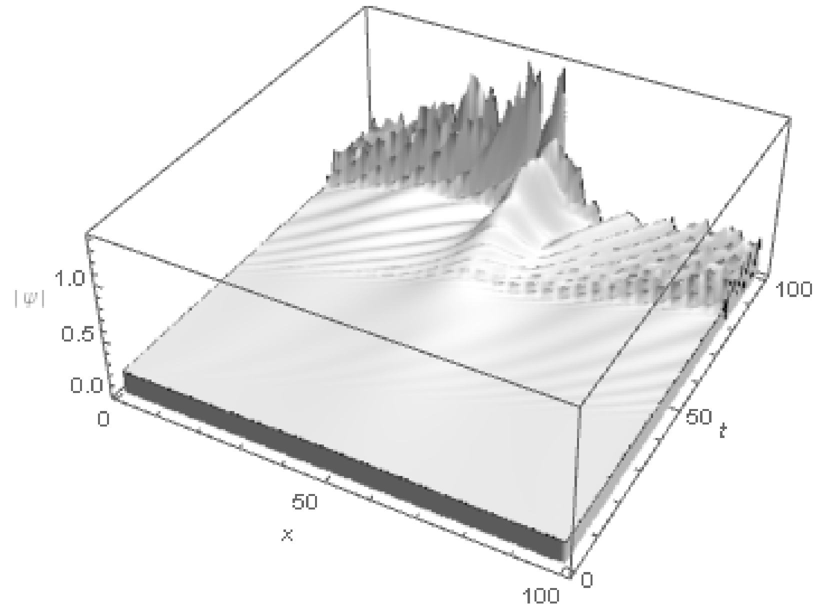

Figure 3: The absolute of field amplitude for ε=0.2, L=100, and g=1. Eq. 3 is solved numerically with the initial form of field given in eq. 2. rogue waves accumulate left side.

Although some chaotic fluctuations happen, as can be seen from the figures, if the forward difference is more significant than the backward, the waves accumulate to one side. If the backward difference is more significant than the forward, the waves accumulate to another side.

Conclusions

In this paper, we have studied discrete rogue waves with different hopping amplitudes. We show that choosing hopping amplitude a bit bigger in forward and backward direction correlates with discrete rogue waves' sites. According to hopping amplitude, discrete rogue waves accumulate to one side. It is vital to control where discrete rogue waves occur. That is why hopping amplitudes behave like control parameters. It is an open question that can be proven by experimental studies.

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