Dynamic similarities of rotors with rubbing blades

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Abstract: The non-linear behaviour of rubbing cylindrical rotors have been studied in several papers. In such systems rich dynamics have been found for frequencies above the natural frequency. Below natural frequency the solution was found to be stationary. In this paper the influence of blades is studied. A Jeffcott rotor with three, five, ten and fifty blades is used and the contacts are described by large displacement beam theory. The model shows that bladed turbines have a similar dynamic behaviour if scaled properly. This imply that general conclusions for a simple system can be extracted to a more complex one. However it is also shown that the forces and amplitudes are not easily scaled and therefore if the values are wanted one have to simulate the actual system.

Keywords: rotor, dynamic, impact, rubbing, beam, blade.

1. Introduction

In rotor dynamics there are several situations when non-linear problems can occur. One such example is rub-impact which is a highly non-linear phenomenon. The problem is of industrial interest since there are several applications where rub-impact is the main cause for unwanted vibrations e. g. gas turbines, centrifuges, compressors and generators. It has been reported that 10.2% of 275 reported jet engine failures during 1962 to 1975 were caused was rubbing between rotating and stationary parts [1]. Several studies have been performed on the Jeffcott rotor with this kind of rubbing impacts. Some of them are described below, with focus on findings and development of methods.

The jump phenomenon and the influence of radial clearance were studied analytically in [2]. A modified Harmonic Balance method has been used to predict the occurrence and analyse the stability of quasi-periodic motion [3]. In [4] Fourier series and Floquet theory was used for analysis of global bifurcation and stability. They also reported three routes to chaos; from stable periodic through period doubling bifurcations, grazing bifurcation and a sudden transition from periodic motion to chaos. The stability for the case of full annular rub and cross coupling stiffness was analysed in [5]. Chaos has been reported to exist over large parameter ranges and different solutions can coexist [6]. In [7] approximate analytical solutions was developed for non-linear dynamical responses and in [8] the harmonic balance method was used to

Received: 4 April 2012 / Accepted: 15 September 2012

ISSN 2241-0503

calculate periodic responses of the non-linear system. In most cases the contact was modelled with an increased stiffness and Coulomb friction. However, other frictional models has also been considered [9]. In all models with annular rub, the system reaches a stationary point below the natural frequency of the rotor. It is only above the natural frequency where complex dynamics and multiple solutions are found [10]. In these models the rotor and stator was assumed to be circular. Bladed rotors in aero engines have also been studied with complex FEM models [12]. Due to the complexity of these models only short time sequences has been analysed and therefore the dynamics on parameter ranges is still unknown.

In a recent paper an attempt was made to model a rubbing Jeffcott rotor with three blades where the displacements were described by large displacement beam theory [13]. The model showed that when blades are included rich dynamics was found below the natural frequency of the rotor. Simulation time was also short which made the model suitable for numerical analysis. In this paper the same model is used but the dynamic influence of the number of blades is studied. The target is to evaluate similarities and differences in the dynamic when the number of blades are changed.

2. The Model

The model of the Jeffcott rotor is shown in Figure 1 for a case with three blades. The mass of the rotor, m, is supported by the shaft with stiffness k and damping c. The rotor is amplitude limited by the stator which has a radius R. The rotor is described by a point mass m in the centre and n mass less beams of length L, E and area moment of inertia I. The rotor is rotating with the angular velocity ω. In Figure (C) the geometry of the contact is shown. When a blade is in contact the beam will be deformed transversally Δ and axially δ. Both deformations are necessary in order to keep the beam inside the limit radius R. The contact force is described by a radial force P normal to the circle pointing from the contact point towards o and a tangential force μP, where μ is a coefficient of friction. To simplify the analysis it is assumed that $\omega t\text{-}\phi$ is small which imply that P is an axial force and μP a tangential force on the beam.

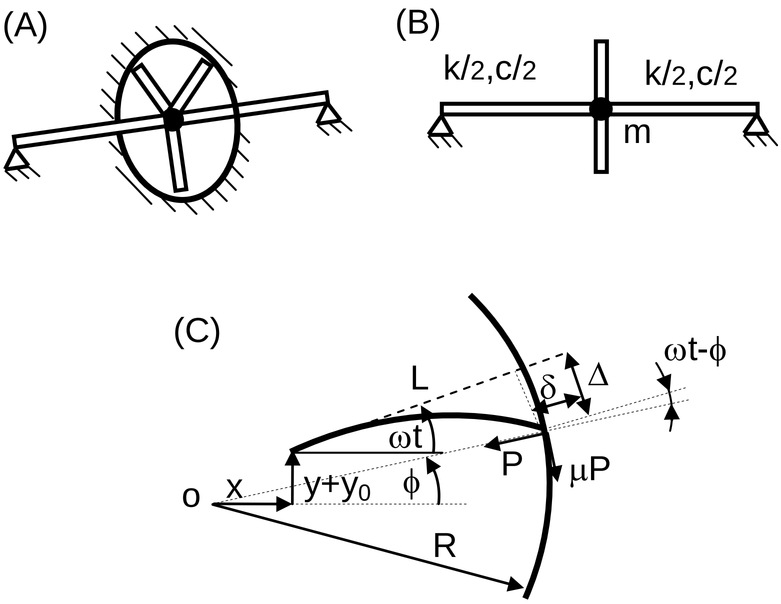

Fig. 1. Rub impact model of the Jeffcott rotor. View of the whole system (A), side view (B) and geometry of the contact (C).

If o is the centre of the stator, the vector to the contact point of the beam can be described as

$$\bar{r} = \left(x + (L-\delta)\cos(\omega t) + \Delta\sin(\omega t)\right)\bar{i} + \left(y + y_0 + (L-\delta)\sin(\omega t) - \Delta\cos(\omega t)\right)\bar{j}$$

where $\bar{i}$ and $\bar{j}$ are unit vectors in x and y direction respectively. The displacement y_0 is an initial misalignment of the rotor. When the rotor is in contact, the rotor is limited by the stator so that

$$\left|\bar{r}\right| = R$$

The contact force is described by the radial force P pointing toward o and a tangential component given by a coefficient of friction μP. As noted above, it is assumed that the contact angle $\phi = \omega t$ and therefore the forces on the beam are given by an axial compression force P and a transversal force μP. From beam theory the deformation of cantilevered beam is given by the equation

$$w''(\varepsilon) = \frac{P}{EI}\left(1 + (w')^2\right)^{3/2}\left((\Delta - w(\varepsilon)) + \mu P(L - \varepsilon - \delta)\right)$$

ε. The beam is assumed clamped at $\varepsilon=0$ and subjected to the forces P and μP on the free end ($\varepsilon=L-\delta$). At the free end, the beam will be displaced by the forces axially δ and transversally Δ. By numerical integration the values of P□ Δ and δ can be found which satisfies $\left|\bar{r}\right| = R$. When a blade is in contact the forces in x and y directions are;

$$Fx = P\frac{\bar{r}\bullet\bar{i}}{R} - \mu P\frac{\bar{r}\bullet\bar{j}}{R} \quad ,$$

$$Fy = P\frac{\bar{r}\bullet\bar{j}}{R} + \mu P\frac{\bar{r}\bullet\bar{i}}{R} \quad .$$

For n blades there will be one set of such forces (Fx_i, Fy_i) for each blade i, hence the equation of motion for the Jeffcott rotor then becomes

$$\ddot{x} + 2\varsigma\omega_n\dot{x} + \omega_n^2 x = -\sum_{i=1}^{n} Fx_i \big/ m$$

$$\ddot{y} + 2\varsigma\omega_n\dot{y} + \omega_n^2 y = -\sum_{i=1}^{n} Fy_i \big/ m$$

In this paper four cases will be studied with different number of blades. In Figure 2 the four cases are shown where (a) is three blades, (b) five blades, (c) ten blades and (d) fifty blades.

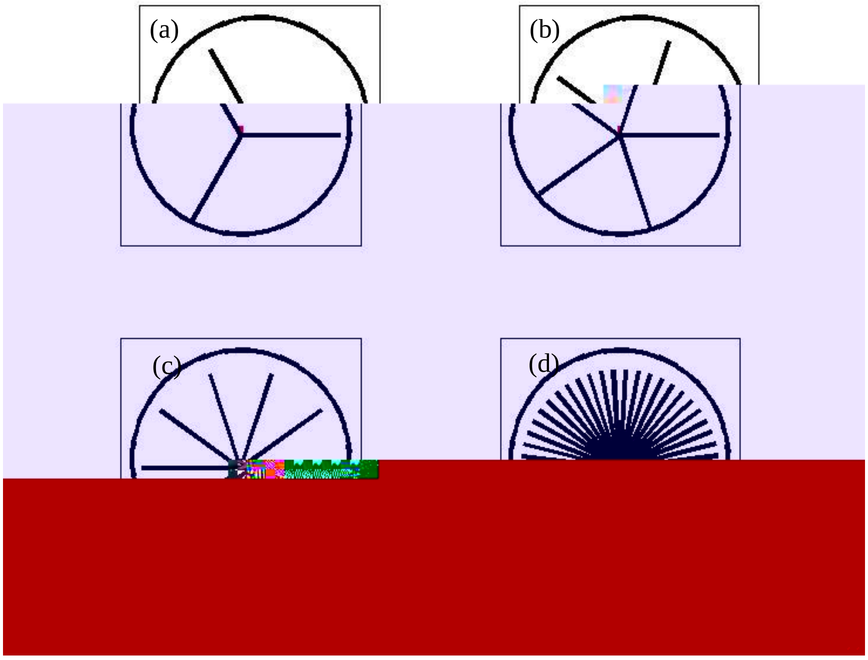

Fig. 2. Rub impact model for different number of blades. (a) three blades, (b) five blades, (c) ten blades and (d) fifty blades.

3. Method

In this paper different bifurcation diagrams are used to evaluate the system. In these simulations a fourth order Runge-Kutta integration with adjustable time-step is implemented in an in-house code written in Fortran. In the bifurcation

frequency ω. For the suggested model there are 5 dimensions in the state space:

displacements x, y, velocities $\dot{x}, \dot{y}$ and the phase $\phi = \omega t$. Since the phase can be restricted to the interval *[0, 2π]* it can be described as a circle S^1 with the period $2\pi/\omega$. Thus points in the 5 dimensional state space are given by $(x, \dot{x}, y, \dot{y}, \theta) \in R^4 \times S^1$

this state space for a constant value of the phase θ_p. In this paper this constant value is chosen to 2π

$$\sum = \left\{ x, \dot{x}, y, \dot{y}, \theta_p \,\middle|\, \theta_p = 2\pi \right\}.$$

A rubbing system is selected with *R 1/22-! 1/2-! 1/2-!* $_0$*=0.010001, E=2.06 21*^{11}*-! 1/12 1/112*^{3}*/12, m=1* and ω_n*=10 [SI units]*. The values for EI corresponds to a blade in the shape of a rectangular steel beam with height *0.001[m]* and width *0.01[m]*. In Figure 3 the bifurcation diagrams are shown for the four cases with three, five, ten and fifty blades. The displacement in *y* is scaled with the clearance (*y/(R-L)*). The figures shows rich dynamics below the natural frequency with an increasing number of chaotic regions for increasing number of blades.

Since the frequency of the impacts will increase with the number of blades, an attempt is made to find similarities in the dynamics by scaling the excitation frequency. In Figure 4 the same cases are shown for the case when the frequency axis is scaled with *n/3*. This is done in order to find similar excitation frequency as in (a) for the other cases. The gray dotted line indicates the maximum displacement for each frequency. In Figure 5 the maximum contact force is plotted for each case.

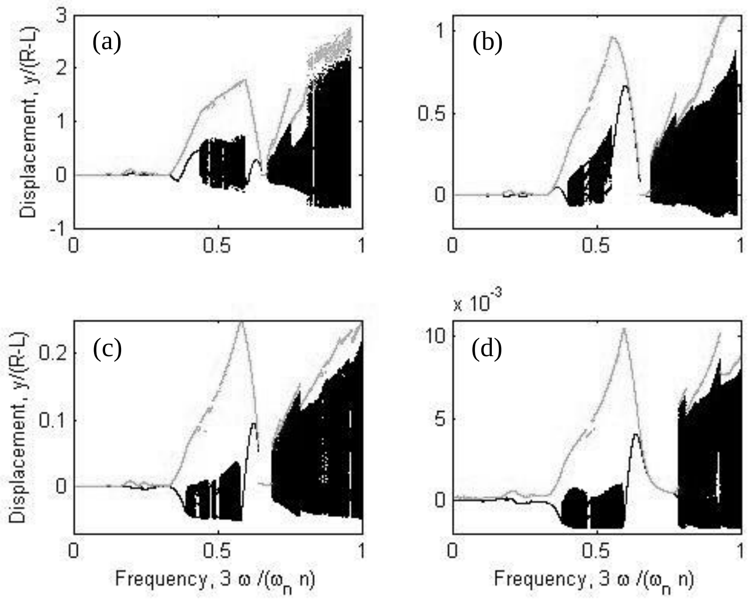

Fig. 4. Scaled bifurcation diagrams for w=0-1 for different number of blades. (a) three blades, (b) five blades, (c) ten blades and (d) fifty blade.

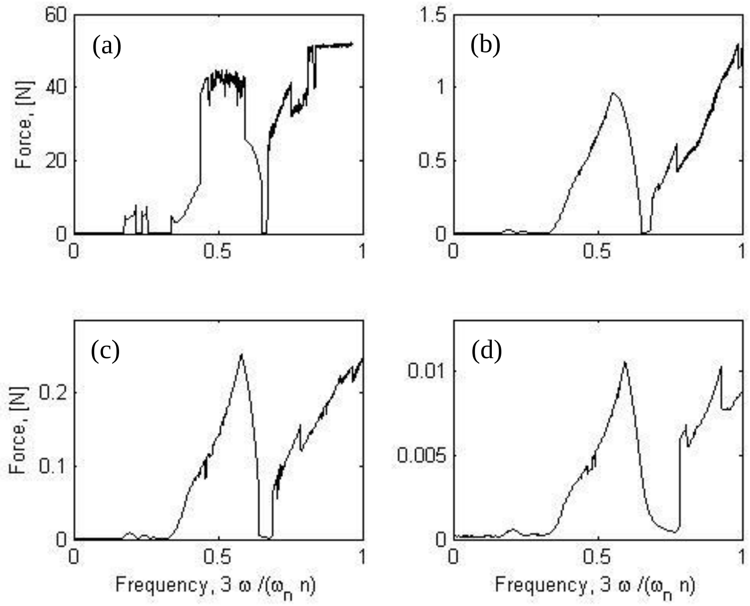

Fig. 5. Maximum contact force for different number of blades. (a) three blades, (b) five blades, (c) ten blades and (d) fifty blade

4. Discussion and Conclusion

Rubbing rotors have been studied extensively but mainly with models describing the rotor and the stator as cylinders. In several industrial applications the rotor consists of blades which conditions at contact significantly differs from the perfect circle. The target of this paper is to evaluate similarities and differences for rubbing bladed rotors with different number of blades. Four cases are compared namely three, five, ten and fifty blades. In the bifurcation diagrams in Figure 3 it is shown that the amplitude decreases when the number of blades is increased. It is also shown that the number of regions with long periodic or chaotic behaviour increases for more blades. For three blades the system becomes unstable for frequencies close to the natural frequency of the system. For the other number of blades no such instability was found.

Since the excitation frequency increases with the number of blades, the frequency axis is scaled with, *n/3* (number of blades/3). Then, it was shown that the dynamics for scaled frequency was similar for the systems with peak amplitudes and chaotic regions in the same range. For low frequencies the amplitude is low but for an excitation frequency larger then ω_n/n the amplitude starts to grow. At about $0.4\omega_n/n$ a chaotic region is found and the vibration reaches a peek at the end of the chaotic region at about $0.6\omega_n/n$. Then a short periodic interval of low amplitude appears followed by a new chaotic interval with increased amplitudes. The forces of Figure 5 shows peaks at the same position as the maximum vibration amplitudes. The actual value of maximum force or the maximum amplitude could not be scaled but it was shown that they will decrease with the number of blades.

At low frequencies ($<2.4\omega_n/n$), the simulations indicates that general dynamics such as areas of high amplitudes and forces can be predicted together with location of the two first regions of chaotic motion. But for higher frequencies the dynamic will differ due to the number of blades.

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