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Chaos and regularization in stellar wind

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Abstract. It is well known that the power stellar wind of OB and Wolf-Rayet (WR) stars consist of the numerous dense inhomogenuities (clumps) and more rarefied homogeneous interclump medium. Clumps are randomly distributed in the whole volume of stellar wind and have arbitrary sizes and fluxes. Formation of the clumps appears to be the chaotic process connected with internal instability of the radiation dominated stellar wind. Our modelling of the line profiles in spectra of OB and WR stars with clumped wind show that initially chaotic ensemble of the clumps can be regularized. As a result of that regularization the clusters of clumps which manifest themselves as strong bumps on the line profiles cam be developed.
Keywords: chaos, stellar wind, clumps, regularization.

1 Introduction

Twenty-first-century theoretical physics is coming out of the chaos revolution [2]. Astrophysics, as a part of physics is also a field for an application of the chaos theory [21].

Chaotic structures can be found in the Solar System [5], in the arrays of orbits of exoplanets [18] and in the atmospheres of the late-types AGB and post-AGB stars [6,7].

Atmospheres of early-type stars quite a long time were considered as homogeneous spherically symmetric flows [13]. In the beginning of 80th the first indications of the presence of high density regions (blobs or clumps) in the atmospheres (winds) of the early-type (hot) stars both from the theory and the observations were revealed [1,4].

This inhomogeneous wind were described in the *clump model* [1]. In this model a stellar wind is proposed to be composed of the numerous dense clumps and low density interclump medium. Total number of the clumps can exceeds 10^3. The ions of the low and moderate ionization stages are located dominantly in the clumps while the interclump medium seems to be strongly ionized.

A separate clump forms a small detail in the line profiles in the spectrum of early-type stars. All ensembles of the clumps in the winds of these stars form a

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stochastic line profile variability (LPV) in the stellar spectra. A stochastic character of the LPV allows us to conclude that clumps born and dissipate chaotically. A chaotic ensemble of clumps in the atmosphere may be described in the framework of the Stochastic clump model proposed by Kostenko & Kholtyhin [11] and Kudryashova & Kholtygin [12] and close to the model of discrete wind emission elements proposed by Lepine & Moffat [14].

Spectral observations of Wolf-Rayet (WR) and OB stars show that together with dissipative processes in the stellar atmospheres resulting in a chaotization of the stellar wind and a formation a stochastic wind structure, some kinds of a regularization processes leading to the formation of quasi-regular structures in the wind can be also effective. In the present paper we consider these evidences of the wind regularizations.

2 Chaos in stellar wind: stochastic clump ensemble

In our stochastic clump model [11,12] we suppose that there is no way to know where in the wind, when and how the next clump can appear. Ith means that one can only say about the probability for clump to born in the fixed wind volume and has a determined values of the mass, size, fluxes in the lines and other parameters. For each clump these values are defined through the distribution function $N_{\rm cl}(M_{\rm cl}, R_{\rm cl}, \theta, \varphi)$ of clumps on masses $M_{\rm cl}$, sizes $R_{\rm cl}$ and other parameters. Here $M_{\rm cl}$ is a mass of the clump, $R_{\rm cl}$ is its radius, θ is an angle between the direction of the clump motion and line of sight and φ is an azimuthal angle of the clump in the coordinate system where the origin of coordinates is located in the center of star and the Z-axis coincides withy the line of sight.

The total flux in the line formed by a clumped atmosphere in a frequency interval $[\nu, \nu + d\nu$ can be presented as

$$F(\nu)\nu = F^{\rm icl}(\nu)\nu + F^{\rm cl}(\nu)\nu + F^{\rm cl-icl}(\nu)\nu\,. \tag{1}$$

Here the value of $F^{\rm icl}(\nu)$ is the part of the line flux formed by the homogeneous interclump medium only, $F^{\rm cl}(\nu)$ is the clumps contribution and $F^{\rm cl-icl}(\nu)$ refers to the contribution of the clump - interclump medium interactions to the line profile.

Due to of the large velocity gradients in the stellar wind the contributions of the separate clumps into the total line flux can be considered independently and a part of the total line flux formed by clumps

$$F^{\rm cl}(\nu) = \sum_i F^i_\nu = \int_{(4\pi)} \int_{M_{\rm min}}^{M_{\rm max}} \int_{R_*}^{R_{\rm wind}} N_{\rm cl}(M_{\rm cl}, R_{\rm cl}, \theta, \varphi) F^i_{\rm cl}(\nu)\, d\Omega\, dM_{\rm cl}\, dR_{\rm cl}\,, \tag{2}$$

where $M_{\rm min}$ and $M_{\rm max}$ are the minimal and maximal masses of the clumps in the ensemble. A function $F^i_{\rm cl}(\nu)d$ describes a flux, which a clump with a number i emits in the frequency interval $[\nu, \nu + d\nu]$ in the solid angle $d\Omega = 2\pi sin\theta\, d\theta\, d\varphi$. R_* is a stellar radius and $R_{\rm wind}$ is the wind radius.

As it was shown by Kostenko & Kholtygin [11] the contribution of the interclump medium into the total intensity of most of the lines in the early-type star spectra (e.g. lines of ions CIII, HeI-II, etc.) is small. The interaction of clumps with the interclump medium gives contribution mainly in the X-ray region and weakly impacts on the profiles of optical and UV lines considered. It means that the intensity of these lines are mainly determined by chaotic clumps in the wind.

Studies of LPV for O and WR stars (Kaper et al. [15], Lépine & Moffat [14]) specify that clumps are mainly formed in a narrow area of an atmosphere near the stellar core. It means that distribution of clumps on distances from the stellar core, masses and directions can be considered independently:

$$N_{\rm cl}(M_{\rm cl}, R_{\rm cl}, \Omega) = N_m(M_{\rm cl})\, N_r(R_{\rm cl})\, N_{\rm cl}(\Omega). \tag{3}$$

Early-type stars are the powerful sources of X-Ray emission (e.g., Oskinova et al. [20]). For explanation both the UV optical an X-Ray spectra of these stars Kholtygin at al. [8] propose the *3-phase model* of the early-type star winds. In this model is supposed that wind can be in 3 phase states: homogeneous warm wind with a mean temperature $\approx 10^5\,K$, cold clumps with $T \approx 10^4\,K$ and hot clumps (hot zones with a temperature T up to 10^8 K. Warm wind and cold clumps emit in an optical and UV range, whereas a radiation of hot zones are mainly in a X-Ray region.

For WR stars clumps give the *main contribution* in the line emission, but for OB stars clumps give the smaller one. There exist a phase transitions between hot and cold phases. Cold clumps can be heated by shocks up to 10^8 K (Bychkov & Aleksandrova [3]), whereas hot zones cool very fast with cooling time is less than 1 min and the hot clumps became th cold clumps again soon after their heating. The agreement of the characteristic times of optical and X-Ray variability supports the *3-phase model* (see Oskinova et al. [19] and reference therein for details).

3 Modelling the clump ensemble

To model the chaotic distribution of clumps in the stellar winds we need to know the distributions $N_m(M_{\rm cl})$, $N_r(R_{\rm cl})$ and $N_{\rm cl}(\Omega)$.

We present a distribution of clumps on masses in atmospheres of early-type stars as $N_m(M_{\rm cl}) \sim (M_{\rm cl})^{-\gamma}$ and adopt the values of $\gamma \approx 2.0$ (see arguments presented by Kudryashova & Kholtygin [12]).

For modelling the distribution $N_r(R_{\rm cl})$ we suppose that clumps are born randomly near the stellar core, the total clump number in the atmosphere is constant and their distribution on radius R is determined by a relation $R^2 N_r(R)(R) V_{\rm cl}(R) = const$. For a dependence of the clump velocity $V_{\rm cl}(R)$ on the distance R from the center of star we adopt standard β-law:

$$V_{\rm cl}(R) = V_0^{\rm cl} + (V_\infty^{\rm cl} - V_0^{\rm cl}) \left(1 - \frac{R_*}{R}\right)^{\beta} . \tag{4}$$

Here $V_0^{\rm cl}$ is the formal clump velocity at a level $R = R_*$, $V_\infty^{\rm cl}$ is the terminal clump velocity at $R \gg R_*$, and a typical value of a parameter $\beta = 0.5-1$. We use mainly the spherical-symmetric distribution $N_\Omega(\Omega)$ of directions of clumps.

We assume that each clump forms a detail of the line profile (subpeak) with gauss distribution of intensity. Dependencies of a total flux in the different lines formed by separate clump at the distance R to star were calculated by Kostenko & Kholtygin [11]. As in a paper by Lépine ([16]) we suppose that the full fluxes of subpeaks $F_i \propto \sigma_i^2$, where σ_i^2 is a velocity dispersion inside a clump with number i.

Follow Kudryashova & Kholtygin [12] we suppose that mean clump *lifetime* is determined via a relation

$$T^{\rm cl} = T_{\rm cl}^{\rm max}\,(F_{\rm cl}^{\rm max}/F_{\rm cl})^\gamma\,, \tag{5}$$

where $T_{\rm cl}^{\rm max}$ is a *lifetime* of a clump wich have a maximal flux $F_{\rm cl}^{\rm max}$of the considered line and $\gamma \approx 1$ (see Lépine [16] for details. The *lifetime* of a clump is an interval between two moments of times. The first one is a moment when a clump is formed an emit in the line. The second is a moment when the clump can exists by does not emit in the considered line. It means that in a common case the *lifetime* of clump depends of the line whic we consider.

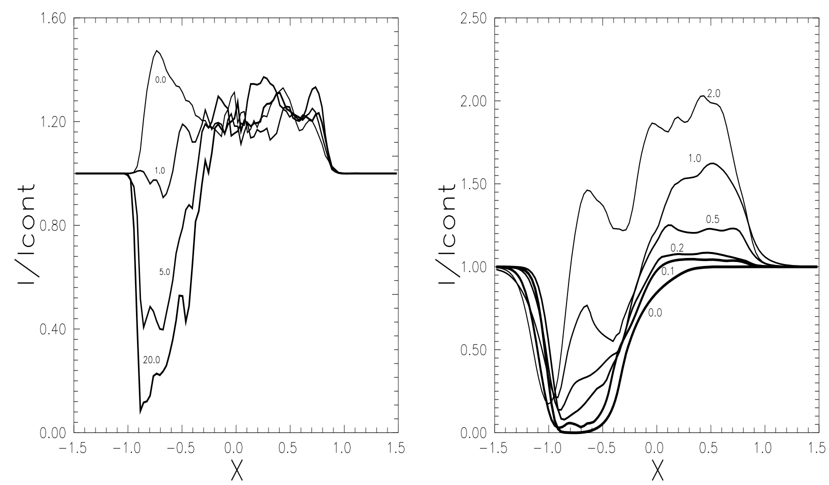

Fig. 1. Left panel: a typical mean line profiles in a dependence on $\tau_{\rm cl}^{\rm max}$ = 0.0, 1.0, 5.0 and 20.0 and for $\zeta = 0.5$. **Right panel**: the same as in the left panel, but for a value of the parametr $\zeta = 0.0$, $0.1\,0.2\,0.5, 1.0$ and 2.0 for $\tau_{\rm cl}^{\rm max}$=10.

The resonance lines of ions of the most elements in the atmospheres of the early-type stars have the strong absorption components. This absorption can appear when a large clump is on the line of sight and screen the emission of the stellar core. We use the next procedure to take into account the absorption of the stellar emission by clumps. Suppose that there are a number of clumps which on the line of sight can absorb the radiation of the stellar photosphere and the and a total optical depth for absorption of the continuum radiation can be presented as a sum of all optical depths of all such clumps.

To calculate an optical depth $\tau_i(\nu)$ of a clump with a number i in the central frequency of the line we use the scaled relations

$$\tau_i(0) = \tau_{\rm cl}^{\rm max}(F_i/F_{\rm max})^{\mu_\tau},$$

where $\tau_{\rm cl}^{\rm max}$ is an optical depth of a clump with a maximal line flux $F_{\rm max}$. From calculations by Kostenko & Kholtygin [11] of the ionization structure of the early-type stars we conclude that parameter $\mu_\tau \approx 2$.

4 Line profile calculations for the clumped wind

For the sake of the simplicity hereinafter will plot the calculated line profile in the dimensionless frequencies

$$x = (\nu - \nu_0)/\varDelta\nu_\infty\,, \tag{6}$$

where $\varDelta\nu_\infty = \nu_0(V_\infty/c)$ is the total line width, c is the light velocity, ν_0 is the central frequency of the line.

It should me mention that the relation (2) gives us the *instantaneous* line profile only, whereas the observed line profiles are the mean of all instantaneous profiles over the whole interval of the observations. For evaluating the *quasi-observed* line profile we average all *instantaneous* line profiles over the typical time of observations of one line profile $\varDelta T$. The typical values of $\varDelta T = 10 -$ 30 min.

Main parameters of the stochastic model are σ_{max}, a velocity dispersion in a clump with a maximal flux, ε, a ratio of a minimal and a maximal fluxes of line formed by an ensemble of clumps and τ_{cl}^{max}, the optical depth of a clump with the maximal flux. To normalize the line profile at the level of the continuum we introduce a parameter $\zeta = F_{\rm line}/F_{\rm cont}$, where $F_{\rm line}$ is the total flux emiited in the emission component of the line and $F_{\rm cont}$ is the flux in the continuum within the frequencies of the line.

For example we plot a dependence of mean model line profile versus $T_{\rm cl}^{\rm max}$ in Fig. 1 for a resonance doublet CIV $\lambda\,1548{,}1550$.

The LPV can be clearly seen in the case of using the difference line profiles (individual profiles minus mean line profile). For obtaining the difference model profile we calculate the averaged *quasi-observed* line profiles over the whole period of observation $T_{\rm obs}$. For an illustration we plot the typical difference line profiles in the stochastic clump model in Fig. 2. The dashed lines show the displacement of subpeaks on the line profiles from the center to the wings of the line.

This displacement reflects the acceleration of the clumps in the wind and can be seen in Fig. 3 where we plot the dynamical spectra for line CIV $\lambda\,1548{,}1550$ LPV for typical parameters of a clump ensemble at a total duration $T^{\rm full} = 10^h$ of *quasi-observations.*

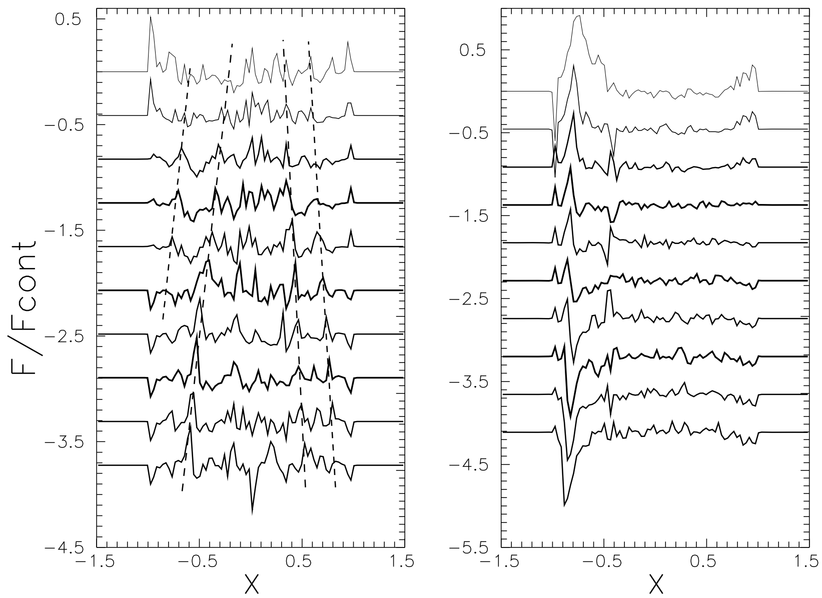

Fig. 2. Left panel: difference line profiles for the pure emission line ($\tau_{\max} = 0$) for the stochastic clump model with parameters $\varepsilon = 10^{-5}$ and $\sigma_{\max} = 0.20$.The time interval between the successive profiles is 30 min. **Right panel**: the same as in the left panel but for opaque clumps with $\tau_{\max} = 25$.

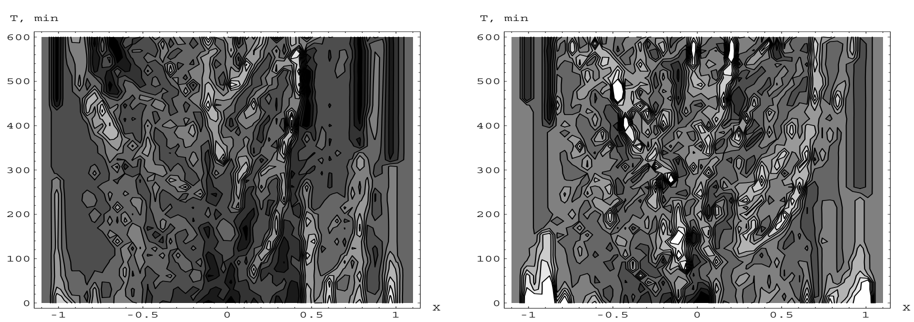

Fig. 3. Left panel: dynamical spectra in the Stochastic Clump Model for parameters $\sigma_{max} = 0.20$, $\varepsilon = 10^{-3}$ and a parameter $\tau_{\rm cl}^{\max} = 0$ and for 10^h of total time of "quasi-observations". **Right panel**: the sam as in the left panel, but for $\tau_{\rm cl}^{\max} = 0$.

5 Using wavelets for testing clumps

For OB stars the clump contribution, connected with small-scale structures in the stellar wind, in the total line profile variations is not so significant as for WR ones. For this stars the share of the regular variations of the line profiles, connected with the large-scale structures in the stellar wind, is significant. It means that we have to use the more effective methods for testing a clump contribution in the LPV.

The most convenient tool for a decomposition the clump contribution in the LPV is the wavelet analysis. The wavelet transform of the analyzed function $f(x)$ (in our case it is a difference line profile) is

$$W(s,u) = \frac{1}{s}\int\limits_{-\infty}^{\infty} f(x)\psi\left(\frac{x-u}{s}\right)dx \;=\; \int\limits_{-\infty}^{\infty} f(x)\,\psi_{su}(x)dx\,. \tag{7}$$

where $\psi(x)$ is the mother wavelet, s is a scale. In our case the most suitable is the so-called MHAT wavelet $\psi(x)$=$(1-x^2)\exp(-x^2/2)$, which has a narrow energy spectrum. The MHAT wavelet is proportional to the second derivative of a Gaussian and can be used to select the gauss-like features in the differential line profiles.

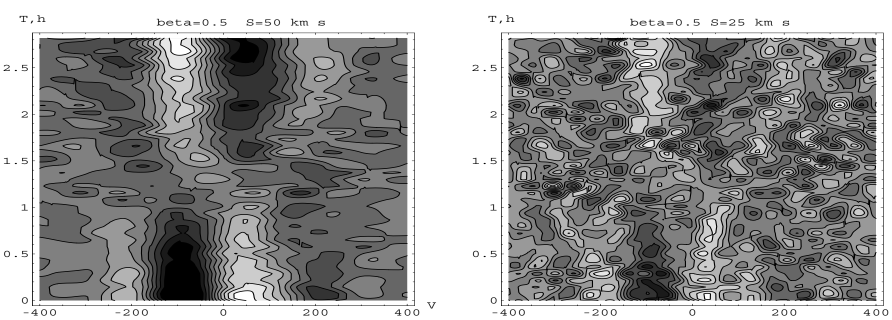

Fig. 4. Left panel: dynamical wavelet spectra for line HeII $\lambda\,4686$ in a spectra of star δ Ori A for the scale $S = 50\,$km/s. **Rightt panel**: the same as in the left panel, but for $S = 25\,$km/s

Kholtygin et al. ([9]) described a obtaining the *dynamical wavelet spectra* for lines in spectra of early-type stars. Those spectra are the wavelet transform of the difference spectra for the analyzed line in the velocity V space in a dependence of the time of observation t and for the fixed scale s. In this case, the scale variable s is expressed in km/s.

In Fig. 4 we plot the dynamical wavelet spectra for line HeII $\lambda\,4686$ in spectra of O star δ Ori A for the scales $S = 50$ and 25 km/s. Details of our observations are described by Kholtygin et al. [9]).

For small scales in an interval $S = 1-5\,$km/s the dynamical wavelet spectra is determined by the noise contribution mainly and do not plot in the Fig. 4. In the same time for large scale $S = 50\,$km/s mainly regular variations in the *dynamical wavelet spectra* can be detected, as it can be seen in Fig. 4 (left panel). For intermediate values of the scales S we can detect in the dynamical wavelet spectra both the stochastic variations connected with clumps and regular variations induced by the large scale structures. Both types of variations are seen in Fig. 4 (right panel).

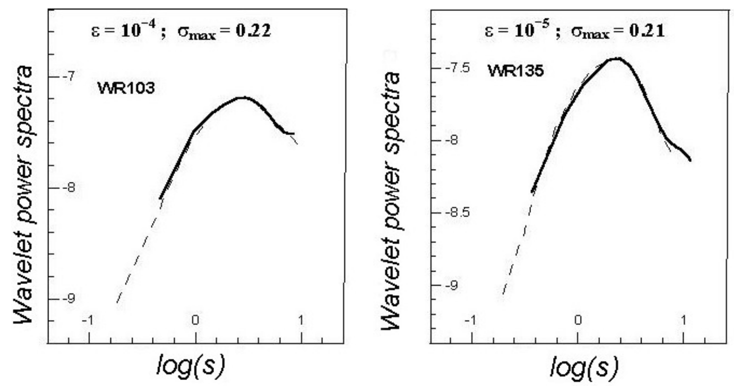

Fig. 5. Left panel: mean wavelet power spectra for lines CIII λ 5696 and HeII λ 5511 in a spectra of star WR 103 as a function of a scale S (solid line) in a comparison with wavelet power spectra for mean model profiles of these lines (dashed lines). **Right panel**: the same as in the left panel, but for WR 135.

6 Regularization of the chaotic clump ensemble

To study what is a real structure of the clumps in the wind of early-type stars we compare the wavelet power spectra of the difference line profile in the spectra of selected WR stars with calculated for model profiles in the stichastic clump model. The methodic how to calculate the wavelet power spectra is described by Kudryashova and Kholtygin [12] and by Kholtygin [10].

The wavelet power spectra for 8 WR stars were taken from a paper by Lépine et al. [17]. The quality of the fit of the model and obtained from the real line profiles wavelet power spectra is good as it can be see in Fig. 5.

Table 1. Parameters of the clump ensembles for selected WR stars

Star	Sp. Class	ζ	V_∞(km/s)	ε	$\sigma_{\max}$
WR 103	WC9	17.0	1190	10^{-4}	0.22
WR 111	WC5	1.7	2415	10^{-5}	0.25
WR 134	WN6	2.0	1905	10^{-3}	0.20
WR 135	WC8	10.0	1405	10^{-5}	0.21
WR 136	WN6	2.3	1605	10^{-5}	0.20
WR 137	WC7+OB	2.5	2550	10^{-5}	0.22
WR 138	WN5+OB	0.4	1345	10^{-5}	0.23
WR 140	WC7+O4-5	1.25	2900	10^{-4}	0.035

The parameters of the stochastic clump model which provide the best fit are presented in the Table 1. It should be mention that the values of $\sigma_{\max}$ for all WR stars excluding the binary system WR 140 are very close. It means that the parameters of the clump ensembles and their structure are also close.

The velocity dispersion $\sigma_{\max}$ in the clumps are rather large as it can be sin from the Table 1. For typical for WR stars terminal velocities $v_\infty = 1500 - 2000$ km/s the value of $\sigma_{\max}$=200-600 km/s for clumps with the maximal fluxes in the considered line. The sizes of such clumps can be as large as $4R_*$ as it follows from the simple estimations.

It may be concluded that the detai;ls of the line profiles with very large velocity dispersion can not be formed by a separate clump but by cluster of the smaller clumps with close values of the radial velocities and probably with close locations in the wind. It means, in turn, that the initially chaotic ensemble of the clumps can be regularized and the regular structure of clumps can appear.

7 Conclusion

From an analysis the structure of winds of the early-type stars we can conclude:

1. The line profiles in spectra of early-type stars and their variations can be descibed in the *stochastic clump model*.

2. The initially stochastic clump ensemble does not remain totally chaotic. The large cluster of clumps which forms the large details of the line profiles are formed in the wind.

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