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Exploring Sullivan's Health Status Index of Mortality and Morbidity

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Sullivan proposed the most used index of mortality and morbidity, a relatively simple and easy to apply method as an extension of the classical life table. In his study he had also estimated the disability free life expectancy for USA the year 1965. Although the estimated healthy life years lost from disability by Sullivan are only a small proportion of the recent estimates we further check the related estimates done in 1971 because in these days the disability estimates were focused mainly on the severe disability causes resulting in no active years of life. We have tested Sullivan's results for USA (1965) with one of our hitting time models and found significant differences for the healthy age at birth but very good approach for the healthy age at 65 years of age. Related study was published for New Zealand (2006) and the results are in a very good agreement with ours. In the latter case we have estimated the active life expectancy for 34 countries from the human mortality database. The advantage of our method is that we use only population and death data without the use of results from health questionnaires.

Introduction

Daniel F. Sullivan introduced a remarkable method for estimating the healthy life expectancy based on the classical Life Table. The seminal paper on "A Single Index of Mortality and Morbidity" was published in 1971 in Health Services and Mental Health Administration (HSMHA) Health Reports. Reading his paper you understand the deep insight on the subject from the author leading to his remarkable work. Furthermore, few years ago (1966), he had done an extensive analysis on the definition and estimation of the population health status. Sanders (1964) explored the measurement of community health levels. Chiang (1965) work directed towards mathematical models for the health status and George W. Torrance (1976) proposed a unified mathematical approach to the health status index models including a detailed bibliographic search of the existing models.
Between all models presented the Sullivan model was already a quite promising model from the applied point of view. Sullivan in his 1971 work already had estimated the loss of healthy life years for USA by using his method. This was a

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very important point to promote his health status index. It was an index providing results. Not a theoretical one.

The missing point when launching questionnaires is that the expected replies will based on people's experience on what we call health; good health, bad health or death. Replies express our experience on health based on the data already known by individuals. These data include deaths and population and the inevitable decline of the known population by means people of the same or similar age to be in good or bad condition or dead. That we ask people is already included in our databases for the existing population by year of age and the related number of deaths at every year of age.

However, a question arises regarding the health status or the health state of individuals. As we have only population and death data where is the information for the health state hidden?

The reply is as simple as the replies received from people when we launch questionnaires. People have similar data from a closed of course environment. The health state estimated comes mainly as a comparison with the main event; death and less as a measure of disabilities. Of course people have a way to understand the way disabilities lead to a lower and lower health status and finally death. But why does not exist a mathematical model or method to produce similar results as the replies of people to questionnaires? As people make simple estimates we can search for not a very complicated model. A simple but yet quite important was proposed by Torrance (1976). The basis was a simple procedure expressing by 1 the alive and by 0 death persons. He considered the state of functioning of an organism by adding intermediate cases with levels of disability. This was a model mainly directed towards the direction of collecting disability data via a health system or questionnaires.

However, the most important point of his study was the definition of the Health Status over time t of a population denoted by $H(t)$. He suggests that this Health Status will come as a summation of the related health status $h(t)$ of every individual into the population with a related weight when appropriate.

As the health status of an individual is a stochastic process the next point should be the derivation of the health status of a population as the probability density function of the health status stochastic process from the barrier which could be set as zero for simplicity. However, Torrance (1976) probably was not aware for the developments in this direction already done in a very theoretical level from 1971 to 1976 (for an analysis see [2, 4-15]). Even today only few researchers enter into the complicated world of the first exit or hitting time processes. Instead Torrance and mainly Sullivan preferred the leading system used by actuaries and demographers the last four centuries that is the Classical Life Tables. They thus established a quite strong and relatively easy to apply system which turned out to be the preferred method to estimate the health status or health status index in very many countries and for several time periods with a cost of an enormous statistical survey system all over the world.

Two main points where missing:

1. The proposal of a hitting time model expressing the health status of a population something done by Janssen and Skiadas (1995) and the publications by Skiadas and Skiadas from 2007 until today (5-15).
2. The finding of a method to estimate the health status of a population from death and population data only, thus providing a useful tool for comparisons between countries and regions avoiding the bias resulting from the way people with different cultures and way of living respond to the health questionnaires.

The task of this paper is to insert a Sullivan like method to the already proposed Health State Life Table thus providing an extra tool for estimating the healthy life expectancy and the loss of healthy life years without using data from questionnaires. The simplest first exit time model used was proposed by Skiadas and Skiadas (2010) [8, 9]. The model is expressed by the following formula for the hitting time probability density function $g(t)$ at time t

$$g(t) = k(l + (c-1)(bt)^c)(t)^{-3/2} e^{-\frac{(l-(bt)^c)^2}{2t}}$$

where b, l, c, k are parameters. The Health State Function is

$$H(t)=l-(bt)^c$$

This is a declining function for the *health state of a population* as we have called the analogous *health status of population* in terms of Sullivan. The slight different terminology was accepted to distinguish the estimates done by using models and modeling from the estimates based on questionnaires. However, both terms will tend to express the same as both methods tend to provide similar results. The first attempt is presented in this paper. We use the model for the Health State of a population to estimate the fraction of people with disabilities per year of age, thus enabling the estimation of the Active Live Expectancy (ALE) via a Health State Life Table. This is the classical life table in which we have added a Sullivan like part for the healthy life expectancy estimation. Both abridged and complete Health State Life Tables were constructed. Due to space limits only the abridged table is included (see Table I).

The proportion of the loss of healthy life years is estimated with the method illustrated in Figures 1 and 2. Figure 1 illustrates the case of USA females the year 1965. The blue curve is the estimated from the hitting time procedure health state function $H(t)$ of the population. The line AB expresses the optimum case (no deaths) with no disabilities and other health declining phenomena whereas the curve ADC expresses the real situation.

That is immediately clear is that the area ABCDA represents the total loss of health of the population whereas the area ADCOA expresses the Healthy region. As a consequence the fraction ABCDA/ABCOA provides the net loss of health of the population from zero to 110 years of age. This is the important information enabling us to estimate the Active Life Expectancy (ALE) and the

Loss of Active Life Years (LALY) according to the terminology used by the Ministry of Health and Statistics New Zealand (2008) in a published discussion paper [3]. For our application we use the classical Life Table expanded for the estimation of the Active Life Expectancy.

TABLE I

Abridged Health State Life Table (0 - 85+) for New Zealand (males 2006)											
The Abridged Life Table Including Life Expectancy and Active Life Expectancy Estimation											
Life Expectancy Estimation							Active Life Expectancy Estimation				
Age	Mortality	100.000 population Sx and Number of Deaths dx		Survivorship	Tx=S(Lx)	Life Expectancy	Fraction of Loss of Active Life Years	Survivorship without disability	Txd=S(Lxd)	Active Life Expectancy	Loss of Active Life Years
x	qx	Sx	dx	Lx	Tx	ex	px_active	Lxd_total	Txd_total	ALE	LALY
0	0,0059	100000	589	99.470	7.803.206	78,0	0,000	99.470	7.488.886	74,9	3,1
1	0,0011	99411	112	397.419	7.703.736	77,5	0,000	397.419	7.389.416	74,3	3,2
5	0,0008	99299	79	496.295	7.306.317	73,6	0,000	496.294	6.991.997	70,4	3,2
10	0,0008	99219	80	495.895	6.810.022	68,6	0,000	495.885	6.495.703	65,5	3,2
15	0,0044	99139	436	494.604	6.314.127	63,7	0,000	494.555	5.999.818	60,5	3,2
20	0,0045	98703	440	492.413	5.819.523	59,0	0,000	492.246	5.505.263	55,8	3,2
25	0,0047	98262	465	490.150	5.327.110	54,2	0,001	489.702	5.013.017	51,0	3,2
30	0,0051	97798	501	487.736	4.836.960	49,5	0,002	486.714	4.523.315	46,3	3,2
35	0,0059	97297	571	485.057	4.349.224	44,7	0,004	482.978	4.036.601	41,5	3,2
40	0,0079	96726	765	481.716	3.864.167	39,9	0,008	477.851	3.553.623	36,7	3,2
45	0,0125	95961	1.198	476.810	3.382.451	35,2	0,014	470.119	3.075.773	32,1	3,2
50	0,0186	94763	1.758	469.421	2.905.641	30,7	0,023	458.520	2.605.654	27,5	3,2
55	0,0300	93005	2.792	458.046	2.436.220	26,2	0,037	441.222	2.147.133	23,1	3,1
60	0,0492	90213	4.435	439.979	1.978.174	21,9	0,056	415.364	1.705.911	18,9	3,0
65	0,0772	85778	6.620	412.341	1.538.195	17,9	0,083	378.311	1.290.548	15,0	2,9
70	0,1266	79158	10.020	370.740	1.125.854	14,2	0,118	326.830	912.237	11,5	2,7
75	0,2034	69138	14.064	310.531	755.114	10,9	0,166	258.992	585.407	8,5	2,5
80	0,3246	55074	17.874	230.685	444.583	8,1	0,228	178.145	326.415	5,9	2,1
85	1	37200	37.200	213.899	213.899	5,8	0,307	148.270	148.270	4,0	1,8
			100.000				*Developed by Christos H Skiadas 19 April 2014*				

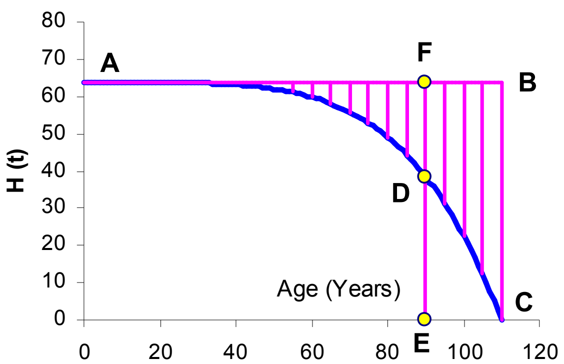

Figure 1. Health State of USA Population (females, 1965)

The estimates at an age level $x=t$ provide the formula for the fraction of the loss of active life years p_x

$$p_x=(DF)/(EF)$$

It is clear that $p_x=0$ at zero age and 1 at the end of the life time. The resulting fraction of the loss of active life years during age is presented in Figure 2.

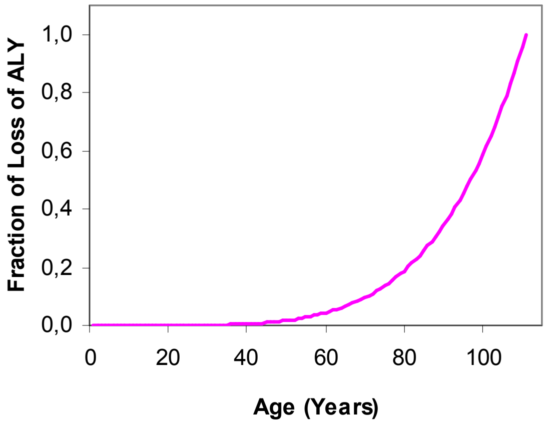

Figure 2. The fraction of the loss of active life years

TABLE II						
Comparisons of our model with Sullivan's results						
	Life Expectancy, Healthy Life Expectancy and Loss of Healthy Life Years at Birth (USA 1965)					
	Sullivan	Model	Sullivan	Model	Sullivan	Model
Both	70.2	70.1	64.9	66.5	5.3	3.6
Males	66.8	66.7	61.6	62.7	5.2	4.0
Females	73.7	73.8	68.4	70.6	5.3	3.2
	Life Expectancy, Healthy Life Expectancy and Loss of Healthy Life Years at Age 65 (USA 1965)					
	Sullivan	Model	Sullivan	Model	Sullivan	Model
Both	14.6	14.6	11.3	11.4	3.3	3.2
Males	12.9	12.8	9.4	9.5	3.5	3.3
Females	16.2	16.3	13.1	13.3	3.1	3.0

Our method is tested with the results presented in Sullivan's paper for USA 1965. Table II summarizes the results and Table III includes the parameter estimates for the model used. For the healthy life expectancy at birth our method underestimates Sullivan's results, whereas the findings for both methods are almost identical for the healthy life expectancy at age 65 for males, females and both.

TABLE III						
Four Parameter Hitting Time Model						
	Model Parameters				Statistics	
USA 1965	b	l	k	c	SSE	R^2
Both	0.0250	13.41	0.367	4.00	0.000654	0.944
Males	0.0293	13.96	0.366	3.45	0.000820	0.926
Females	0.0200	13.85	0.367	4.65	0.000494	0.965

TABLE IV						
Comparisons of our model with New Zealand's results						
	Life Expectancy, Healthy Life Expectancy and Loss of Healthy Life Years at Birth for ALE (Active Life Expectancy)					
	Life Expectancy at Birth (LE)		Active Life Expectancy (ALE)		Years without Active Life	
Males 2006	New Zealand	Model	New Zealand	Model	New Zealand	Model
0	78.1	78.0	74.9	74.9	3.2	3.1
15	63.7	63.7	60.7	60.5	3.0	3.2
25	54.2	54.2	51.3	51.0	2.9	3.2
45	35.3	35.2	32.5	32.1	2.8	3.1
65	18.0	17.9	15.5	15.0	2.5	2.9
Females 2006	New Zealand	Model	New Zealand	Model	New Zealand	Model
0	82.1	82.0	78.4	78.9	3.7	3.1
15	67.6	67.5	64.0	64.4	3.6	3.1
25	57.8	57.7	54.4	54.6	3.4	3.1
45	38.5	38.3	35.2	35.2	3.3	3.1
65	20.6	20.4	17.5	17.5	3.1	2.9

We had another opportunity to test our model results with relatively more recent findings for New Zealand for Active Life Expectancy for males and females in 2006. The comparative study is presented in Table IV. The findings are in perfect agreement with the related study for New Zealand [3].

Discussion

The estimation for the active life expectancy with our method provides very good results because the method is based on the dramatic loss of health during the few last years of the life span easily provided by the model. Similar results we have explored in several countries for 2006 (see Table V). Other transformations will make possible the estimation of other disability life periods including of healthy life years from moderate or light disability causes.

TABLE V - Active Life Expectancy Estimation

Country	Males 2006			Females 2006		
	LE	ALE	LALY	LE	ALE	LALY
Australia	79.1	75.7	3.4	84.1	80.4	3.7
Austria	77.0	74.0	3.0	83.1	79.8	3.3
Belarus	63.3	59.4	3.9	75.3	72.4	2.9
Belgium	76.5	73.6	2.9	82.5	79.3	3.3
Bulgaria	69.1	65.4	3.7	76.1	73.6	2.5
Canada	78.2	74.7	3.5	83.2	79.3	3.9
Czech Republic	73.3	70.0	3.3	79.9	77.0	2.9
Denmark	75.8	72.2	3.6	80.8	76.7	4.1
Estonia	67.3	63.0	4.3	78.5	75.7	2.8
Finland	75.7	72.7	3.0	83.1	80.0	3.1
France	77.3	74.1	3.2	84.8	81.0	3.8
Germany	76.9	73.6	3.3	82.7	79.2	3.5
Hungary	69.0	64.2	4.8	77.5	74.5	3.0
Ireland	77.2	73.5	3.7	82.1	78.0	4.1
Israel	78.4	74.1	4.3	82.3	77.8	4.5
Italy	78.5	74.9	3.6	84.4	80.2	4.2
Japan	78.9	75.1	3.8	86.4	82.0	4.4
Latvia	65.6	60.8	4.8	76.3	73.8	2.5
Lithuania	65.2	60.5	4.7	76.9	74.5	2.4
Luxemburg	76.9	73.8	3.1	82.6	79.8	2.8
Netherlands	77.6	74.2	3.4	82.1	79.0	3.1
New Zealand	78.0	74.7	3.3	82.5	79.1	3.4
Norway	78.1	75.2	2.9	83.0	80.0	3.0
Poland	70.7	66.6	4.1	79.4	76.3	3.1
Portugal	75.3	72.5	2.8	82.3	78.4	3.9
Russia	60.2	56.4	3.8	73.1	70.6	2.5
Slovakia	70.3	66.4	3.9	78.3	75.3	3.0
Slovenia	74.2	70.8	3.4	81.7	78.2	3.5
Spain	77.4	74.1	3.3	84.4	80.4	4.0
Sweden	78.7	75.7	3.0	83.2	80.0	3.2
Switzerland	79.1	75.7	3.4	84.6	80.9	3.7
UK	77.1	73.5	3.6	81.7	77.7	4.0
Ukraine	62.2	58.3	3.9	73.6	71.1	2.5
USA	75.3	71.9	3.4	80.7	76.9	3.8

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