Chaos In a Modified Cardiorespiratory Model

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Abstract: A new modified cardiorespiratory model based on the famous DeBoer beat-to-beat model and Zaslavsky map (which describes dynamics of the respiratory system as a generator of central type) was studied in details. In this case the respiratory tract was firstly modeled by the self-oscillating system under the impulsive influence of heartbeat. The steady-state regimes of the modified model are investigated by methods of the dynamical system theory. The regular (periodic and quasi-periodic) and chaotic regimes typical for functioning of the cardiosystem are found and studied.
Keywords: A beat-to-beat model, Cardiorespiratory system, DeBoer model, Zaslavsky map, Nonlinear dynamics, Chaotic regimes.

1. Introduction

The human cardiovascular system closly interacts with different organs and systems of organism. Realized self-oscillations in a cardiovascular system are under an activity of practically entire organism (see [2-5, 9-11]). Physiological rhythms are not isolated processes. There are numerous interactions of rhythms between itself and with an internal and external environment. Cardiac and respiratory rhythms form up during embryo development, and even the brief break of these rhythms after a birth results in death.

Existence of breathing and heart rhythm synchronization effect, found experimentally in the cardiovascular system both for healthy people and with pathologies, is well-proven in work Toledo [10] in 2002. It is well known, the dynamic process of mutual synchronization can be realized only in a case of presence of a subsystem mechanical interaction. Therefore, the indicated effect display testifies the presence of both direct and feedback interactions between the cardiovascular and respiratory systems.

A heart system and organism of man in general have one of major descriptions of activity, such as a blood pressure dynamics. His time-history, along with electrocardiogram (ECG), is an important information generator for research and diagnostics of laws and pathologies of the cardiovascular system. The task of mathematical model construction, describing the dynamics of arterial blood pressure, is far from completion. Complications of such design are related to the necessity of taking into account of influence on the cardiac rhythms not only the cardiovascular system but also other organs and systems of organism, in particular a respiratory system.

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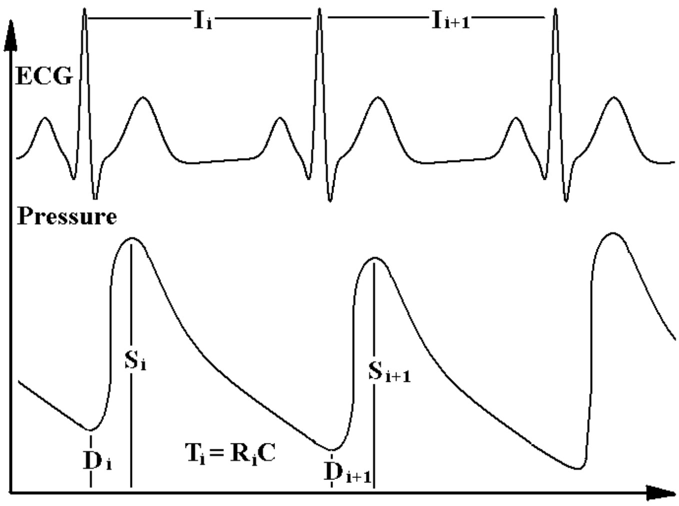


Fig. 1. Characteristics of the heartbeat in DeBoer model.

2. The mathematical model of a direct and reverse interactions

The DeBoer model of a cardiovascular system is under direct action of a respiratory systems (what corresponds to experimental data) [3]. This model was substantially developed in future. The sinus node responsiveness (and other detailed factors) is taking into account in the work of Seidel and Herzel [9] (the so-called SH-model). In this model chaotic dynamics was found in dynamics of a cardiosystem.

The models of both DeBoer and SH only considered direct respiratory influence on heartbeats. The SH-model got further development [5], where an effect of heartbeat and the resultant changes in the baroreceptor afferent activitiy to the SH-model are added and the cardiorespiratory sinchronization found due to this modification. Interaction of blood pressure and amplitudes of breathing oscillations revealed in accordance with principles of optimum control in the DeBoer model is investigated in the Grinchenko-Rudnitsky model [2]. This model allowed, in particular, to explain appearance of a peak on the Meyer frequency in the spectrums of pressure oscillations and synchronization of cardiac and respirator rhythms.

However, this model does not consider the reverse mechanical influence effect of the heartbeat changes on a breathing phase (frequency). In the present study, we add to the DeBoer model a self-oscillating system (which describes dynamics of the respiratory system as a generator of central type [4]) which is under impulsive influence of heartbeat.

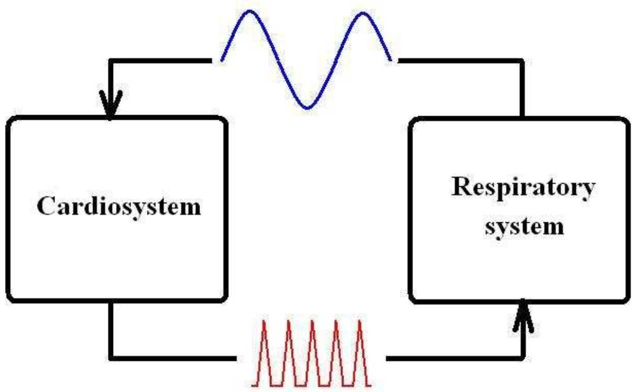


Fig. 2. Interaction of the cardiovascular and respiratory system

The DeBoer model describes the followings main characteristics of the heartbeat (see Figure 1) system: systolic pressure S , diastolic pressure D, R-R interval I and arterial time constant T (in a state of rest for a healthy man S=120 mmHg, D=80 mmHg, I=800 ms, T=1500 ms). This mathematical model is a system of five discrete nonlinear maps. This model contains only a direct mechanical influence of the respirator system on the cardiosystem and can be written in the form:

$$D_i' = S_{i-1}' \exp\left(-\frac{2}{3}\frac{I_{i-1}'}{T_{i-1}'}\right),$$

$$S_i' = D_i' + \gamma \frac{T_0}{S_0} I_{i-1}' + \frac{A}{S_0}\sin\left(2\pi f T_0 t_i\right) + \frac{c_2}{S_0},$$

$$I_i' = G_v \frac{S_0}{T_0}\hat{S}_{i-\tau_v}' + G_\beta \frac{S_0}{T_0} F(\hat{S}', \tau_\beta) + \frac{c_3}{T_0},$$

$$T_i' = 1 + G_\alpha \frac{S_0}{T_0} - G_\alpha \frac{S_0}{T_0} F(\hat{S}', \tau_\alpha),$$

$$\hat{S}_i' = 1 + \frac{18}{S_0}\arctan\frac{S_0(S_i'-1)}{18},$$

where $i \ge 1$, $D' = D / S_0$, $S' = S / S_0$, $\hat{S}' = \hat{S} / S_0$, $I' = I / T_0$, $T' = T / T_0$, $F(\hat{S}, \tau) = 1/9(\hat{S}_{i-\tau-2} + 2\hat{S}_{i-\tau-1} + 3\hat{S}_{i-\tau} + 2\hat{S}_{i-\tau+1} + \hat{S}_{i-\tau+2})$, $t_i = \sum_{k=0}^{i-1} I_k'$ is a real time, A=3 mmHg is a breathing amplitude, f=0.25 Hz is a breathing frequency, $c_2 = S_0 - D_0 - \gamma I_0$, $c_3 = I_0 - S_0(G_v + G_\beta)$, $\gamma = 0.016$ mmHg, $G_\alpha = 18$

ms/mmHg, $G_\beta = 9$ ms/mmHg, $G_v = 9$ ms/mmHg, $\tau_\alpha = \tau_\beta = 4$, $\tau_v = 0$, is equal to 0 if frequency of heartbeat is less then 75 beat/min, and τ_v is equal to 1, if frequency is more then 75 beat/min.

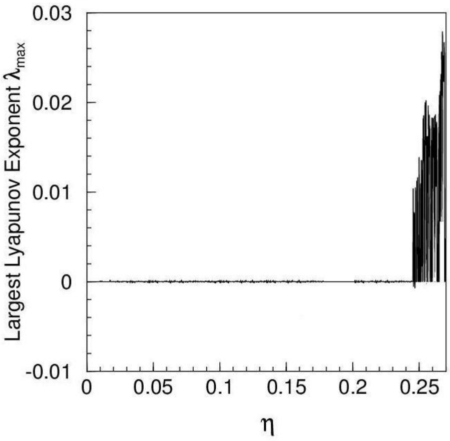
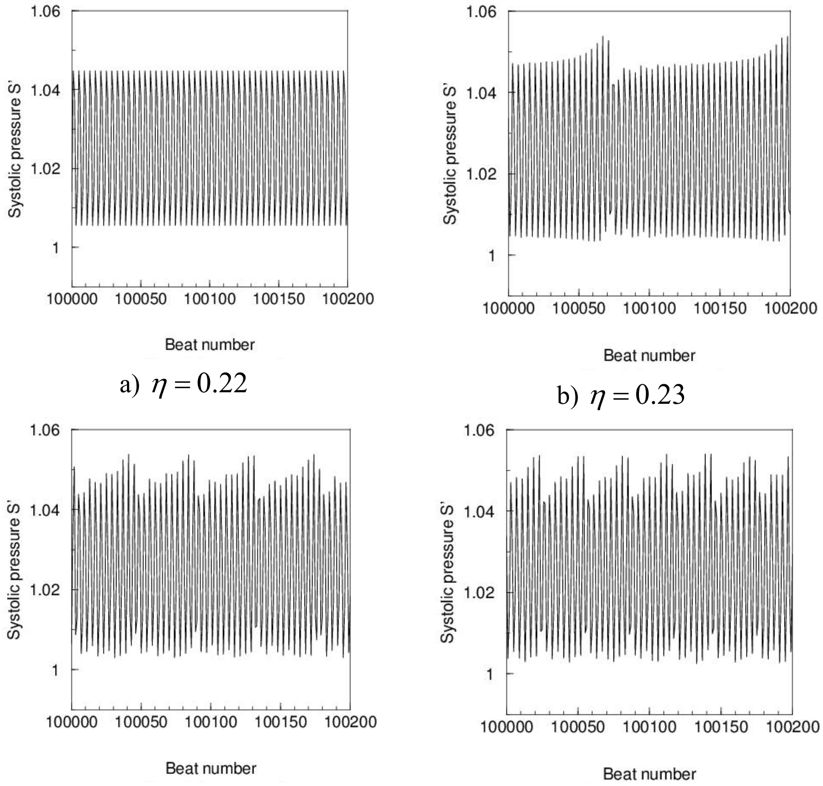


Fig. 3.Largest Lyapunov exponent of the modified system

We suppose that a healthy man at rest breathes periodically with a permanent frequency and an amplitude of motions of thorax. In that case a breathing process can be described as the self-oscillating system [4], which has a steady limit cicle. Thus for the mathematical modeling of a such system equations of the Zaslavskiy map could be used. Famous Zaslavsky map is the system of equations [8, 12] which describes the dynamics of an amplitude r_n and a phase φ_n of the system (in which periodic self-oscillations with a frequency ω are realized) which is under T-periodic impulsive action of constant intensity η . Te system has the following form:

$$r_{n+1} = \left(r_n + \eta \sin \varphi_n\right) \exp\{-\kappa T\},$$
$$\varphi_{n+1} = \varphi_n + \omega T + \nu\left(r_n + \eta \sin \varphi_n\right) \frac{1 - \exp\{-\kappa T\}}{\kappa},$$

where κ, ν are constant parameters.



a) $\eta = 0.22$ b) $\eta = 0.23$

c) $\eta = 0.24$ d) $\eta = 0.25$

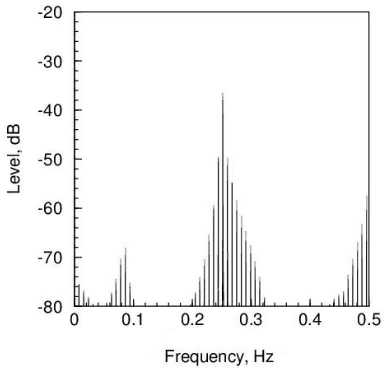
Fig. 4. Simulated systolic pressure data (cases a, b, c and d)

In our approach these equations are used to describe changes of an amplitude and phase of a respiratory system effect for every R-R interval with an intensity proportional to systolic pressure: $-\eta(S_n - S_0)$

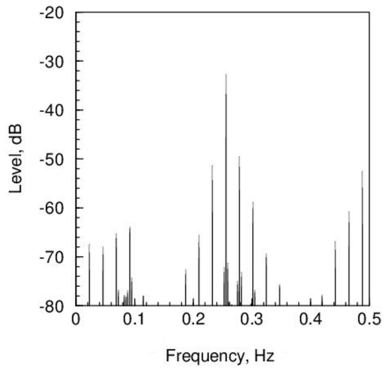
$$r_{n+1} = \left(r_n - \eta(S_n - S_0)\sin\varphi_n\right)\exp\{-\kappa I_n\},$$
$$\varphi_{n+1} = \varphi_n + 2\pi f I_n + \nu\left(r_n - \eta(S_n - S_0)\sin\varphi_n\right)\frac{1-\exp\{-\kappa I_n\}}{\kappa},$$

where I is R-R interval, $\eta > 0,\ \kappa,\ \nu$ are constant parameters of interaction.

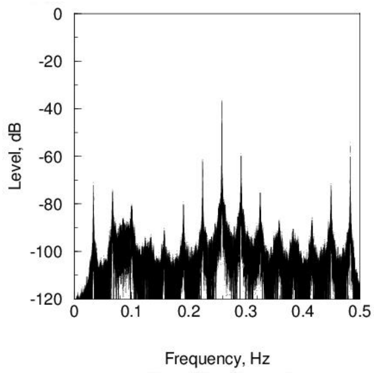
Thus, we study the dynamics of the modified model of cardiorespiratory system, which consists of the DeBoer model with direct respiratory influence $(A + r_i)\sin\varphi_i$, and with reverse influence modeled by the Zaslavskiy map system (see Figure 2).



a) $\eta = 0.23$



b) $\eta = 0.24$



c) $\eta = 0.25$

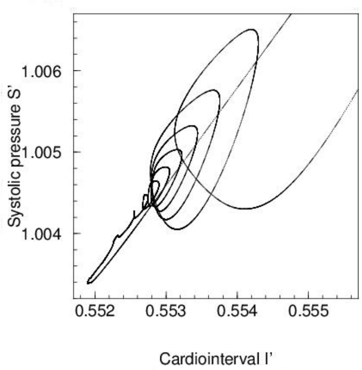
Fig. 5. Power spectra computed from systolic pressure data (cases a, b and c)

3. Numerical simulations results

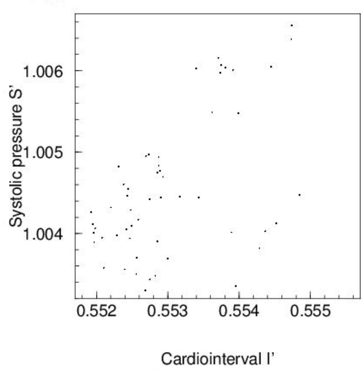
In accordance with physiology of healthy man, the followings values of variables and constants are used in our numerical simulations: $I'[0] = 0.53$, $S'[-j] = 1.08$, $j = 0,...,6$, $r'[0] = 0$, $\varphi'[0] = 0$, $\kappa = 0.001$ 1/ms, $\nu = 0.001$ 1/msmmHg. In order to study steady-state regimes first of all the largest Lyapunov exponent [1, 6, 7] was found. The dependence of the largest Lyapunov exponent of the modified system on values of the bifurcation parameter η is shown in Figure 3. The dynamics of the system changes with increasing of this parameter. There is the region where Lyapunov exponent positive ($\eta > 0.245$) that means transition to chaos occurs. We emphasize that η describes intensity of heart influence on a respiratory system. The next Figure 4 illustrates a behaviour of systolic pressure data in the modified model. Power spectra computed from these data are shown in Figure 5. The spectrum in Figure 5.a and in Figure 5.b have discrete peaks which are situated equidistantly with a frequency difference. So that, graphs indicate that there are regular regimes in the modified system.

Finally, for the steady-state regimes, when the largest Lyapunov exponent is positive and the chaotic regime is realized, the power spectrum is continuous (Figure 5.c).
Phase portrait projections on the plane of the simulated systolic pressure and R-R interval data are presented in Figure 6. The phase portrait in the Figure 6.a represents a singular solid curve and corresponds to quasiperiodic regime. There are only several points in the phase portrait in Figure 6.b which means that at $\eta = 0.24$ the modified system has regular periodic regime. And in Figure 6.c when $\eta = 0.25$ the phase portrait has numerous lines (the number of which increases in time) and corresponds to chaotic steady-state regime. So we have found such steady-state basic regimes as:

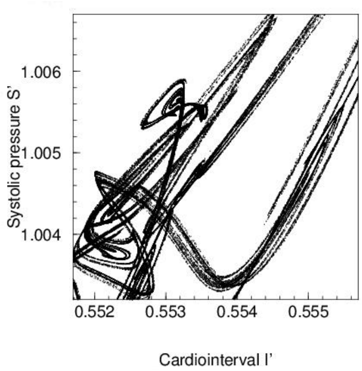
1. at $\eta = 0.22$, periodic regime (Figure 4.a);
2. at $\eta = 0.23$, quasiperiodic regime (Figure 4.b, Figure 5.a, Figure 6.a);
3. at $\eta = 0.24$, periodic regime (Figure 4.c, Figure 5.b, Figure 6.b);
4. at $\eta = 0.25$, chaotic regime (Figure 4.d, Figure 5.c, Figure 6.c).



a) $\eta = 0.23$



b) $\eta = 0.24$



c) $\eta = 0.25$

Fig. 6. The parts of phase portraits simulated systolic pressure and R-R interval data (cases a, b and c)

4. Conclusions

On the basis of the DeBoer model an interaction of the heartbeat and the respiratory system as dissipative Zaslavskiy map is studied and the modified model of cardiosystem is built out. This model takes into account both direct and reverse influence of subsystems – cardiovascular and respiratory.

The methods of modern theory of the dynamical systems are used to study laws of the steady-state regimes of the modified model. Firstly the chaotic regimes were found out. Analysis of bifurcational curves of the largest Lyapunov exponent, projections of phase portraits, temporal realizations and power spectrums allowed to investigate the basic laws of dynamics of the model. The dynamics of heartbeat and respiratory systems are in good correspondence with experimental information of healthy man. Found irregularities of phase trajectories of the modified model depend on intensity of heart rhythm influence on breathing, what is well known characteristic for the dynamics of the cardiovascular system of healthy man.

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