Chaotic Modeling and Simulation (CMSIM) 4: 477-488, 2016

First observation of Quasi-Chaos in Erbium doped fiber ring laser

S. Zafar Ali

Department of Electrical Engineering, Air University, E-9 Islamabad, Pakistan
email ***zafarali@mail.au.edu.pk***

Abstract

This work reports the first numerical evidence of existence of pseudo or quasi-chaos in a loss modulated EDFRL true Additive Chaos Modulation (ACM) scheme, thus possibly adding fifth region of operation in non-autonomous nonlinear systems. Quasi-chaos apparently looks like chaos but actually converges to same time and physical phase space trajectory, even with widely separated initial conditions, behaviour exactly opposite to the basic essence of chaos i.e. sensitive dependence on initial conditions (SDIC). Subject quasi-chaos was earlier believed to be pure chaos since the output passed qualitative visual tests of chaos like aperiodicity in time domain, rich spectral content in frequency domain, direct observation test in phase space, and fast decreasing autocorrelation function. Even quasi-chaos gives positive Lyapunov Exponent (LE), using TISEAN, in time delayed pseudo phase space built by time delaying lasing E field. Thus a complete knowledge of numerical model and driving conditions is a must to validate existence of a pure or quasi-chaos. EDFRL Chaos Message Masking(CMS) configuration is also shown here producing a pure chaos, for comparison, with desired sensitivity to initial conditions, besides passing all above mentioned visual tests of chaos and a positive LE spectrum. Emergence of quasi-chaos will have far reaching implications in chaos applications.

I Introduction

Chaos is the third most important discovery of 21^{st} century being actively researched in multiple disciplines in theoretical and applied contexts and new aspects of chaos are still forthcoming [1]. An improved knowledge of chaos will help better understanding of various important phenomenon including heartbeat and neuron signals which are inherently chaotic. Optical chaos produced by different types of lasers is a well-researched field which met

Received revised: 14 August 2016 / Accepted: 21 October 2016
 ISSN 2241-0503

successful experimental demonstration in Athens [2]. EDFRL shows rich dynamical behaviour and have proven to be a useful platform to study nonlinear dynamics and chaos [3-10] in addition to other practical applications. There are three main schemes for generation of chaos in EDFRL i.e. loop nonlinearities [3], cavity loss modulation [4-9] and pump modulation [10]. The detailed study of chaos generation dynamics of EDFRL with five key parameters i.e. cavity loss, cavity gain, modulation index, pump power and modulating frequency variation was done earlier [4,6], showing how EDFRL switches, with the change of above mentioned control parameters, between four possible regions of operation i.e. periodic, quasi-periodic, stable and chaotic. It was next shown [7] that using square and other complex loss modulating signals the LE of EDFRL chaos increases thereby raising the degree of unpredictability and security. Later [8] it was found that pulsed chaos gives better LE than non-pulsed chaotic oscillations in EDFRL. It was pointed out later [9] that the original EDFRL model with chaos message masking (CMA/CMS) as proposed by Luo [4] and later studied in detail [6] has message signal also being added into the loop which makes it true ACM scheme instead of reported CMA / CMS. The detailed study of effect of message parameters i.e. message frequency, amplitude and phase on EDFRL chaos dynamics therefore became necessary which was carried out in next work [9]. It was shown there [9] that chaos is produced only once the modulating and message frequencies are not integral multiple of each other which shows that the two frequencies interplay with each other to give shown results.

The detailed study carried out in this work was triggered by an unusual observation during simulations that different initial conditions did not produce different chaos as expected but all chaos seemed to be forced to single trajectory. This is a violation of SDIC which is main defining attribute of chaos and therefore this output needs to be termed as something other than chaos, say quasi-chaos, because apparently it behaves like chaos unless numerically subjected to different initial conditions. Negating SDIC in turn negates long term unpredictability. Previously, it was believed and reported [4-9] to be pure chaos because it mimicked all behaviour of chaos and qualifies qualitative tests as well as LE test in time delayed pseudo phase space using TISEAN routines [11]. Once message signal is removed from the loop, EDFRL starts producing pure chaos for same set of parameters and modulating signal.

The paper is organised as follows. Section I is introduction and literature review, followed by Section II which gives mathematical models and optical circuits of both configurations studied in this paper. Section III on simulations shows convergence of quasi-chaos of ACM to same trajectory irrespective of IC and divergence of trajectories in CMS configuration even for small IC deviations. Section IV discusses these simulation results and Section V concludes the results and indicates their implications.

II Mathematical model

In this section are given the optical circuits and corresponding mathematical models of loss modulated EDFRL true ACM and true CMA/CMS schemes respectively in Fig.1 and Eq. (1) and (2), 'true' emphasizing their corrected versions [9]. It is obvious from figures and equations that ACM has message sine wave being added into the loop modifying chaos dynamics, as studied in detail [6,9] while CMS is devoid of message and its effects on chaos dynamics.

$$\dot{E}_{LA} = -k_a(E_{LA} - c_a S_{in}) + g_a E_{LA} D_A + \xi_{LA} \tag{1a}$$

$$\dot{D}_A = -\frac{1}{\tau}[(1 + I_{PA} + E_{LA}^2)D_A - I_{PA} + 1] \tag{1b}$$

$$k_a = k_{a0}(1 + m_a \sin(\omega_a t)) \tag{1c}$$

$$S_{in} = S_0(1 - m_s \sin(\omega_s t)) \tag{1d}$$

$$\dot{E}_{LA} = -k_a E_{LA} + g_a E_{LA} D_A + \xi_{LA} \tag{2a}$$

$$\dot{D}_A = -\frac{1}{\tau}[(1 + I_{PA} + E_{LA}^2)D_A - I_{PA} + 1] \tag{2b}$$

$$k_a = k_{a0}(1 + m_a \sin(\omega_a t)) \tag{2c}$$

where "." denotes time derivative, E_{LA} is the lasing field strength, D_A is population inversion density, τ is the of Erbium meta-stable state decay time, ξ_{LA} is the spontaneous emission factor, I_{PA} is the pump power, k_{a0} is the cavity loss (decay rate), g_a is the cavity gain,, m_a is the modulation index, ω_a is the angular loss modulating frequency, S_0 is the message amplitude and ω_s is the message frequency. The various sets of parameters for which chaos is produced is extensively discussed earlier [6].

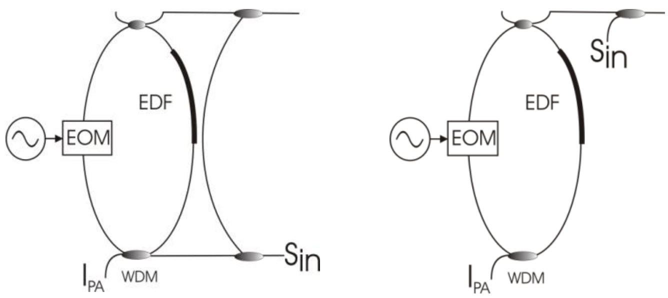


Fig.1 Cavity loss modulation in EDFRL
(a) True ACM producing quasi-chaos

(b) True CMA/CMS producing pure chaos

The values of model parameters used in simulations in following section are same as early [6] and given in Table.1 below. The only difference is there is no message S_0 in CMS model.

Table 1 EDFRL parameters-default values and range of variation

Parameter	**Symbol**	**Value**
Pump power	I_{PA}	10 mw
Modulation index	m_a	0.03
Decay rate	k_{a0}	3.3x10^7
Gain	g_a	6.6 x10^7
Message amplitude	S_0	1
Modulating frequency	ω_a	3.5x10^5
Message frequency	ω_s	3.14x10^5
Message Coupling strength	c_a	0.01

III Simulations results

Fig.2 shows the convergence to same trajectory of differently starting chaotic trajectories in time and phase plots for same set of EDFRL parameters as given in Table.1 as well as driving conditions in loss modulated ACM model. It may be noted that one set of initial conditions i.e. E_{LA0}=0 and D_{A0}=0.47 is taken as a reference to compare the convergence time of other initial conditions to the reference trajectory. It can be observed in time domain plots of Fig. 2(a) where the initial conditions are E_{LA0}=12 and D_{A0}=0.496 that it converges to the defined reference trajectory in approx. 0.75 msec. The arrows in time domain mark significantly different amplitudes in the chaotic pulses initially. However, it can be observed that amplitude difference is very small in next pulses after which the two chaotic trajectories converge to same value exactly overlapping in time and phase plots. In Fig. 2(b) once the initial conditions are E_{LA0}=5 and D_{A0}=0.496, resulting chaos converge to the reference trajectory in about 1.75 msec. It is to be noted that the distance of initial conditions from reference initial conditions is more in Fig. 2(a) than in Fig. 2(b), yet convergence time is smaller in first figure than the second. Thus the convergence time is not proportional to the distance of initial conditions from the reference initial condition. The arrows in phase plots indicate that the initial conditions for the two waveforms are taken far away from each other yet the red phase plot converges to the green one after few turns in phase plot.

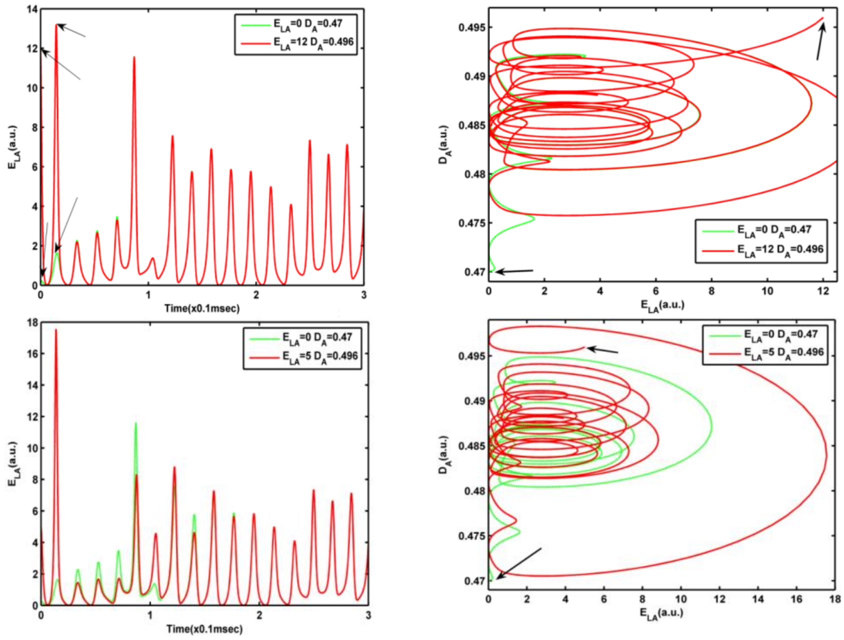


Fig.2 ACM Quasi-chaos convergence for different initial conditions.

(a) E_{LA}=0, D_A=0.47 (green) and E_{LA}=12, D_A=0.496 (red)

(b) E_{LA}=0, D_A=0.47 (green) and E_{LA}=5 , D_A=0.496 (red)

Once convergence of quasi-chaos to same trajectory is established above simulations of the ACM model, the CMS model is simulated next, using Table.1 parameter values again, with different initial conditions to dig for either converging or diverging behaviour with different initial conditions but same parameters eliminating message sine wave. It is found that even slightly varying initial conditions as labelled in Fig.3 time domain plots make the pure chaos diverge in longer run. Also the time of start of divergence of trajectories as marked by arrow in Fig.3 increases with the decrease in difference in initial conditions. The latter behaviour is quite as expected for a pure chaos which is SDIC as well as long-term unpredictable.

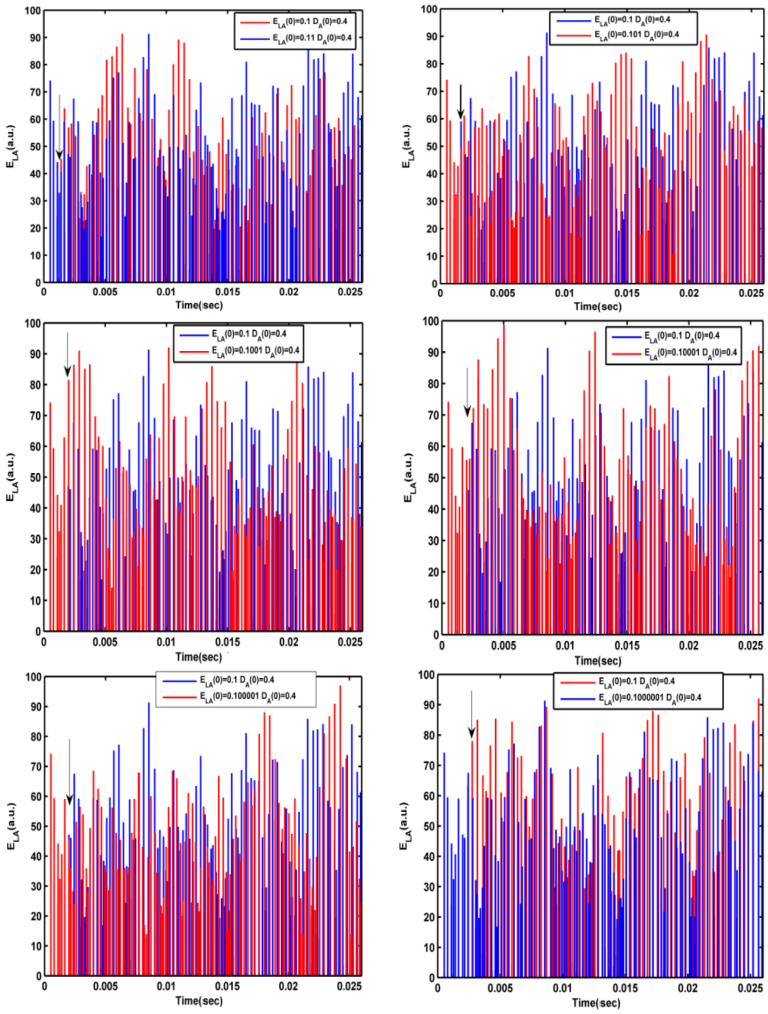


Fig.3 CMS Pure chaos showing SDIC for different initial conditions.

(a) E_{LA0}=0.1, D_{A0}=0.4 (red) and E_{LA0}=0.11, D_{A0}=0.4 (blue)
(b) E_{LA0}=0.1, D_{A0}=0.4 (blue) and E_{LA0}=0.101, D_{A0}=0.4 (red)
(c) E_{LA0}=0.1, D_{A0}=0.4 (blue) and E_{LA0}=0.1001, D_{A0}=0.4 (red)
(d) E_{LA0}=0.1, D_{A0}=0.4 (blue) and E_{LA0}=0.10001, D_{A0}=0.4 (red)
(e) E_{LA0}=0.1, D_{A0}=0.4 (blue) and E_{LA0}=0.100001, D_{A0}=0.4 (red)
(f) E_{LA0}=0.1, D_{A0}=0.4 (red) and E_{LA0}=0.1000001, D_{A0}=0.4 (blue)

The various plots (time, phase space, frequency, and autocorrelation) are shown in Fig.4 to compare quasi and pure chaos, in left and right coloums, as produced by ACM and CMS configurations respectively with system parameters kept same except elimination of message in latter. The quasi-chaos time plots in Fig.4(a) (left) has pulse time period equal to time period of modulating sine wave while pure chaos plot (on right) is chaotic in time interval as well as pulses amplitude. The pulses in quasi-chaos are bunched while pure chaos has no bunching, while the dynamic range of pure chaos appears better than its counterpart. Fig.4(b) shows the phase space of both chaos and it can be seen that both phase space plots are strange attractors indicating chaos as per direct observation method [6]. The apparent crossing of phase space lines will not be there if time is added as third dimension because the third assumed differential eqaution will be simply t'=1. It can be seen in Fig4.(c) that the phase space of quasi-chaos is less fractal as compared to that of pure chaos. Also quasi-chaos is denser at lower pulse amplitudes while pure chaos is denser at higher pulse amplitudes making phase space plots denser on inner and outer sides respectively. Fig.4(c) shows frequency spectrum of quasi and pure chaos with latter being richer and more random in spectral lines as compared to former. Also the modulating frequency and its harmonics are visible only in the case of quasi-chaos frequecny spectrum because here the pulse repetition time is being decided by modulating signal itself as reported earlier [6] also.The autocorrelation diagrams of both chaos are compared in Fig.4(d) and it is found that pure chaos has a sharper decay of autocorrelation than its counterpart while the autocorrelation function has nonzero lower values in quasi-chaos due to humps beneath pulses in time domain. But above all the most important observation is that quasi-chaos is in fact difficult to detect from the diagrams as shown in Fig.4 till it is discovered by actually testing initial conditions as in this work. Now once it is discovered the clues of quasi-chaos can be outlined, the most important being the fixed time period instead of chaotic time period and visibility of modulating frequency and harmonics in frequency domain.

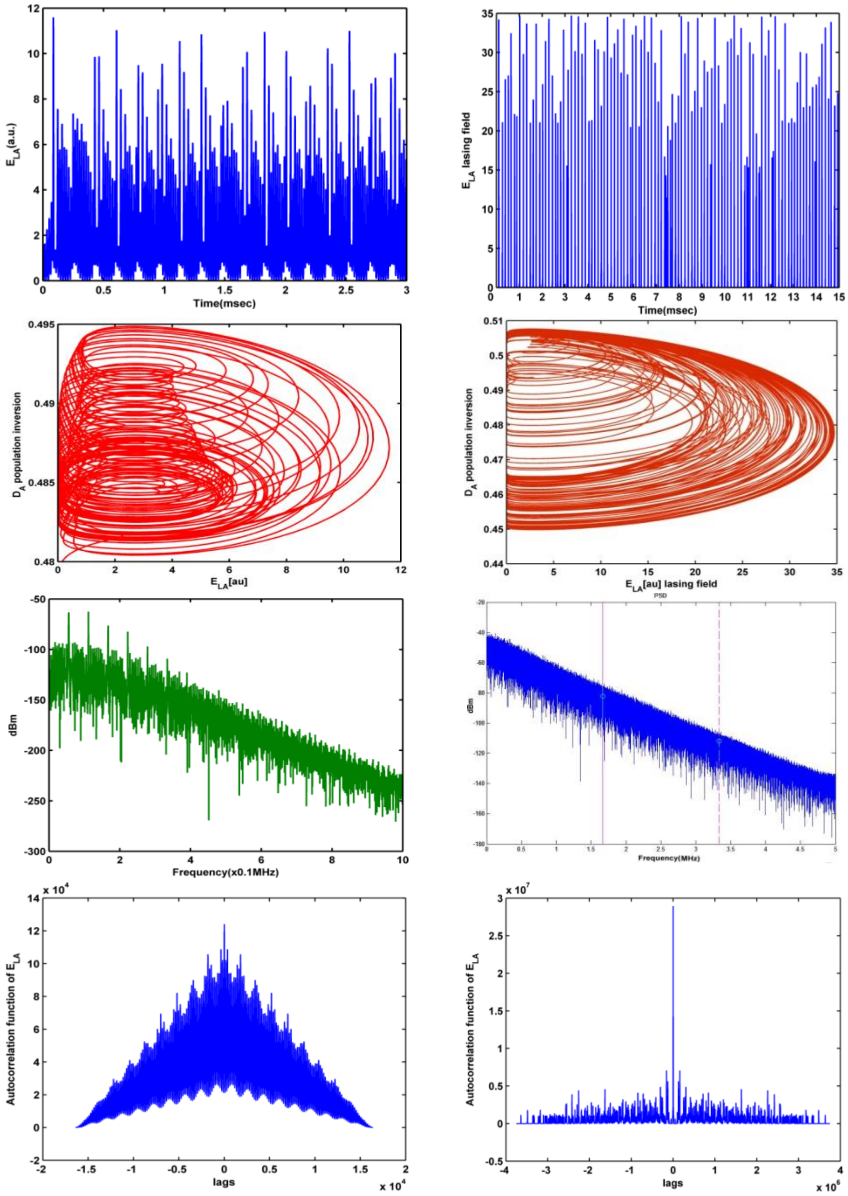
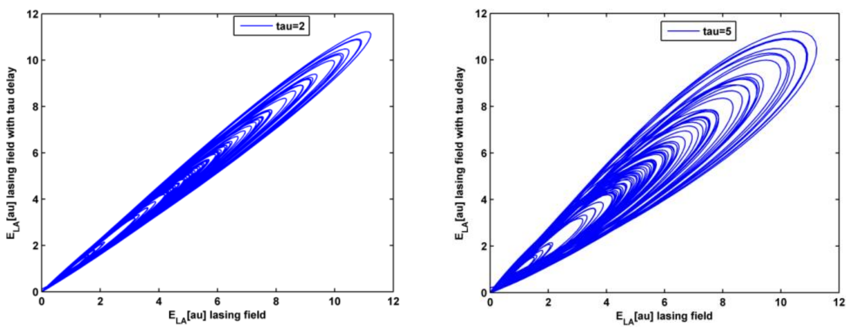


Fig.4 Comparison of quasi-chaos(left) with pure chaos(right) for loss modulation EDFRL

(a) Time domain, (b) Phase space, (c)Frequency domain, (d)Autocorrelation

The LE spectrums of quasi chaos generated from ACM and reported earlier for sine[6] and square [7] modulating signals were based on time series analysis of pseudo phase space generated by time delaying E_{LA} lasing. The reason why quasi-chaos also showed positive LE there, is that SDIC is not violated in pseudo time delayed phase space of lasing field. This fact is shown here in Fig. 5 for two such values of time delay (in samples) i.e. tau=2 and tau=5 samples, that these attractors are fractal in nature. Fractal nature of phase space means a strange attractor which shall give positive LE result with TISEAN. It implies that LE calculation using TISEAN is not a sufficient test to differentiate pure chaos from quasi-chaos and subjecting the system to different initial conditions numerically or experimentally is a must.



(a) **(b)**

Fig.5 Time delayed (tau) pseudo phase plots of quasi-chaos
(a) tau=2 (b) tau=5

IV. Discussion

The simulations were carried out to establish the presence of quasi-chaos in ACM and pure chaos in CMS. First it is shown that ACM loss modulated EDFRL produces an apparent chaos which converges to single trajectory making it a quasi- instead of a pure chaos. It is also observed for quasi-chaos that convergence time is not strictly dependent on deviation of initial conditions. As soon as the message is removed to reconfigure it as CMS, keeping all other parameters same including loss modulating sine wave, the convergence is immediately replaced by divergence of pure chaos trajectories. This divergence of trajectories is observed even for very small deviations in initial conditions as is expected for a pure chaos. It is noted that the time for the start of this divergence decreases with the increase in deviation of initial conditions, also as expected. Hence it is indicated that message signal is adding an extra periodic perturbation in the cavity which is interacting with the multiplicative perturbation of loss modulating sine wave, thus producing quasi instead of pure chaos. Next the LE spectrums of quasi and pure chaos are calculated using time series analysis with well-known TISEAN routines and plotted side by side. Most surprisingly, quasi-chaos gives positive LE since TISEAN uses time delayed pseudo phase space of lasing E field. The time delayed phase space is found fractal for two time delays. Thus LE calculation using time series analysis by TISEAN is not a sufficient test to identify pure chaos as shown in this study, because it misjudges quasi-chaos in EDFRL also as pure chaos. The most valid test of chaos is physically subjecting the system or its numerical model to different initial conditions and look for either convergence or divergence of trajectories for quasi and pure chaos respectively. This observation of pseudo or quasi chaos adds fifth region of operation in nonlinear systems; the other well-known four regions being periodic, quasi-periodic, stable and chaotic already reported and studied in detail for loss modulated EDFRL [6].

V. Conclusions

This work was motivated by an unusual observation of converging behaviour in temporal and physical phase space of apparently chaotic trajectories, for different initial conditions, in ACM loss modulated EDFRL. Such behaviour is totally new and cannot be categorised as any of four known modes of operation in nonlinear systems i.e. periodic, quasi-periodic, stable and chaos. Therefore, it is termed as quasi-chaos, as it looks like chaos, but violates the basic definition of chaos i.e. sensitive dependence on initial conditions. The findings here confirm the presence of convergence to same quasi-chaotic trajectories even for very widely separated initial conditions. Previously, this output from this EDFRL configuration was considered to be pure chaos since the output passed all qualitative visual tests of chaos; like aperiodicity in time domain, rich spectral content in frequency domain, direct observation test in phase space, and fast decreasing autocorrelation function. In this work all above plots are placed side by side for making comparisons between pure and quasi-chaos. However, EDFRL quasi-chaos surprisingly gives positive lyapunov exponents with TISEAN; as TISEAN routines are based on time delayed pseudo phase space of observed time series data, which is found fractal in this work, for the lasing E field. At the same time, it in no case implies that loss modulation scheme in EDFRL is not capable of producing a pure chaos. For comparison purposes, EDFRL is also shown to be able to produce a pure chaos, just by eliminating the message from the loop (CMS scheme), exhibiting desired sensitivity to minute changes in initial conditions. Pure chaos, as expected, passes all above mentioned visual tests of chaos and also giving positive LE using TISEAN. The only main test of quasi-chaos is thus numerically subjecting the system model to different initial conditions.

This is an important discovery which has five main implications on chaos theory and its engineering applications. Firstly, a complete knowledge of numerical model and driving conditions is a must to validate existence of either pure or quasi-chaos. Secondly, finding quasi-chaos would not be possible in an experimental work on EDFRL, because of inaccessibility of population inversion initial condition. Secondly, fractal nature of time delayed pseudo phase space is responsible for positive lyapunov exponent calculations, using TISEAN, misinterpreting quasi-chaos as pure chaos in earlier works. Fourth, chaos synchronisation of quasi-chaotic systems is also artificial i.e. ACM EDFRL receiver is in fact not synchronised to corresponding transmitter because of any seed being fed, since both outputs readily get converged to same trajectory, independent of their initial conditions, in reality, due to their quasi-chaotic nature. Fifth, pure chaos will always prove as chaos in qualitative and quantitative tests, but quasi-chaos will spoof itself as pure chaos until complete model is available for trying different initial conditions; output time series shall not be the only thing available. Other possible implications of this discovery are presently under study and will be reported shortly.

VI. References

1. M. Virte, K. Panajotov, H. Thienpont and M. Sciamanna, "Deterministic polarization chaos from a laser diode" Nat. Photonics, **7**(1) ,60-65 (2013).

2. A. Argyris, D. Syvridis, L. Larger, V. A. Lodi, P. Colet, I. Fischer, J. G. Ojalvo, C. R. Mirasso, L. Pesquera and K. A. Shore, "Chaos-based communications at high bit rates using commercial fibre-optic links", Nature, **438** (17),343-346(2005).

3. H. D. I. Abarbanel, M. B. Kennel, M. Buhl and C. T. Lewis, "Chaotic Dynamics in Erbium-doped fiber ring lasers", Phy. Rev. A, **6**(3), 2360-2374(1999).

4. L. G. Luo and P. L. Chu , "Optical secure communications with chaotic Erbium doped fiber lasers", J. Opt. Soc. Am. B, **15**(10), 2524-2530 (1998).

5. Y. Imai, H. Murakawa and T. Imoto, "Chaos synchronisation characteristics in Erbium-doped fiber laser systems", Opt. Commun., **217**(1-6), 415-420 (2003).

6. S. Z. Ali, M. K. Islam and M. Zafrullah "Effect of Parametric variation on generation and enhancement of chaos in erbium-doped fiber-ring lasers" Opt. Engg., **49**(10), 105002-1-12(2010).

7. S. Z. Ali, M. K. Islam and M. Zafrullah "Generation of higher degree Chaos by controlling harmonics of the modulating signal in EDFRL", Optik **122**(21) , 1903-1909 (2011).

8. S. Z. Ali, M. K. Islam and M. Zafrullah "Comparative Analysis of Chaotic Properties of Optical Chaos Generators", Optik **123**(11), 950-955 (2012).

9. S. Z. Ali, M. K. Islam and M. Zafrullah, "Effect of message parameters in additive chaos modulation in Erbium Doped Fiber Ring Laser ", Optik **124**(18), 3746-3750 (2013).

10. A. N. Pisarchik, A. V. Kir'yanov and Y. O. Barmenkov "Dynamics of an Erbium-doped fiber laser with pump modulation : theory and experiment" J. Opt. Soc. Am., **22**(10), 2107- 2114 (2005).

11. R. Hegger, H. Kantz, and T. Schreiber, "Practical implementation of nonlinear time series methods: The TISEAN package", Chaos **9**(2), 413-435(1999).