Looking for chaos in copper historical prices

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Abstract. Knowing the inherent characteristics of a price time series is the preface of forecasting future prices. This study tries to use chaos theory to investigate the behavior of copper prices. Keeping the mathematical definition of chaos as a standard frame, the known chaos tests were applied based on a discipline. The Lyapunov exponent and the BDS test (Brock, Deckert, Scheinkman, 1986) approved the evidence of nonlinear behavior and possible evidence of chaos for the gathered data. However, the close returns test, as a direct test of chaos, rejected the presence of chaos in copper return series. Finally, a GARCH (generalized autoregressive conditional heteroskedasticity) type model was fitted on the series under study to capture this nonlinearity. The BDS test results indicated that there was no longer any nonlinearity in the filtered data. It was concluded that the stochastic methods should be applied to predict future copper prices. **Keywords:** Chaos, the BDS test, Lyapunov exponent, the close returns test, nonlinear dynamics.

1 Introduction

The chaos tests look for a deterministic behavior through seemingly turbulent structures. A number of researchers have applied these tests for different financial time series to examine whether their series are chaotic. Many of them, such as Frank and Stengos [1], Hsieh [2], Panas and Ninni [3], and Scarlet et al [4], have established the presence of chaos in the economic time series. Many others, such as J.T. Barkoulas et al [5], Cecen and Erkal [6], Frank et al [7], Gilmore [8], Adrangi et al [9], Yousefpoor et al [10], though have found evidence of non-linear structure in their time series under study, they believe it is a non-linear stochastic behavior not a deterministic one. Knowing the inherent characteristics of a time series is important because it leads the researcher at the next step to select the best model of forecasting. As mentioned by Aihara [11],

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if a data set be chaotic, a chaotic neural network can be applied to model its dynamics.

The common chaos tests that are introduced in chaos literature have been applied to study these characteristics. But, these tests need an excessive number of data that are not usually available for financial time series. This is even worse for yearly time series. Ramsey [12] has shown that the correlation dimension will be misleading if the number of examined data be small. Panas [13] used only metric chaos tests and showed that the behavior of copper returns (from January 1989 to December 2000) was mostly driven by unpredictable stochastic variables. He mentioned that there was no priority between different chaos tests and recommended that a careful investigation should be done into validity of these tests. Yousefpoor et al [10] offered a systematic approach to select the chaos tests. They kept the mathematical definition of chaos (in sense of Devaney) as a direction to choose the tests.

In this paper we tried to implement the chaos tests on daily copper prices in a systematic order on the basis of mathematical definition of chaos. Alike researches such as J.T. Barkoulas et al. [5], we applied the topological approach, by using the close return test, which attempts to detect more fundamental properties of a chaotic system and, on the other hand, it is not misleading for relatively small data sets. To do so, a number of copper historical prices were gathered from London Metal Exchange market from January 1997 to October 2007 (number of observations=2735) to be examined.

This paper is organized as follows. Section 2 presents chaos theory basis. Section 3 presents the empirical results of each test. Section 4 draws conclusions.

2 Chaos theory basis

The chaos simply defines as a non-linear deterministic dynamic which seemingly behaves like a stochastic structure. The interest is to find the function that governs these dynamics. The motion of a time series like $(x_0, x_1, x_2, ..., x_{n-1}, x_n)$ that seems stochastic may be governed by a chaotic function like $f(x_{n-1}) = x_n$. Having this function, one will be able to develop the series for future $(x_{n+1}, x_{n+2}, ...)$. Yousefpoor et al [10] mentioned the mathematical definitions of a chaotic function which briefly conveys all properties that a chaotic function possesses:

Let V be an interval, it is said that $f : V \to V$ is chaotic on V if:

- I. f has sensitive dependence on initial conditions;
- II. *f* is transitive;
- III. Periodic points are dense in V.

The rapid divergence of solutions, which are close together initially, is called sensitive dependence on initial conditions.

Transitivity implies that the orbit of some interval I_0 is dense in [0, 1].¹ The set I_0 is said to be dense in [0, 1] if for each interval $I \subseteq [0, 1]$ there is a point of

 I_0 which is in I.

Periodic points are all unstable and this means that as soon as an orbit comes close to a periodic point, it will be pushed away somewhere else.

There is a special case that each of these conditions implies the other two [15].² Yousefpoor et al [10] mentioned that it might be impossible to recognize this special case and it is impossible to rely on investigating one condition in practice. Nevertheless, they regarded conditions (I) and (III) mentioned in main definition of chaos. It is notable that no test is found for condition (II) of chaos mathematical definition. They also mentioned that all chaos prerequisites must simultaneously be established for a time series to be chaotic.

Our goal is also performing introduced chaos tests (based on mathematical definition of chaos) to check null hypothesis of chaos for copper daily prices. In other words, if our data do not satisfy just one of conditions (I) and (II), they are not chaotic.

3 Empirical results

There are many chaos tests that have been used for chaos detection in different researches. But, Knowing the conditions we are looking for, we selected the largest Lyapunov exponent and the close return tests which examine conditions (I) and (III) of mathematical definition of chaos in order. The BDS test is also used to detect what kind of dependency exists among data. In this section, the empirical results are arranged according to our research development.

At the first step we calculated the copper return series. It is generally believed in financial affairs that, against return series, the price time series are not stationary. Stationary is important because it assures that statistical properties of a time series do not change over time. Thus, we changed daily copper price series to a stationary one by first difference $r_t = \ln(p_t - p_{t-1})$. Using the augmented Dickey–Fuller [16] and Phillips–Perron [17] tests, it was approved that this return series is a stationary one (Table 1).

¹ A sequence of iterates like $(x_0, x_1, x_2, ..., x_{n-1}, x_n)$, is called the orbit of x_0 under f in dynamical system approach.

In special case of symmetric one-hump mapping, each of these conditions implies the other row.





Table 1. Stationary test of first differenced time series

Test	1% critical value	5% critical value	10% critical value	Value of statistic
ADF	-2.57	-1.94	-1.61	-38.28
PP	-2.57	-1.94	-1.61	-55.38

3.1. Looking for nonlinear dependence

As mentioned previously, chaotic data exhibit nonlinear behavior. A numeric test which is widely used for nonlinearity testing is the BDS test (Brock, Deckert, and Scheinkman, [18]). The test uses correlation function (also called the correlation integral) as the test statistic. This choice is in contrast with the Grassberger-Procaccia [19] test, which uses the correlation dimension. The correlation function is needed in deriving the correlation dimension, but the two are not the same.

Since the derived distribution of the correlation dimension is unknown, the BDS test uses the correlation function as the test statistic. The asymptotic distribution of the correlation function is known under the null hypothesis of whiteness (independent and identically distributed observations). As a result, the BDS test can be used to produce a formal statistical test of whiteness against dependence. However, the sampling distribution of the BDS test statistic is not known under the null of chaos [20].

The BDS test can be used to produce a test of linearity against the broad alternative of nonlinearity, whether or not chaotic. Fitting a linear model of time series and removing linear structure, The BDS test then can be used to determine whether there is evidence of remaining dependence in the data. If all linear dependence has already been removed, then any remaining dependence should be nonlinear [20].

Consider a time series of observations $x_t, t = 1, ..., T$. Use this series of scalars to create an 'embedding'. In other words construct a series of '*M*-histories' as $x_t^M = (x_t, x_{t+1}, ..., x_{t+M-1})$. This converts the series of scalars into a slightly shorter series of vectors with overlapping entries. One uses this stack of vectors to carry out the analysis. Suppose that the true, but unknown, system which generated the observations is *n*-dimensional. Then provided $M \ge 2n+1$ generically the *M*-histories recreate the dynamics of the underlying system. This result, due to Takens [21], permits one to use the M-histories to analyze the system's dynamics. The spatial correlations amongst the points (*M*-histories) are measured by calculating the correlation integral, $C^M(\varepsilon)$ [7].

Setting s = 2 in the following equation, Euclidian norm is achieved:

$$L_{s} = \left[\sum_{k=0}^{M-1} \left| X_{i-k} - X_{j-k} \right|^{s} \right]^{1/s}$$
(1)

Doing so, the correlation integral is calculated using the following equations:

$$C(M,\varepsilon,T) = C^{M}(\varepsilon) = \frac{2}{T_{m}(T_{m}-1)} \left[\sum_{1 \le i \le j \le T} \left(\prod_{k=0}^{M-1} I_{s}(X_{i+k},X_{j+k}) \right) \right]$$
(2)

$$T_m = T - M + 1 \tag{3}$$

The statistic of the BDS test is then given as:

$$BDS = \sqrt{T} \left[C(M, \varepsilon, T) - C(1, \varepsilon, T)^M \right] / \sigma(M, \varepsilon, T)$$
(4)

$$\sigma_M^2(\varepsilon) = 4 \left[K^M + 2 \sum_{j=1}^{M-1} K^{M-j} C^{2j} + (M-1)^2 C^{2M} - M^2 K C^{2M-2} \right]$$
(5)

$$K = K(\varepsilon) \approx K(\varepsilon, T) = \frac{6}{T_m(T_m - 1)(T_m - 2)} \sum_{t < s < r} I_\ell(x_t, x_s) I_\ell(x_s, x_r) \quad (6)$$

In which T represents the length of data and I is Heaviside step function. The distribution of the BDS statistic will well be approximated when a sample has 500 or more observations; the M is selected to be 5 or lower; and ε is selected to be between 0.5 and 2 S.D. of data [22].

According to the BDS test, if x_t is *IID* with a non-degenerate distribution, then $C(M, \varepsilon, T) \rightarrow C(1, \varepsilon, T)^M$, as $T \rightarrow \infty$.

Removing linear dependencies of a time series, the BDS test is carried out on

the residuals to check null hypothesis of being *IID*. If the BDS statistics do not equal to zero, the null hypothesis will be rejected and presence of a nonlinear

dependency will be established. Because our efforts were unproductive to fit a linear time series model on our data, we carried out the BDS test on our original data. ³ Table 2 reports the results of the BDS test for our original data:

BDS		M				
		2	3	4	5	
	0.5	9.48	10.81	11.76	13.54	
ϵ/σ	1	9.05	10.42	11.42	12.37	
	1.5	9.25	10.75	11.74	12.53	
	2	8.81	10.34	11.44	12.10	

Table 2 .BDS statistics for original data (first difference)

From this table it can be inferred that there is some kind of nonlinearity through our data. In the next two sections we inspect (based on chaos definition) whether this nonlinearity is of chaos.

3.2. Testing the condition (I)

The method of the Lyapunov exponent can be employed to determine whether the process generating a time series is chaotic. The approach is based on the idea that the distance between two points is described by the largest Lyapunov exponent. The Lyapunov exponent measures the average rate of contraction (when negative) or expansion (when positive) of trajectories starting nearby on an entire attractor.⁴ The exponents can be positive or negative, but at least one exponent must be positive for an attractor to be classified as chaotic. If the distance of two nearby trajectories grows exponentially (on average), this is evidence of chaos because it shows that the process exhibits sensitive dependence to initial conditions. Thus, where *L* is the largest Lyapunov exponent, the criteria are [23]:

Stochasticity if L < 0; (7)

Chaos if L > 0 (8)

Here, we employed Kurths and Herzel [24] algorithm to estimate the Lyapunov exponent. The approach is started by constructing *M*-histories in order to reconstruct the system. All nearby pairs are selected from amongst the *M*-histories. All selected (X_i^M, X_j^M) should satisfy following condition:

³ The Ljung and Box test was used to fit a linear time series model on our data. But, there was only a weak autocorrelation and, on the other hand, the adjusted R^2 statistic refused the fit goodness of suggested models.

⁴ The trajectory of a chaotic time series is attracted to a part of state space. This part of state space is called 'attractor'.

$$r_0(M;i,j) = \left\| x_i^M - X_j^M \right\| < \varepsilon$$
(9)

In which, ε is a small positive number and $\|.\|$ is a metric. We will use the Euclidean measure of distance. In eq. (9) we have selected the nearby points in the *M*-histories. Using the next equation, the original nearby points are followed further n steps forward in time:

$$r_n(M;i,j) = \left\| x_{i+n}^M - X_{j+n}^M \right\|$$
(10)

Now, the following ratio is calculated:

$$d_n(M;,i,j) = r_n(M;i,j) / r_0(m;,i,j)$$
(11)

If the nearby points have separated, $d_n(M; i, j)$ will be larger than one. Finally one aggregates over the $d_n(M; i, j)$ to get an aggregate statistic [7]:

$$L(M,n) = \sum_{i \neq j} \ln d_n(M;i,j) / N(N-1)$$
(12)

Our results of L(M, n) calculation are reported in table 3.

. (1 1	М		
L(M,	n)	2	3	4	5	6
	0.5	1.9	2.48	1.46	1.22	1.03
ε/σ	1	1.9	1.21	1.08	1.07	1.55
	1.5	1.9	1.21	1.08	0.68	0.66

Table 3. Lyapunov exponents

As it can be seen, all Lyapunov exponents are positive. Though, some researchers have construed positive Lyapunov exponents as chaos, there are many cases that nonlinear stochastic systems have raised positive Lyapunov exponents. For example, see Yousefpoor et al. [10] and Damming and Mitschke [25].

As a matter of fact, having a positive Lyapunov exponent is a necessary but not sufficient evidence of chaos. So, the other prerequisites mentioned in the chaos definition should be investigated necessarily.

3.3. Testing the condition (III)

Each attractor contains a large number of unstable orbits of many periodicities; that are 'dense'. This density implies periodic points. In order to test the periodic points Gilmore [26] presented a new topological test which is named 'close return test'. This topological test is a qualitative one and has many advantages over metric tests (e.g. correlation dimension). ⁵ For instance, it is applicable to relatively small data sets in economics and finance.⁶

A quantitative form of this test has also presented by Gilmore [8]. Alike the BDS test, this quantitative form detects departures from *IID* and, therefore, is disregarded here.

The starting point for implementing the topological algorithm is the time series $\{x_i\}$ without an embedding. If one of the observations x_i occurs near a periodic orbit, then subsequent observations will evolve near that orbit for a while before a sufficiently long time, they will return to the neighborhood of x_i after some interval, T, where T indicates the length of the orbit, measured in units of the sampling rate. This means that $|x_i - x_{i+T}|$ will be small. Further, x_{i+1} will be near x_{i+1+T} , x_{i+2} will be near x_{i+2+T} and so on. Thus it makes sense to look for a series of consecutive data elements for which $|x_i - x_{i+T}|$ is small.

To detect these regions of 'close returns' in a data set a color-coded graph can be constructed. All differences $|x_i - x_{i+T}|$ are computed. If $|x_i - x_{i+T}| < \varepsilon$, the result is coded black; if $|x_i - x_{i+T}| \ge \varepsilon$, the result is coded white. A threshold value, ε , is determined as a small percentage (usually 2% to 5%) of the largest difference between any two values across the data set. The horizontal axis of the graph indicates the observation number, *i*, where i = (1, 2, ..., N), and the vertical axis is designated as *t*, where t = (1, 2, ..., N - i). Close returns in the data set are indicated by horizontal line segments. However, if the data set is stochastic, a generally uniform array of black dots will appear [8, 26].

Keeping close return plots of a chaotic map^7 and of a pseudorandom series, presented by Gilmore [8], as our standard (figures 2 and 3), we compared our blue coded close return plots (500 by 300) with them. Figures 4 to 8 illustrate the close return plots of our series.

⁵ The metric approach is characterized by study of distances between points on a strange attractor.

⁶ The other advantages of this test are described in Gilmore [26].

⁷ The Henon map (Henon, [27]).





Fig. 8. Close return plot of 2000 to 2400 observations.

As it can be seen, our plots are more like the close return plot of pseudorandom series than that of chaotic map. Thus, the condition (III) of chaos definition is not established and our series under study is not chaotic. This result, however, does not take our study to an end. Because, the kind of nonlinearity we found through our data should be known. The idea is removing this nonlinearity from our data via a GARCH-type model; and, then, using the BDS test to examine its adequacy for the established nonlinear structure.

3.4. Residual evaluation of a GARCH-type model

Some researches extract the standardized residuals from an appropriate ARCHtype model and, then, test for chaos on the standardized residuals. Chen [28] showed that filtering may affect the dimensionality of the original data and filtered data may mimic chaotic data. To avoid this, at first step, we decided to test for chaos after removing just linear dependencies. As mentioned before, however, we couldn't fit a satisfactory linear time series model on our data and we worked on original data set. Having construed a nonlinear- non chaotic structure in our time series, we would implement the BDS test on the residuals of a GARCH-type model to check whether the fitted model could describe this structure.

Using Engle [29] test it was approved that a GARCH (2,1) model could be appropriate to filter this nonlinearity from our series. Table 5 shows the chi-square statistic significant at 5% level for Studentized residuals of this model with null hypothesis of *IID*.

Table 4. GARCH (2,1) model					
Parameter	Value	Std. Error	T-ratio		
μ	3.4e-004	2.6e-004	1.33		
$lpha_{_0}$	4.1e-006	1.13e-006	3.63		
$lpha_{_1}$	0.073212	0.01053	6.95		
eta_1	0.32931	0.14084	2.34		
$oldsymbol{eta}_2$	0.57895	0.13391	4.32		

Table 5. Engle test for residuals	of GARCH model.	No ARCH effect.
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lags	Engle Statistic	5% critical value
5	6.30	11.07
10	7.50	18.30
20	20.10	31.41

The BDS test was also performed on the standardized residuals of this model. It is notable that Brock et al. [18] demonstrated that the distribution of this test changes when applied on the residuals ARCH and GARCH-type filters. The authors suggest bootstrapping the null distribution to obtain the critical values for the statistic when applying it to standardized residuals from these models. Hence, The BDS statistics are evaluated against critical values obtained from Monte Carlo simulation. Table 6 shows that the residuals of the fitted model are *IID* (the null hypothesis is approved) and this model can describe the nonlinear dynamics found in our time series.

Tuble 0. BDB stutistics for intered data						
BDS		<i>M</i>				
		2	3	4	5	
	0.5	0.96	1	1.08	1.18	
s/a	1	0.793	0.912	0.856	0.8	
670	1.5	1	1.3	1.42	1.43	
	2	1.27	1.84	2.11	2.17	
Critical values for		М				
BDS statistic		2	3	4	5	
	0.5	2.60	2.62	2.90	3.28	
ε/σ	1	2.58	2.40	2.47	2.55	
270	1.5	2.68	2.56	2.53	2.53	
	2	3.33	3.12	2.93	2.82	

Table 6. BDS statistics for filtered data

4 Conclusions

We kept the mathematical definition of chaos as a standard frame to select and implement chaos tests on copper return series. Previous studies have considered only metric tests for such series. Here, we applied the close return test which is a topologic one. This test is a direct test of chaos and applicable for small data sets.

It's also notable that some researchers believe that if the residuals of a stochastic model be *IID*, the chaos tests will not be required and these models describe the relationship among data. As mentioned above, filtering may cause a misunderstanding of chaos in some cases. To avoid this, the suggestion is implementing chaos tests on both original and filtered data sets.

Following the above policies, we couldn't establish a chaotic behavior in copper return series. The suggestions for forecasting copper prices include using stochastic time series models or running the Monte Carlo simulation.

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