Behavior of Coupled Fractal Structures and Their Attractors in the Model Nanosystem

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Abstract: The behavior of the deformation field of coupled fractal structures in the multilayer nanosystems is investigated. On the example of coupled fractal surfaces (elliptic and hyperbolic type) it is shown that the behavior of the deformation field is determined by mutual influence of stochastic processes on each other. Features of behavior of attractors (singular points) of the deformation field for these structures is investigated. When changing the governing parameters, there are possible effects of alteration and moving of the fractal structures relative to each other.
Keywords: Fractal Bulk Structures, Coupled Systems, Attractors, Deformation Field, Multilayer Nanosystem.

1 Introduction

Recently, various nonlinearities in periodic structures and metamaterials have been actively investigated [1]. Further development was given to nanophysics, condensed matter physics (Tosi [2], Schneider [3]), neural networks (Heyman [4]), which use nanoobjects as active objects. The most important task is to study the mutual influence, collective interactions, dynamics of such nanoobjects (V. Abramov [5, 6]). In this case, there are possible the effects of various topological phase transitions associated with the shape and dimension of nanoobjects, the formation of new coupled structures (clusters). One example of this kind can be model multilayer nanosystems and bulk fractal structures appearing there (V. Abramov [7], Abramova [8-10]).
The relevance of such studies is confirmed by the award of the Nobel Prize in Physics for 2016 to M. Kosterlitz, D. Thouless, D. Haldane for theoretical discoveries of topological phase transitions and topological phases of matter.
In this paper, the effects associated with the shape of objects are investigated using attractors of both separated and coupled structures. When modelling the fractal bulk structures in the multilayer nanosystem, bulk lattice nodes that are attractors can be as active elements (C.H. Skiadas [11]). These attractors form a surface of active elements of the deformation field (displacement field). In order to determine the position of singular points (attractors) of a displacement field in a single active layer it is necessary to cross this surface by a plane. As a result it is possible to find isolines of singular points in a separate layer.
In this paper, the dynamics of the interactions of both separated and related structures is modelled by organizing separate stochastic processes in the coupled

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system. In this case, many physical properties (for example, the deformation field (V. Abramov [7], Abramova [8, 9])) in coupled fractal structures differ from the properties of separate fractal objects.

The aim of this paper is to study the mutual influence of structures and their attractors, separate stochastic processes on each other in a coupled fractal structure.

2 Mutual influence of elliptic type structures and their attractors in a coupled fractal system

A coupled fractal structure which consists of two separate fractal structures ($\mathrm{i}=1,2$) is considered. This coupled structure is in the bulk discrete lattice $N_1 \times N_2 \times N_3$, whose nodes are given integers n, m, j.

The nonlinear equation for the dimensionless displacement function u of lattice node of this coupled fractal structure is given in the form (Abramova [8])

$$u = \sum_{\mathrm{i}=1}^{2} R_{\mathrm{i}} (1-\alpha)(1-2sn^2(u-u_0,k)) / Q_{\mathrm{i}} \ ; \quad p_{0\mathrm{i}} = p'_{0\mathrm{i}} + p'_{1\mathrm{i}} n + p'_{2\mathrm{i}} m + p'_{3\mathrm{i}} j \ ; \qquad (1)$$

$$Q_{\mathrm{i}} = p_{0\mathrm{i}} - b_{1\mathrm{i}} (n-n_{0\mathrm{i}})^2 / n_{c\mathrm{i}}^2 - b_{2\mathrm{i}} (m-m_{0\mathrm{i}})^2 / m_{c\mathrm{i}}^2 - b_{3\mathrm{i}} (j-j_{0\mathrm{i}})^2 / j_{c\mathrm{i}}^2 \, . \qquad (2)$$

In expressions (1), (2) index i corresponds to the number of a separate fractal structure; α is the fractal dimension of the deformation field u along the Oz-axis ($\alpha \in [0,1]$); u_0 is the constant (critical) displacement; k is the modulus of the elliptic sine; parameters $p'_{0\mathrm{i}}$, $p'_{1\mathrm{i}}$, $p'_{2\mathrm{i}}$, $p'_{3\mathrm{i}}$, $b_{1\mathrm{i}}$, $b_{2\mathrm{i}}$, $b_{3\mathrm{i}}$, $n_{0\mathrm{i}}$, $n_{c\mathrm{i}}$, $m_{0\mathrm{i}}$, $m_{c\mathrm{i}}$, $j_{0\mathrm{i}}$, $j_{c\mathrm{i}}$ characterize different fractal structures.

Parameters R_{i} determine the orientation of the deformation fields of separate structures in the coupled system. In general case these parameters may depend on the layer index j and the dimensionless time t. Functions Q_{i} take into account the interaction of the nodes of both in the main plane of the discrete rectangular lattice as well as interplane interactions.

Singular points (attractors) of the deformation field of the multilayer nanosystem are located on the surface, the core of which is determined by condition

$$Q_1 \cdot Q_2 = 0 \, . \qquad (3)$$

If surface (3) is crossed plane $j = j_k$, we obtain the equation of the isolines.

Nonlinear equations (1), (2) can be solved by iteration method on any of indices n, m, j. If one of these indices is considered fixed, then the result of the iteration will be the displacement function, which is a stochastic surface, depending on the other two indices.

In this work the iterative procedure on index n simulates a stochastic process on a rectangular discrete lattice with a size $N_1 \times N_2$, $n = \overline{1, N_1}$; $m = \overline{1, N_2}$,

$N_1 = 240$, $N_2 = 180$.

Separate fractal bulk structures (FBS) are chosen in the form of: a real fractal elliptic cylinder (FEC) and an imaginary fractal elliptical cylinder, whose structure allows interpretation as a fractal quantum dot (FQD). The equations of surfaces of the considered structures do not depend on index j, thus the parameters $b_{3\mathrm{i}} = 0$ in the functions Q_{i}.

Based on these separate structures, we form coupled systems FEC1-FEC2 (fig. 1-3), FQD1–FEC2 and FEC1–FQD2 (fig. 4). The main parameters for FBS were the following: $\alpha = 0.5$; $u_0 = 29.537$; $k = 0.5$.

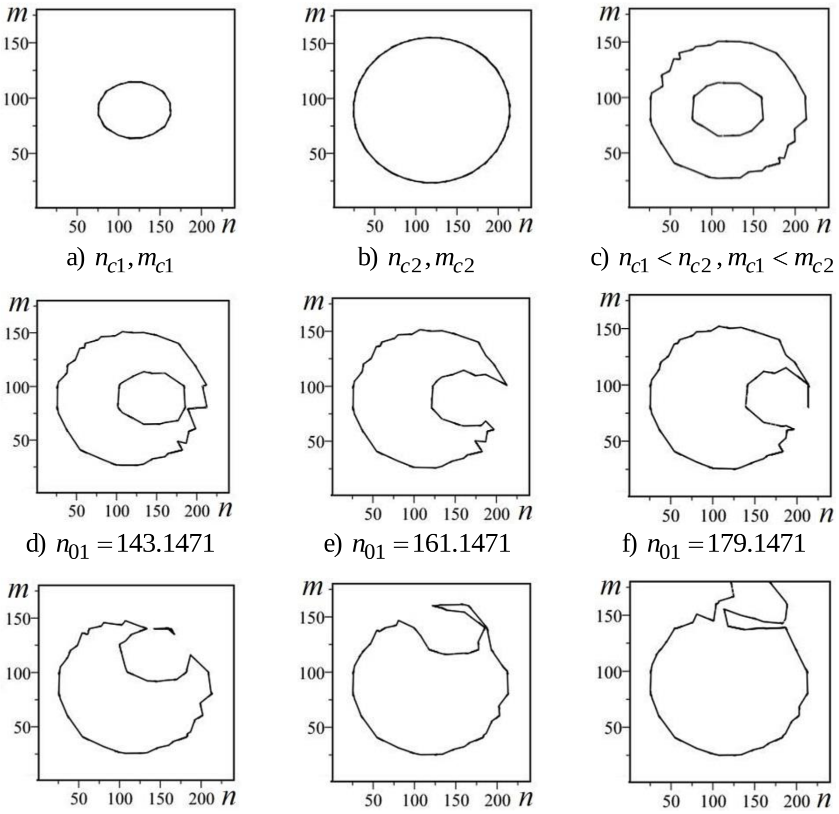
For the first (FEC1) and second (FEC2) fractal elliptical cylinders parameter $p'_{0\mathrm{i}}$ is $p'_{0\mathrm{i}} = 1.0123$; for the first (FQD1) and second (FQD2) fractal quantum dots parameter $p'_{0\mathrm{i}}$ is $p'_{0\mathrm{i}} = -1.0123$.

For these structures the following parameters were same $p'_{1\mathrm{i}} = p'_{2\mathrm{i}} = p'_{3\mathrm{i}} = 0$; $b_{1\mathrm{i}} = b_{2\mathrm{i}} = 1$; $j_{0\mathrm{i}} = 31.5279$; $j_{c\mathrm{i}} = 11.8247$; i=1,2 ; the parameters describing the semi-axes were different $n_{c1} = 44.4793$; $m_{c1} = 25.7295$; $n_{c2} = 94.4793$; $m_{c2} = 65.7295$.

The parameters $(n_{0\mathrm{i}}, m_{0\mathrm{i}})$ determining the fixed positions of the centres of gravity of the separate structures in the coupled fractal system were the same or different. The orientation of the deformation fields of separate structures in a coupled system was determined by the choice of parameters R_{i}.

On fig. 1 attractors (isolines of singular points cores) of such coupled fractal elliptic cylinders with coinciding ($n_{01} = n_{02} = 119.1471$; $m_{01} = m_{02} = 89.3267$; fig. 1 c) and shifted (fig. 1 d-i) centres of gravity are shown. For separate structures of FEC1 (for $R_1 = 1$, $R_2 = 0$), FEC2 (for $R_1 = 0$, $R_2 = 1$) the attractors (fig. 1 a, b) are located on curves of elliptic type. For the coupled fractal structure of FEC1-FEC2, the centres of gravity of FEC1 and FEC2 coincide and the inequalities ($n_{c1} < n_{c2}$, $m_{c1} < m_{c2}$) are satisfied for the semi-axes, therefore the structure of the attractor FEC1 is located inside the structure of the attractor FEC2 (fig. 1 c). In this case, the influence of the attractors on each other is observed: on the elliptical type curves there appear distortions in comparison with the smooth curves in fig. 1 a, b.

The shift of the centre of gravity in the internal structure of the FEC1 with a fixed centre of gravity near the external structure of the FEC2 results in a essential alteration of the structure of the attractors of the coupled system (fig. 1 d-i). For fig. 1 d-f parameters $m_{0\mathrm{i}}$, n_{02} stay the same and parameter n_{01} changes, thus the FEC1 moves inside the FEC2 along axis On. For attractors on fig. 1 g-i parameter $n_{01} = 145.1471$ does not change and the parameter m_{01} changes, thus the FEC1 moves inside the FEC2 along axis Om.



a) n_{c1}, m_{c1} b) n_{c2}, m_{c2} c) $n_{c1} < n_{c2}, m_{c1} < m_{c2}$

d) $n_{01} = 143.1471$ e) $n_{01} = 161.1471$ f) $n_{01} = 179.1471$

g) $m_{01} = 115.3267$ h) $m_{01} = 139.3267$ i) $m_{01} = 159.3267$

Fig. 1. Attractors of separate FEC1 (a), FEC2 (b) and coupled FEC1- FEC2 (c-i) structures with coinciding (c) and shifted (d-i) centres of gravity.

Fig. 2 shows the cross-sections of the deformation field $u \in [0;1]$ (top view) of fractal elliptic structures, whose attractors are shown in fig. 1. The deformation fields both of separate (fig. 2 a, b) and coupled (fig. 2 c-i) structures are stochastic surfaces with pronounced features behavior near the attractors from fig. 1.

For the separate structure of FEC1 (fig. 2 a), the first stochastic process was realized with $R_1 = 1, R_2 = 0$. For the separate structure of FEC2 (fig. 2 b), the second stochastic process was realized with $R_1 = 0, R_2 = 1$.

For a coupled system, the both processes were realized together with $R_1 = 1, R_2 = 1$ (fig. 2 c-i).

The realization of the first and second stochastic processes is determined by the choice of the values of the semi-axes n_{ci}, m_{ci} of the separate structures in a coupled system. These semi-axis values are related with the characteristic correlation lengths of the stochastic processes.

Inside the elliptical type (FEC) regions a pronounced stochastic behavior of the deformation field is observed (fig. 2 a, b). The presence of "tails" to the right of the FEC1 (fig. 2 a), to the left and to the right of FEC2 (fig. 2 b) is due to the iterative process along variable n.

The appearance of "tails" to the left of the FEC2 (fig. 2 b) is related by the greater correlation length of the second stochastic process ($n_{c2} > n_{c1}$, $m_{c2} > m_{c1}$). In this case, for coupled structures (fig. 2 c-i) there is a pronounced influence of stochastic processes on each other, the shift of internal FEC1 relative to external FEC2.

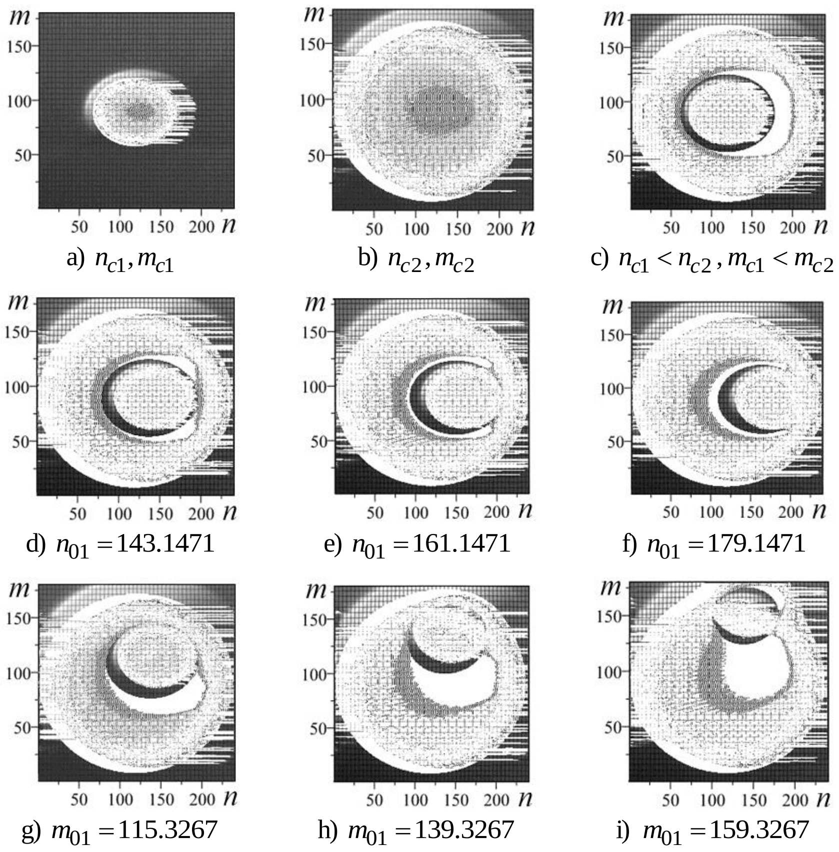
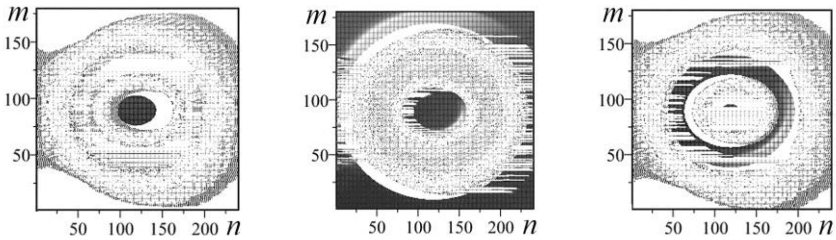


Fig. 2. Cross-sections $u \in [0;1]$ (top view) of the separate FEC1 (a), FEC2 (b) and coupled FEC1- FEC2 (c-i) structures with coinciding (c) and shifted (d-i) centres of gravity for $R_1 = 1, R_2 = 1$.

Change in the orientation of the deformation field of separate structures (signs for parameters R_1, R_2) for a coupled structure from fig. 2 c with coinciding centres of gravity leads to the essential alteration of the deformation field of the coupled structure (fig. 3).

In this case, an internal region with a regular behavior of the deformation field is possible (fig. 3 a, b). Note that the behavior of the attractor from fig. 1 c is preserved and does not depend on the change of signs R_1, R_2.



a) $R_1 = 1, R_2 = -1$ b) $R_1 = -1, R_2 = 1$ c) $R_1 = -1, R_2 = -1$

Fig. 3. Cross-sections $u \in [0;1]$ (top view) of coupled structures FEC1- FEC2 with coinciding centres of gravity for different signs R_1, R_2.

Next, we consider coupled structures FQD1-FEC2 and FEC1-FQD2, in which one of the attractors is real, the other is imaginary.

An example of separate structures with an imaginary attractor are quantum dots FQD1, FQD2 and with the real attractor are FEC2, FEC1.

The presence of an imaginary attractor in FQD1 influences the behavior of the real attractor of FEC2 of the whole coupled structure FQD1-FEC2 (fig. 4 a).

In this case, for the coupled structure there are distortions on a curve of elliptic type as compared with smooth curves from fig. 1 a, b, the internal curve (fig. 1 c) disappears.

The cross-sections of the deformation field $u \in [0;1]$ (top view) of the FQD1-FEC2 structure for different signs of parameters R_1, R_2 are given on fig. 4 b,c,d,e.

An appearance of internal regions with a regular (fig. 4 b, e) and structured (fig. 4 c, d) behavior of the deformation field are observed.

The cross-sections of the deformation field $u \in [0;1]$ of another structure FEC1-FQD2 for different signs of the parameters R_1, R_2 are given on fig. 4 f,g,h,i.

The main features of the behavior of the deformation field (the presence of internal regions with regular (fig. 4 f, i) and structured (fig. 4 g, h) behavior) are preserved.

However, external regions with regular behavior appear (fig. 4 f,g,h,i), there is an appearance of an "inflow" (distortion of regular behavior) near the structure of FEC1- FQD2 (fig. 4 f).

This is due to the presence of an imaginary attractor and the influence of the stochastic process from FQD2 to FEC1.

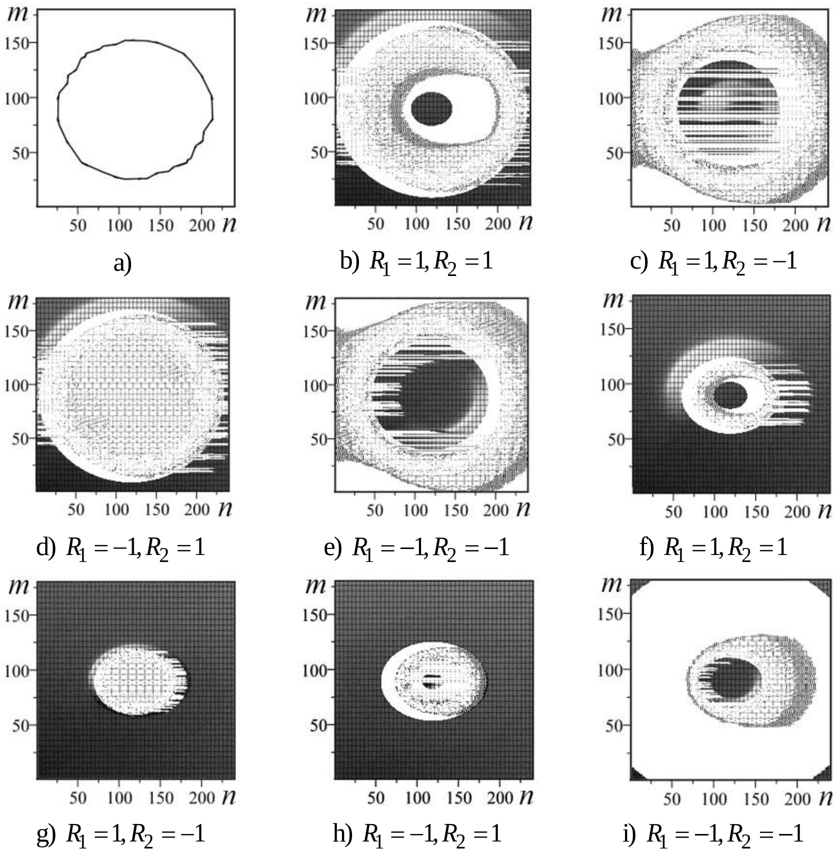
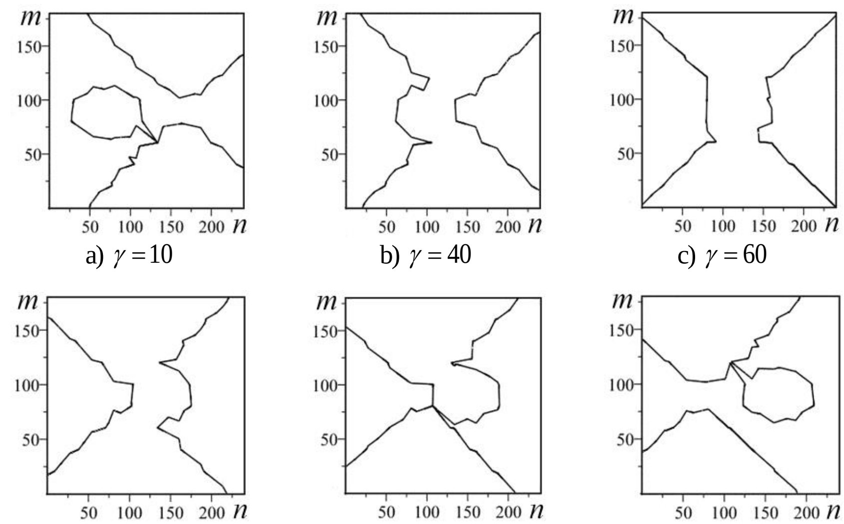


Fig. 4. Cross-section $u \in [0;1]$ (top view) of coupled FQD1-FEC2 (b,c,d,e), FEC1-FQD2 (f,j,h,i) structures with coinciding centres of gravity for different R_1, R_2. Attractor (a) of coupled FQD1-FEC2 structure.

3 Behavior of coupled structures of elliptic and hyperbolic types

Next, we investigate the behavior of coupled fractal structures based on the separate structures of the elliptic (FEC) and hyperbolic (FHC) types: FEC3-FHC1 and FEC4-FHC2 with variable centres of gravity.
The behavior of the attractors (fig. 5) and the deformation field (fig. 6-8) of such structures has its features as opposed to structures of only elliptic type.
For the FEC3 parameters were the following: $p'_{01} = 1.0123$; $b_{11} = b_{21} = 1$; $n_{01} = 59.1471 + \gamma$; $n_{c1} = 44.4793$; $m_{c1} = 25.7295$. For the FHC1 parameters

were the following: $p'_{02} = 1.0123$; $b_{12} = -1$; $b_{22} = 1$; $n_{02} = 179.1471 - \gamma$; $n_{c2} = 17.4793$; $m_{c2} = 12.7295$. Other parameters for these structures were the same: $p'_{1i} = p'_{2i} = p'_{3i} = 0$; $b_{3i} = 0$; $m_{0i} = 89.3267$; $j_{0i} = 31.5279$; $j_{ci} = 11.8247$ ($i = 1, 2$). Variable parameter γ describes the shift of the centres of gravity of coupled structures along axis On.



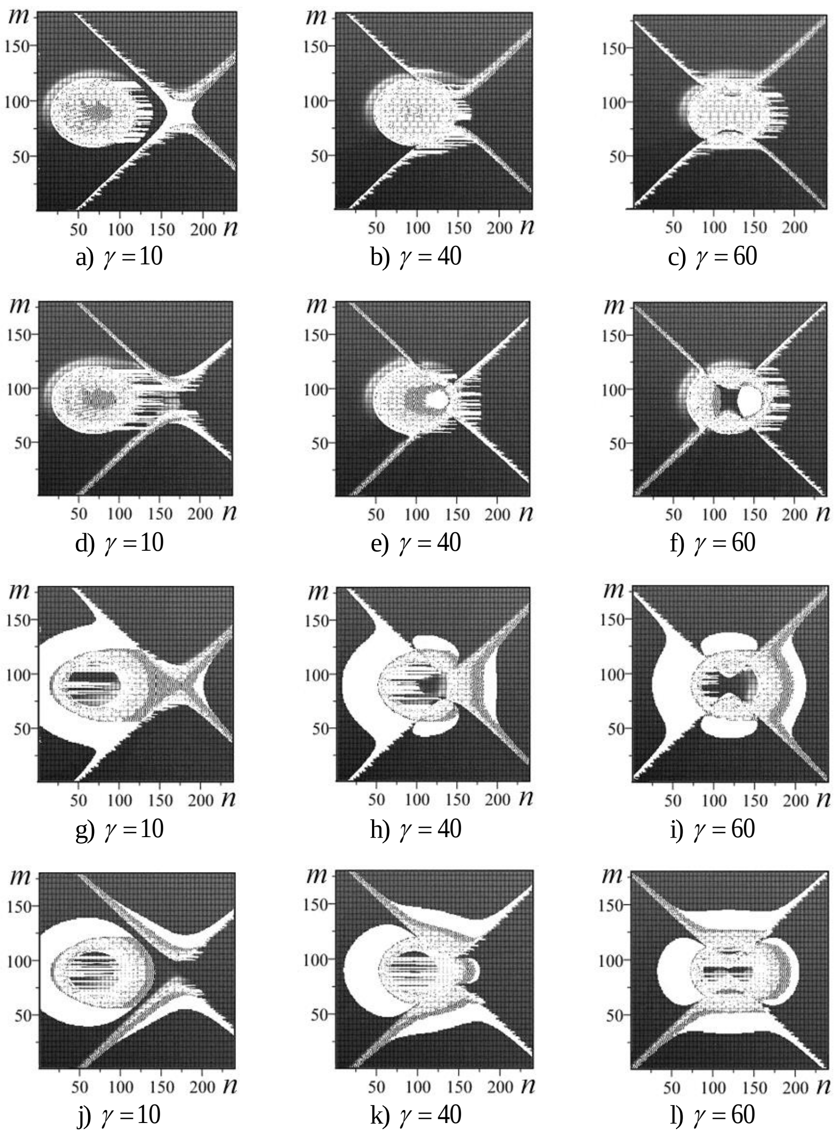
a) $\gamma = 10$ b) $\gamma = 40$ c) $\gamma = 60$

d) $\gamma = 80$ e) $\gamma = 90$ f) $\gamma = 110$

Fig. 5. Attractors of coupled FEC3-FHC1 structures with variable centres of gravity.

Changing parameter γ from 0 to 120, we make approach of the structures FEC3 and FHC1 (moving centres of gravity along axis On, fig. 5-7).
At value $\gamma = 60$ the centres of gravity of FEC3 and FHC1 coincide (fig. 5 c).
The change of signs of parameters R_i determines a different orientation of the deformation fields of separate structures, which leads to the essential alteration of the deformation field of the coupled structure (fig. 6-7).
At a fixed value of the centres of gravity of separate structures (for example $\gamma = 10$) the structure of the attractor of the coupled system (fig. 5 a) does not depend on the change of signs of R_i.
However, in this case, the structure of the deformation field changes essentially (fig. 6 a,d,g,j), which is due to the influence of different stochastic processes of different orientations on each other. If the signs of parameters R_i do not change (for example, $R_1 = 1, R_2 = 1$) and parameter γ takes different values, then the mutual orientation of the stochastic processes is conserved (fig. 6 a-c, 7 a-c) and

the structure of the attractors essentially changes (fig. 5).
In this case, the deformation field changes (fig. 6 a-c, 7 a-c), which is associated with the appearance of a different type of non-linearity of stochastic processes due to the shift of the centres of gravity of separate structures.



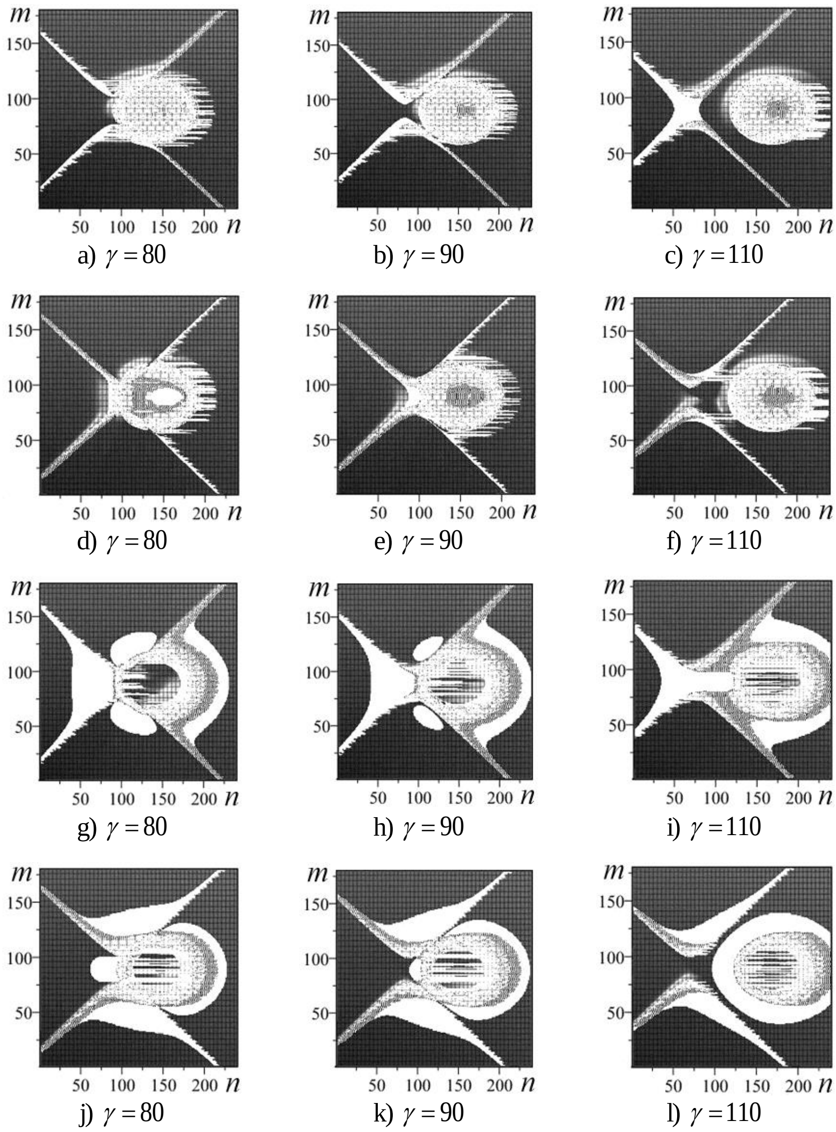
a) $\gamma = 10$ b) $\gamma = 40$ c) $\gamma = 60$

d) $\gamma = 10$ e) $\gamma = 40$ f) $\gamma = 60$

g) $\gamma = 10$ h) $\gamma = 40$ i) $\gamma = 60$

j) $\gamma = 10$ k) $\gamma = 40$ l) $\gamma = 60$

Fig. 6. Cross-section $u \in [0;1]$ (top view) of coupled FEC3-FHC1 structures with variable centres of gravity: (a,b,c) - $R_1 = 1, R_2 = 1$; (d,e,f) - $R_1 = 1, R_2 = -1$; (g,h,i) - $R_1 = -1, R_2 = 1$; (j,k,l) - $R_1 = -1, R_2 = -1$.



a) $\gamma = 80$ b) $\gamma = 90$ c) $\gamma = 110$

d) $\gamma = 80$ e) $\gamma = 90$ f) $\gamma = 110$

g) $\gamma = 80$ h) $\gamma = 90$ i) $\gamma = 110$

j) $\gamma = 80$ k) $\gamma = 90$ l) $\gamma = 110$

Fig. 7. Cross-section $u \in [0;1]$ (top view) of coupled FEC3-FHC1 structures with variable centres of gravity: (a,b,c) - $R_1 = 1, R_2 = 1$; (d,e,f) - $R_1 = 1, R_2 = -1$; (g,h,i) -

$R_1 = -1, R_2 = 1$; (j,k,l) - $R_1 = -1, R_2 = -1$.

When choosing another (along the diagonal) law of change of the centres of gravity of structure FEC4-FHC2 there are essential differences in the behavior of attractors and cross-sections of the displacement function compared to the coupled structure of FEC3-FHC1.
The move of structures FEC4 and FHC2 along the diagonal was described by parameters: $n_{01} = 19.1471 + \gamma$; $m_{01} = 14.3267 + \gamma$; $n_{02} = 219.1471 - \gamma$; $m_{01} = 157.3267 - \gamma$ (fig. 8). Variable parameter γ changed from 0 to 140.
The changes of the structure of the attractors (fig. 8 a-c) and the features of the behavior of the displacement function (fig. 8 d-f) confirm the essentially nonlinear type of stochastic processes in the coupled system.

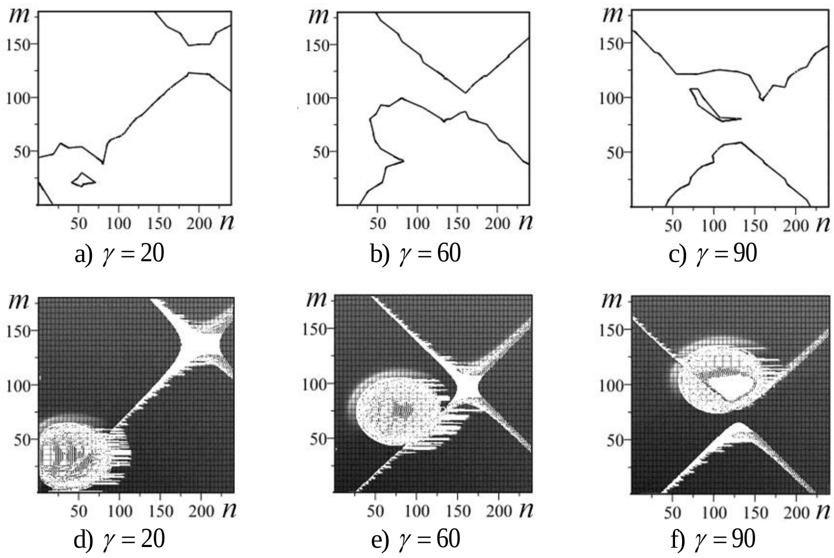


Fig. 8. Attractors and cross-section $u \in [0;1]$ (top view) of coupled FEC4-FHC2 structures with variable centres of gravity for $R_1 = 1, R_2 = 1$.

Conclusions

It is shown that in a model nanosystem the coupled structures based on separate fractal structures can be formed.
Inside the coupled structures (from separate elliptic structures with real attractors, fig. 1-3), regions with regular (cavities with a bottom and cavities without a bottom – holes) and stochastic (fractal holes) behavior of the deformation field are observed. These cavities are surrounded by two smeared boundaries of elliptic type with stochastic behavior of the deformation field. Inside the coupled structures (from

separate elliptic structures with real and imaginary attractors, fig. 4) additionally fine structure in cavities is observed. These cavities are surrounded by a single smeared boundary.
Significant changes in deformation fields for a coupled system from separate structures of elliptic and hyperbolic types are observed (fig. 5-8).
The behavior of attractors (real and imaginary), deformation field, the type of the stochastic process for a separate fractal structure, can be controlled by the choice of governing parameters (values of semi-axis, positions of the centres of gravity).
Additionally, in a coupled fractal structure it becomes possible to govern by a mutual orientation of the deformation fields of separate structures.
Is shown that for a coupled structure, the mutual influence of attractors and separate stochastic processes on each other is significant. The effects of alteration and movement of fractal structures on each to other are possible.
The simulation results can be used to describe various fractal topological phase transitions associated with the shape and dimension of nanoobjects, clusters, and superclusters.

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