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An explanation of stability of extrasolar systems based on the universal stellar law

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Abstract: This work investigates the stability of exoplanetary systems based on the statistical theory of gravitating spheroidal bodies. The statistical theory for a cosmogonical body forming (so-called spheroidal body) has been proposed in our previous works. Starting the conception for forming a spheroidal body inside a gas-dust protoplanetary nebula, this theory solves the problem of gravitational condensation of a gas-dust protoplanetary cloud with a view to planetary formation in its own gravitational field. This work develops the equation of state of an ideal stellar substance based on conception of the universal stellar law (USL) connecting temperature, size and mass of a star. This work also shows that knowledge of some orbital characteristics for multi-planet extrasolar systems refines own parameters of stars based on the combined Kepler 3rd law with universal stellar law (3KL–USL). The proposed 3KL–USL predicts statistical oscillations of circular motion of planets around stars.
Thus, we conclude about a possibility of presence of statistical oscillations of orbital motion, i.e. the oscillations of the major semi-axis and the orbital angular velocity of rotation of planets and bodies around stars. Indeed, this conclusion is completely confirmed by existing the radial and the axial orbital oscillations of bodies for the first time described by Alfvén and Arrhenius.

Keywords: Gas-dust protoplanetary nebula, Spheroidal bodies, Gravitational condensation, Exoplanetary systems, Alfven–Arrhenius' oscillating forces, Stability of planetary orbits, Specific entropy.

1 Introduction

We develop a statistical theory of gravitating spheroidal bodies to explain a stability of the orbital movements of planets as well as the forms of planetary orbits with regard to Alfvén–Arrhenius' oscillating force [1]-[3] in the Solar system and other exoplanetary systems. The statistical theory of formation of gravitating spheroidal bodies has been proposed in our earlier works [4]-[10]. Starting from the concept of forming a spheroidal body inside a gas-dust protoplanetary nebula, this theory solves the problem of gravitational condensation of a gas-dust protoplanetary cloud with a view to planet formation in its own gravitational field and derives the law of planetary distances in the Solar system generalizing the well-known laws [5], [6]. Within framework of

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statistical theory of gravitating spheroidal bodies, a new universal stellar law (USL) connecting temperature, size and mass of each star has been derived [11]. Naturally, the stabilities as well as forms of planetary orbits depend directly on the constancy of a gravitational field level around a star. As Alfvén and Arrhenius noted, "the typical orbits of satellites and planets are circles in certain preferred planes. In satellite systems, the preferred planes tend to coincide with equatorial planes of the central bodies. In the planetary system, the preferred plane is essentially the orbital plane of Jupiter (because this is the biggest planet), which is close to the plane of the ecliptic. The circular motion with period T is usually modified by superimposed oscillations. Radial oscillations (in the preferred plane) with period $\approx T$ change the circle into an ellipse with eccentricity e. Axial oscillations (perpendicular to the preferred plane), also with a period $\approx T$, make the orbit inclined at an angle i to this plane" [1, pp.343-344], [3]. Thus, due to the radial (axial) oscillating force the orbits of moving planets in the Solar system are described by ellipses with focuses in the origin of coordinates and small eccentricities (inclinations). In this connection, the following question appears: *what is the cause of the radial and the axial oscillations as well as the nature of the periodic radial and the periodic axial forces?*

In this work we explain the origin of these oscillating forces [1]-[3] modifying forms of planetary orbits within the framework of the statistical theory of gravitating spheroidal bodies. A justification of the stability of orbital movements of planets as well as the forms of planetary orbits with regard to Alfvén–Arrhenius' oscillating forces in planetary systems is considered. Concretely, we show that a temporal deviation of the gravitational compression function of a spheroidal body (modeling a gas-dust protoplanetary cloud) induces the additional periodic forces making the orbits elliptic ones. Indeed, as alleged earlier that orbits of moving particles inside a flattened rotating and gravitating gas-dust protoplanetary cloud initially are to be circular, however, during evolution of this protoplanetary cloud at formation of protoplanets these orbits can be deformed a little due to collisions with other particles or gravitational influences of forming adjacent planetesimals. In particular, V.S. Safronov marked: «The assumption of initial motion of particles in circular orbits looks natural. At small masses of bodies their gravitational variations were weak, and particles moved in orbits close to the circular ones. In the process of a planet growth, deviations of orbits from the circular increased, and all bodies of a zone had an opportunity to be joined in one planet» [12, p.145].

However, it should be noted that orbits of the moving bodies at *later stages of evolution* of a gas-dust protoplanetary cloud are formed mainly under the influence of its centrally symmetric gravitational field, therefore the solution of Binet's equation determines namely the elliptic (or Keplerian) forms of planetary orbits [13]. Moreover, both Newtonian theory of gravity [14] and consequent Laplacean celestial mechanics [15] explain the elliptic orbits based on centrally symmetric gravitational forces exclusively and do not consider the processes of formation (including collisions, giant impacts, accretions or

gravitational influences of other bodies). This means that such modification (from the circular orbit to the elliptic one) cannot be explained by a process of formation only since the possible reason consists of a temporal deviation of gravitational field of a central body (a star) into a protoplanetary cloud, i.e. stellar corona undergoes periodic small pulsations of compression. These small pulsations of compression induce the radial and the axial oscillations (with the circular frequencies ω_r and ω_z respectively) of orbital body motion. As Alfvén and Arrhenius noted, the motion in a centrally symmetric gravitational field "is degenerate in the sense that $\omega_r = \omega_z$ ...This is due to the fact that there is no preferred direction" [1], [3]. On the contrary, we confirm here that a spatial deviation of the gravitational potential from the centrally symmetric one defines a difference in the values of the radial and the axial orbital oscillations (when $\omega_r \neq \omega_z$) for a rotating ellipsoid-like spheroidal body. That is why an interference of these orbital oscillations can lead to the nonuniform rotation of the stellar layers at different latitudes of a star.
We also show that the stability of parameters of planetary orbits is determined by a constancy of the specific entropy in conformity with the principles of self-organization in complex systems. Therefore, the proposed 3KL–USL explains the stability of planetary orbits in our Solar system and other the multi-planet extrasolar systems as such Kepler-20 [16], HD10180 [17], 55Cnc [18], [19], Alpha Centauri [20], Upsilon Andromedae [21], Gliese 876 [18], etc.

2 The general antidiffusion equation of a slowly evolving process of initial gravitational condensation of a rotating spheroidal body from an infinitely distributed substance

According to the statistical theory [4]-[10] the process of planetary system formation from a protoplanetary cloud to the star with planets can be described by a multi-scale control parameter of dynamical states of a spheroidal body, called the parameter of gravitational condensation α. With point of view of the general statements of the statistical theory of gravitating cosmogonical bodies, the mass density function of a sphere-like (*slowly rotating*) spheroidal body is

$$\rho(r) = \rho_0 \cdot \exp(-\alpha r^2 / 2), \tag{1}$$

where ρ_0 is a density in the center of spheroidal body:

$$\rho_0 = M\left(\alpha / 2\pi\right)^{3/2} \tag{2}$$

and M is a mass of spheroidal body.
As appears from Eq. (1) under an influence of gravitational interactions of particles, there arises a substance mass density inhomogeneous along the radial coordinate r. Because of the mass density value strictly depends on α in Eq.(1) this positive parameter defines a measure of gravitational interactions of

particles in a spheroidal body, therefore it is called *the parameter of gravitational condensation* [4]-[9]. The greatest mass density is concentrated in the interval $[0, r_*]$, where $r_* = 1/\sqrt{\alpha}$ is a point of the mass density bending, outside of which it decreases quickly. It follows from Eq.(1), the isosurface of mass density (isostera) for a spheroidal body is a *sphere*.

The differential equation describing a process of initial gravitational condensation of a *sphere-like* cosmogonical body in a vicinity of mechanical equilibrium has been derived in our earlier works [5], [6], [9]:

$$\frac{\partial \rho}{\partial t} = -\mathrm{G}(t)\nabla^2 \rho \,. \tag{3}$$

where $\mathrm{G}(t) = \frac{1}{2\alpha^2} \cdot \frac{d\alpha}{dt}$ is a *gravitational compression function* (GCF). Since $\alpha = \alpha(t)$ is a monotonically increasing function then $\mathrm{G}(t) > 0$.

Now let us consider the derivation of differential equation for a spheroidal body forming when its isostera is evolving from the sphere to the spheroid. In this connection, using the cylindrical frame of reference (h, ε, z) we shall investigate the mass density function ρ of a *rotating* spheroidal body with a uniform angular velocity Ω in a vicinity of *relative* mechanical equilibrium [5], [6], [8], [9]:

$$\rho(h,z) = \rho_0 \cdot (1-\varepsilon_0^2) \cdot \exp(-\alpha[h^2(1-\varepsilon_0^2) + z^2]/2)\,, \tag{4}$$

where ρ_0 denotes the same density in the center of this spheroidal body in accord with (2) and ε_0 is a constant (of stabilization of the variable ε) called the *eccentricity* of an ellipse. Really, isosurfaces of the mass density (4) are *flattened ellipsoidal* (or *spheroidal* if $|\varepsilon_0| << 1$) ones, and ε_0^2 is a parameter of their flatness [6], [8], [9]. Obviously, when $\varepsilon_0^2 \to 0$ then the equation (4) goes to Eq. (1) for the non-rotational case.

Let us calculate the derivative of ρ with respect to the space coordinates x, y and z as well as the parameters α and ε_0 supposing them as slowly changing (with the time) functions, i.e. $\alpha = \alpha(t)$ and $\varepsilon_0 = \varepsilon_0(t)$:

$$\begin{aligned}\frac{\partial \rho}{\partial \alpha} &= \frac{3}{2} M[\alpha/2\pi]^{1/2} \cdot \frac{1-\varepsilon_0^2}{2\pi} \cdot \exp(-\alpha[h^2(1-\varepsilon_0^2) + z^2]/2) + M[\alpha/2\pi]^{3/2} \times \\ &\times (1-\varepsilon_0^2) \cdot [-\frac{(1-\varepsilon_0^2)h^2 + z^2}{2}] \cdot \exp(-\alpha[h^2(1-\varepsilon_0^2) + z^2]/2) = \\ &= \frac{\rho}{2\alpha}[3 - \alpha(h^2(1-\varepsilon_0^2) + z^2)],\end{aligned} \tag{5a}$$

$$\frac{\partial\rho}{\partial\varepsilon_0}=\rho_0(-2\varepsilon_0)\cdot\exp(-\alpha[h^2(1-\varepsilon_0^2)+z^2]/2)+\rho_0(1-\varepsilon_0^2)\cdot\{-\frac{-\alpha h^2}{2}(-2\varepsilon_0)\}\times$$
$$\times\exp(-\alpha[h^2(1-\varepsilon_0^2)+z^2]/2)=-\frac{2\varepsilon_0}{1-\varepsilon_0^2}\cdot\rho[1-\frac{\alpha}{2}h^2(1-\varepsilon_0^2)]; \tag{5b}$$

$$\nabla_h^2\rho=\frac{1}{h}\cdot\frac{\partial}{\partial h}\left(h\frac{\partial\rho}{\partial h}\right)=-\alpha(1-\varepsilon_0^2)^2\rho_0\cdot\{\frac{1}{h}\cdot\frac{\partial}{\partial h}(h^2\cdot\exp(-\alpha[h^2(1-\varepsilon_0^2)+z^2]/2))\}=$$
$$=-\alpha(1-\varepsilon_0^2)\cdot\rho[2-\alpha(1-\varepsilon_0^2)h^2]; \tag{5c}$$

$$\nabla_z^2\rho=\frac{\partial^2\rho}{\partial z^2}=\rho_0(1-\varepsilon_0^2)\frac{\partial}{\partial z}\{-\alpha z\cdot\exp(-\alpha[h^2(1-\varepsilon_0^2)+z^2]/2)\}=$$
$$=-\alpha\rho_0(1-\varepsilon_0^2)\cdot\exp(-\alpha[h^2(1-\varepsilon_0^2)+z^2]/2)(1-\alpha z^2)=-\alpha\rho\cdot[1-\alpha z^2]; \tag{5d}$$

$$\nabla^2\rho=\nabla_h^2\rho+\nabla_z^2\rho=-\alpha\rho[3-\alpha(h^2(1-\varepsilon_0^2)+z^2)]+2\varepsilon_0^2\alpha\rho[1-\frac{\alpha}{2}(1-\varepsilon_0^2)h^2]. \tag{5e}$$

So, taking into account Eq.(5a) we can see that

$$\rho[3-\alpha(h^2(1-\varepsilon_0^2)+z^2)]=2\alpha\frac{\partial\rho}{\partial\alpha} \tag{6a}$$

and according to Eq.(5b) we find:

$$\rho[1-\frac{\alpha}{2}(1-\varepsilon_0^2)h^2]=-\frac{1-\varepsilon_0^2}{2\varepsilon_0}\cdot\frac{\partial\rho}{\partial\varepsilon_0}. \tag{6b}$$

With regard for Eqs.(6a), (6b) the equation (5e) becomes:

$$\nabla^2\rho=-2\alpha^2\frac{\partial\rho}{\partial\alpha}-\varepsilon_0(1-\varepsilon_0^2)\frac{\partial\rho}{\partial\varepsilon_0}. \tag{7}$$

Now we suppose that an evolution of the mass density of a rotating spheroidal body with the time can be expressed by a composite function of $\alpha=\alpha(t)$ and $\varepsilon_0=\varepsilon_0(t)$. In the first place, we obtain the main antidiffusion equation (3) in the special case of the fixed parameter ε_0 [6], [9]:

$$\frac{\partial\rho}{\partial t}=\frac{\partial\rho}{\partial\alpha}\cdot\frac{d\alpha}{dt}=\left(-\frac{1}{2\alpha^2}\cdot\frac{d\alpha}{dt}\right)\cdot\nabla^2\rho=-\mathrm{G}(t)\cdot\nabla^2\rho, \tag{8a}$$

on the other hand, the following equation is true under consideration of the fixed parameter α :

$$\frac{\partial\rho}{\partial t}=\frac{\partial\rho}{\partial\varepsilon_0}\cdot\frac{d\varepsilon_0}{dt}=\left(-\frac{1}{\alpha\varepsilon_0(1-\varepsilon_0^2)}\cdot\frac{d\varepsilon_0}{dt}\right)\cdot\nabla^2\rho. \tag{8b}$$

With a view to study a temporal evolution of the solution of Eq. (7) we need investigate the functional dependence $\varepsilon_0=\varepsilon_0(\alpha)$ occurring when $\alpha\geq\alpha_c$

(where $\alpha_{\mathrm{c}} = \alpha(t_{\mathrm{c}})$ and t_{c} is a moment of rotation origin). However, it is impossible directly by virtue of the initial statement (at the derivation of the function (4) in the work [6]) about independence α of coordinates h, ε, z. Consequently, the total derivative of the mass density function ρ (with respect to the time) can be represented by the following relation:

$$\frac{d\rho}{dt} = \frac{\partial\rho}{\partial\alpha} \cdot \frac{d\alpha}{dt} + \frac{\partial\rho}{\partial\varepsilon_0} \cdot \frac{d\varepsilon_0}{dt} . \tag{9}$$

To find $\partial\rho / \partial\varepsilon_0$ let us use Eq. (7) at the fixed parameter α whence the desired partial derivative is equal to

$$\frac{\partial\rho}{\partial\varepsilon_0} = -\frac{1}{\alpha\varepsilon_0(1-\varepsilon_0^2)} \nabla^2 \rho . \tag{10}$$

Analogously, if the parameter ε_0 is fixed then the main equation relative to α follows directly from Eq. (7):

$$\frac{\partial\rho}{\partial\alpha} = -\frac{1}{2\alpha^2} \cdot \nabla^2 \rho . \tag{11}$$

Substitution (10) and (11) in Eq. (9) leads to the following *general equation of antidiffusion* with regard to a deformation of spheroidal body in result of its rotation:

$$\frac{d\rho}{dt} = -\tilde{\mathrm{G}}(t) \nabla^2 \rho , \tag{12}$$

where $\tilde{\mathrm{G}}(t)$ is an antidiffusion function, i.e. GCF, taking into account the flattening process of a rotating ellipsoid-like spheroidal body:

$$\tilde{\mathrm{G}}(t) = \frac{1}{2\alpha^2(t)} \cdot \frac{d\alpha}{dt} + \frac{1}{\alpha\varepsilon_0(1-\varepsilon_0^2)} \cdot \frac{d\varepsilon_0}{dt} . \tag{13}$$

In the case of finite values of $\dot{\alpha}$ and $\dot{\varepsilon}_0$ the antidiffusion function $\tilde{\mathrm{G}}(t)$ can increase unlimitedly when $\alpha \to 0$ (at the so-called initial antidiffusion condensation) and when $\varepsilon_0 \to 0$ (at the initial flattening). Therefore, the antidiffusion condensation moment and the flattening moment can be the same, but, in generally, they can be inconsistent. Probably, the flattening appears when the gravitational field arises in a spheroidal body, i.e. in the case if $\alpha(t)$ exceeds its threshold value α_{c} [6]-[9].

When the parameter ε_0 becomes finite $\varepsilon_0 \neq 0$ then a sphere-like spheroidal body begins to deform (to be flattened) that implies a *bifurcation* on a diagram of dynamical states of spheroidal body. As noticed by J.Jeans [22, p.p.188, 190, 191], «on continually varying some parameter (*say* ε_0, *is allowed slowly to* *vary*[1]) we pass through a whole series of continuous configurations of equilibrium, which form what Poincaré has called a "linear series"….Every point on linear series is a configuration of equilibrium; a question which is of the utmost importance in cosmogonical problem is whether this equilibrium is stable or unstable. …Thus we see that there is an *exchange of stabilities* at the point of bifurcation».

In this connection, if we suppose that $\alpha = \alpha(t)$ is a variable of one-dimensional state-space of a spheroidal body then ε_0 can be considered as a *control* parameter [23], [24]. Really, «the conditions of secular stability assume asomewhat different form for a mass rotating freely inspace. In this case the rate of rotation is not constant, but changes as the moment of inertia of the mass changes … Secular stability is lost at a "turning point" or "point of bifurcation"» [22, pp.199-201]. Then Jeans clarified [22, p.p.207, 209] that if the angular velocity« $\Omega = 0$,…so that the configuration must be spherical. If Ω is small, although not actually zero, a spherical surface does not satisfy the condition, the term $\frac{1}{2}\Omega^2(x^2+y^2)$ destroying the spherical symmetry. In this case, as we shall see almost immediately, the configuration is that of an oblate spheroid of small ellipticity…Two linear series of equilibrium configuration, which are spheroidal and ellipsoidal respectively. The configuration which form the first series are commonly known as Maclaurin's spheroid; those which form the second as Jacobi's ellipsoids…».

So, according to Eq.(13) a variation of the spheroidal body form is caused by a dissipation, i.e. by the gravitational energychanging due to the interior energy of gaseous cloud particles. Consequently, according to (9) the flattening process cannot decrease the antidiffusion condensation in the axial direction, but it can reduce the antidiffusion condensation in the rotational plane of a spheroidal body.

As mentioned in our previous works [6]-[9], there is a *threshold (critical) value* α_{c} such as if $\alpha \geq \alpha_{\mathrm{c}}$ then a weak gravitational field (with the gravitational potential φ_{g}) arises in a spheroidal body. To decide this task, according to the formula (1) we found that the gravitational potential of a *sphere-like* (immovable or slowly rotating) spheroidal body is equal [4]:

$$\varphi_{\mathrm{g}}(r) = -\frac{4\pi\gamma\rho_0}{\alpha r}\int_0^r \exp(-\alpha x^2/2)dx = -\frac{\gamma M}{r}\mathrm{erf}\left(r\sqrt{\alpha/2}\right), \tag{14}$$

where $\gamma = 6.673\cdot 10^{-11}\,\mathrm{N\cdot m^2/kg^2}$ is Newtonian gravitational constant, $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-s^2}ds$ is the error function. Since $\lim_{r\to\infty}\mathrm{erf}\left(r\sqrt{\alpha/2}\right) = 1$, then for *large* r the last expression turns into

[1]The author's remark

$$\varphi_{\rm g}(r) = -\frac{\gamma M}{r}. \quad (15)$$

The relation (15), as known, describes the gravitational potential of a field produced by one particle (or a spherical body) of mass M. In the case of *small* r, the function $e^{-\frac{\alpha}{2}r^2} \approx 1 - \frac{\alpha}{2}r^2$ which leads to the transformation of formula (14):

$$\varphi_{\rm g}(r) = -\frac{4\pi\gamma\rho_0}{\alpha r}\int_0^r\left(1-\frac{\alpha}{2}x^2\right)dx = -\frac{4\pi\gamma\rho_0}{\alpha r}\left(r-\frac{\alpha}{6}r^3\right) = \frac{2\pi\gamma\rho_0}{3}\left(r^2-\frac{6}{\alpha}\right). \quad (16)$$

In expression (16), the higher order values of smallness of r were ignored [4], [9], [11]. Thus, expression (16) describes the gravitational potential in the near zone of the field, while Eq. (15) describes that in the remote one of immovable gravitating spheroidal body.

In the case of a rotating spheroidal body, the axial rotation of the spheroidal body creates a flattening of its core, therefore the gravitational potential *in a near zone* of uniformly rotating spheroidal body is described by the following expression in cylindrical coordinates (h, ε, z) [10]:

$$\varphi_{\rm g}(h,z) = 2\pi\gamma\rho_0\frac{1-\varepsilon_0^2}{2\left(1-\varepsilon_0^2\right)^2+1}\left[\left(1-\varepsilon_0^2\right)^2h^2 + z^2 - \frac{4(1-\varepsilon_0^2)+2}{\alpha}\right]. \quad (17)$$

The potential of gravitational field *in a remote zone* of a rotating spheroidal body deviates from the $1/r$ –gravitational field potential (15), so that for large r it estimated by the relation in spherical coordinates (r, θ, ε) [8], [9]:

$$\varphi_g(r,\theta)|_{r>>r_*} = -\sqrt{\frac{2\alpha}{\pi}}\cdot\frac{\gamma M}{r\sqrt{1-\varepsilon_0^2\sin^2\theta}}\cdot\int_0^{r\sqrt{1-\varepsilon_0^2\sin^2\theta}}e^{-\alpha r'^2/2}dr'|_{r>>r_*} \approx -\frac{\gamma M}{r\sqrt{1-\varepsilon_0^2\sin^2\theta}} = -\frac{\gamma M}{r}\left(1+\frac{\varepsilon_0^2}{2}\sin^2\theta\right). \quad (18)$$

Taking into account Eq. (14) we can find that the potential energy of a sphere-like body is described by the expression [4], [9]:

$$E_{\rm g} = -4\gamma\rho_0{}^2[\pi/\alpha]^{5/2} = -\frac{\gamma M^2}{2}\sqrt{\frac{\alpha}{\pi}}, \quad (19)$$

whereas the potential energy of a *rotating* spheroidal body is equal:

$$E_{\rm g} = -\frac{\gamma M^2}{2}\sqrt{\frac{\alpha}{\pi}}\cdot\frac{\sqrt{1-\varepsilon_0^2}}{\varepsilon_0}\,\mathrm{arccot}\,\frac{\sqrt{1-\varepsilon_0^2}}{\varepsilon_0}. \quad (20)$$

Since $\varepsilon_0 << 1$ then Eq. (20) goes over Eq. (19) under the condition $\varepsilon_0 \to 0$. It follows directly from (19) that

$$\alpha = \pi \left(\frac{2E_{\mathrm{g}}}{\gamma M^2} \right)^2 . \tag{21}$$

By analogy with Eq. (14) (see also [4], [9]) the average gravitational potential energy of interaction of a test particle of mass m_0 with a spheroidal body is

$$\overline{E}_{\mathrm{g}} = - \gamma m_0 M \sqrt{\frac{\alpha}{\pi}} , \tag{22}$$

whence

$$\alpha = \pi \left(\frac{\overline{E}_{\mathrm{g}}}{\gamma m_0 M} \right)^2 . \tag{23}$$

With usage Eqs. (19)-(23) the universal stellar law (USL) has been obtained [11].

3 The derivation of the combined Kepler 3rd law with the universal stellar law (3KL-USL) and explanation of stability of planetary orbits through 3KL-USL

For a one-component gaseous cloud (for a spheroidal body formed by a collection N of the similar particles with the masses m_0 so that $M = m_0 N$ is its mass) let us use the virial theorem of Poincaré [22], [25]:

$$2E_{\mathrm{k}} + E_{\mathrm{g}} = 0 , \tag{24}$$

where E_{k} is the total kinetic energy and E_{g} is the total gravitational potential energy of a *steady state* system in the form of a collection of particles moving under no forces except their own mutual gravitational attraction.

According to Boltzmann's molecular kinetic theory the total kinetic energy ofheat movement of particles E_{k} connects with the interior energy U of ideal gas. As noticed by S. Chandrasekhar [26], for a cloud-like configuration of ideal gas the following formula is true:

$$E_{\mathrm{k}} = \frac{3}{2} k_{\mathrm{B}} T N = \frac{3}{2} (\iota - 1) U , \tag{25}$$

where U is the interior energy of a cloud-like configuration of ideal gas, ι is the polytropic exponent, $k_{\mathrm{B}} = 1.38049 \cdot 10^{-23}\ \mathrm{J/K}$ is Boltzmann's constant. It follows directly from virial theorem (24) and the formula (25) that

$$3(\iota - 1)U + E_{\mathrm{g}} = 0 . \tag{26}$$

Let us $E_{\mathrm{g}} + U = E$ be the *total energy* of a cloud-like configuration of ideal gas. Then according to (26) we can obtain that

$$E = -(3\iota - 4)U = \frac{3\iota - 4}{3\iota - 3} E_{\mathrm{g}} . \tag{27a}$$

As shown by Chandrasekhar [26], a gaseous sphere is *stable* when $\iota > 4/3$. Let us ΔE and ΔU be a change of the total energy and a change of the interior energy respectively. Then according to (27a) the quantity of energy lost by *radiation* $-\Delta E$ during the compression process of a cloud-like configuration is equal to

$$-\Delta E = -\frac{3\iota - 4}{3\iota - 3}\Delta E_{\mathrm{g}} > 0, \tag{27b}$$

whereas the *interior energy* increases by the following quantity:

$$\Delta U = -\frac{1}{3\iota - 3}\Delta E_{\mathrm{g}} > 0. \tag{27c}$$

It follows from Eqs. (19), (27a)-(27c), a share of the gravitational potential energy in the form of the work $\left|\Delta E_{\mathrm{g}}\right|$ done by the gravitational compression only partly (in $\frac{3\iota-4}{3(\iota-1)}$ times) is scattered in space through a radiation whereas a remaining part ($1-\frac{3\iota-4}{3(\iota-1)}=\frac{1}{3(\iota-1)}$) is spent to increase the temperature T of a cloud-like gaseous configuration [26]. On the other hand, if a gravitating spheroidal body is an evolutional model of a star then a significant part of its gravitational potential energy in the compression process goes over to the particle heat movement into it. In this connection, the question arises*: how long and why is a stable level of the gravitational field in fixed points of space (for example, around stars) supported?*

The preliminary reply follows from Eqs.(19)-(22) under consideration that the parameter of gravitational compression is changing with the time, i.e. the $\alpha = \alpha(t)$. Namely, owing to the slowly increasing parameter of gravitational compression of a spheroidal body:

$$\alpha(t_2) = \alpha(t_1) + \delta\alpha, \quad t_2 > t_1, \tag{28}$$

the absolute value of gravitational potential energy is also growing:

$$E_{\mathrm{g}}(t_2) = E_{\mathrm{g}}(t_1) + \delta E_{\mathrm{g}}, \quad t_2 > t_1 \tag{29}$$

In other words, according to (19), (21), (23) the speed of dissipation of E_{g} is equal to the speed of α changing in the steady state of virial equilibrium of a spheroidal body. To answer the formulated question quantitatively let us use the universal stellar law (USL) [11].

First of all, let us note that the USL connects the temperature, the size and the mass of a star:

$$\sqrt{\alpha}\cdot\frac{\bar{\mu}_{\mathrm{r}}}{\bar{i}}\cdot\frac{m_{\mathrm{p}}\cdot M}{T} = \kappa, \tag{30}$$

where $\kappa = 3\sqrt{\pi} \cdot \dfrac{k_B}{\gamma} \approx 1.10003963 \cdot 10^{-12} (\mathrm{kg}^2 / \mathrm{K} \cdot \mathrm{m})$ is the universal stellar constant [11], $m_p = 1.67248 \cdot 10^{-27}\,\mathrm{kg}$ is the mass of proton, $\bar{i}$ is an average *relative* number of all degrees of freedom for a particle m_0 of a highly ionized stellar substance with a mean relative molecular weight $\bar{\mu}_r$, i.e. $m_0 = \bar{\mu}_r \cdot m_p$ and $i = 3\bar{i}$ is a number of all degrees of freedom.

By calculating the left part of Eq. (30) for the Sun (taking into account its mass $M_{Sun} = 1.9891 \cdot 10^{30}\,\mathrm{kg}$, the parameter of gravitational condensation $\alpha_{Sun} \approx 2.29701177718 \cdot 10^{-19} (\mathrm{m}^{-2})$ [11], the average temperature of the solar corona $T_{corSun} = 1.5 \cdot 10^6\,\mathrm{K}$ [27] and comparing with the universal stellar constant κ we find a coincidence up to the relative error equal $\delta = 3.37\%$ that testifies the validity of the USL for the Sun. To verify the USL for other stars we need approximation T_{cor} through the effective temperature T_{eff} of stellar surface of these stars, i.e. a modification of the USL.

This modified USL can be testified by calculating the left part of Eq. (30) with usage of parameters for the different types of stars (see Table 1 and Ref. [11]). First of all, we can note the satisfiability of Eq. (30) for the stars belonging to the spectral class G like the Sun: namely, Table 1 shows that the modified USL is carried out with the relative accuracy $\delta = 1.34\%$ for the star Kepler-20 of type G8, it also is carried out with the small relative error $\delta = -0.1\%$ for the star HD10180 of type G1V and with high relative accuracy $\delta = -0.75\%$ for the star HIP14810 of class G5 [11]. On the other hand, the modified USL is carried out with $\delta = -15.87\%$ for the star α Centauri of type K1V which is caused, probably, by an *inexact estimation of temperature of its corona* T_{55Cnc} for the respective class K, i.e. by means of a directly proportional dependence T_{cor} on T_{eff} as in the special case of the Sun of type G2V.Thus, the low accuracy of this law for the stars belonging to the spectral classes F or M, most likely, can be explained by too rough approximation T_{cor} for the more bright or dim stars [11].

Table 1.Verification of the modified USL for stars belonging to the different spectral classes and types

Stars	Spectral class and type	Mass M, kg	Ratio $\overline{\mu}_r / \overline{i}$	Radius R, m	Effective temperature T_{eff}, K	Relative error δ, %
ξ Persei	O7.5III	$7.16076 \cdot 10^{31}$	1/1	$9.737 \cdot 10^{9}$	35000	58.9
τ Scorpii	B0.2V	$2.98365 \cdot 10^{31}$	1/1	$4.52075 \cdot 10^{9}$	29850	56.8
γ Pegasi	B2IV	$1.770299 \cdot 10^{31}$	1/1	$3.3384 \cdot 10^{9}$	21179	51.1
α Andromedae	A3V	$7.16076 \cdot 10^{30}$	1/1	$1.87785 \cdot 10^{9}$	13800	46.1
Sirius A	A1V	$4.017982 \cdot 10^{30}$	1/1	$1.190001 \cdot 10^{9}$	9940	33.7
WASP-12	G0	$2.685285 \cdot 10^{30}$	1/1	$1.091935 \cdot 10^{9}$	6300	23.8
υ Andromedae	F8V	$2.526157 \cdot 10^{30}$	1/1	$1.134361 \cdot 10^{9}$	6212	30
KOI-94	None	$2.486375 \cdot 10^{30}$	1/1	$1.151748 \cdot 10^{9}$	6116	31.1
HD 74156	G0	$2.466484 \cdot 10^{30}$	1/1	$1.09889 \cdot 10^{9}$	6039	27
Kepler-60	None	$2.18801 \cdot 10^{30}$	1/1	$1.04325 \cdot 10^{9}$	5915	30.8
HD 10180	G1V	$2.108446 \cdot 10^{30}$	1/1	$6.955 \cdot 10^{8}$	5911	-0.1
Kepler-33	None	$2.567928 \cdot 10^{30}$	1/1	$1.26581 \cdot 10^{9}$	5904	32.9
HD 155358	G0	$1.829972 \cdot 10^{30}$	1/1	$6.955 \cdot 10^{8}$	5900	12.9
47 Ursae Majoris	G0V	$2.048773 \cdot 10^{30}$	1/1	$8.6242 \cdot 10^{8}$	5892	21
Sun	G2V	$1.9891 \cdot 10^{30}$	1/1	$6.955 \cdot 10^{8}$	5778	3.37
HD 1461	G0V	$2.148228 \cdot 10^{30}$	1/1	$7.615725 \cdot 10^{8}$	5765	4.5
μ Andromedae	G3IV-V	$2.148228 \cdot 10^{30}$	1/1	$8.658975 \cdot 10^{8}$	5700	15
Kepler-11	G	$1.889645 \cdot 10^{30}$	1/1	$7.6505 \cdot 10^{8}$	5680	15.1
HAT-P-13	G4	$2.426702 \cdot 10^{30}$	1/1	$1.08498 \cdot 10^{9}$	5638	22.6
HD 37124	G4V	$1.810081 \cdot 10^{30}$	1/1	$5.7031 \cdot 10^{8}$	5610	-10.4
KOI-730	None	$2.128337 \cdot 10^{30}$	1/1	$7.6505 \cdot 10^{8}$	5590	2.87
61 Virginis	G5V	$1.889645 \cdot 10^{30}$	1/1	$6.5377 \cdot 10^{8}$	5531	-1.99
HIP 14810	G5	$1.969209 \cdot 10^{30}$	1/1	$6.955 \cdot 10^{8}$	5485	-0.75
Kepler-20	G8	$1.814059 \cdot 10^{30}$	1/1	$6.56552 \cdot 10^{8}$	5466	1.34
α Centauri	K1V	$1.857819 \cdot 10^{30}$	1/1	$6.002165 \cdot 10^{8}$	5214	-15.87
55 Cancri	K0IV-V	$1.800136 \cdot 10^{30}$	1/1	$6.558565 \cdot 10^{8}$	5196	-3.1
24 Sextanis	G5	$3.063214 \cdot 10^{30}$	2/1	$3.40795 \cdot 10^{9}$	5098	31.17
18 Delphini	G6III	$4.57493 \cdot 10^{30}$	2/1	$5.91175 \cdot 10^{9}$	4979	39.33
Capella A	K0III	$5.350679 \cdot 10^{30}$	2/1	$8.4851 \cdot 10^{9}$	4940	50.17
14 Andromedae	K0III	$4.37602 \cdot 10^{30}$	2/1	$7.6505 \cdot 10^{9}$	4813	53.61
γ Cephei	K1III-V	$2.78474 \cdot 10^{30}$	2/1	$3.40795 \cdot 10^{9}$	4800	33.55
β Ceti	K0III	$5.56948 \cdot 10^{30}$	2/1	$1.167049 \cdot 10^{10}$	4797	61.17
ξ Aquilae	G9III	$4.37602 \cdot 10^{30}$	2/1	$8.346 \cdot 10^{9}$	4780	57.18
11 Comae	G8 III	$5.37057 \cdot 10^{30}$	2/1	$1.32145 \cdot 10^{10}$	4742	66.54
HIP 57274	K5V	$1.452043 \cdot 10^{30}$	2/2	$4.7294 \cdot 10^{8}$	4640	-29.15
Groombridge34	M1.5V	$8.035964 \cdot 10^{29}$	2/2	$2.635945 \cdot 10^{8}$	3730	-59.5
Gliese 581	M2.5V	$6.16621 \cdot 10^{29}$	2/2	$2.0865 \cdot 10^{8}$	3498	-64.9
Gliese 876	M4V	$6.643594 \cdot 10^{29}$	2/2	$2.5038 \cdot 10^{8}$	3350	-54.59

Supposing the mass of a spheroidal body to be a constant, i.e. $M = \mathrm{const}$, let us apply the USL (30) for two *different states* of a spheroidal body under assumption that $\alpha = \alpha(t)$ is a variable in a dynamical state-space of this spheroidal body:

$$\sqrt{\alpha_1} \cdot \frac{\bar{\mu}_{\mathrm{r}1}}{\bar{i}_1} \cdot \frac{m_{\mathrm{p}} M}{T_1} = \kappa, \tag{31a}$$

$$\sqrt{\alpha_2} \cdot \frac{\bar{\mu}_{\mathrm{r}2}}{\bar{i}_2} \cdot \frac{m_{\mathrm{p}} M}{T_2} = \kappa, \tag{31b}$$

where $\alpha_l = \alpha(t_l)$, $\bar{\mu}_{\mathrm{r}l} = \bar{\mu}_{\mathrm{r}}(t_l)$, $\bar{i}_l = \bar{i}(t_l)$ and $T_l = T(t_l)$, $l = 1, 2$. Dividing Eq.(31b)on Eq.(31a) we obtain:

$$\frac{\sqrt{\alpha_2}}{\sqrt{\alpha_1}} = \frac{T_2}{T_1} \cdot \frac{\bar{i}_2}{\bar{i}_1} \cdot \frac{\bar{\mu}_{\mathrm{r}1}}{\bar{\mu}_{\mathrm{r}2}}. \tag{32}$$

As noted in the work [11] as a rule $\bar{i}_1 = \bar{i}_2 = 1$ for a highly ionized stellar substance, therefore following Eq. (32) we conclude that

$$\frac{\alpha_2}{\alpha_1} = \left(\frac{T_2}{T_1} \cdot \frac{\bar{\mu}_{\mathrm{r}1}}{\bar{\mu}_{\mathrm{r}2}} \right)^2. \tag{33}$$

The ratio shows that the parameter of gravitational condensation α increases when the temperature T of a shell of spheroidal body (called the stellar corona) grows whereas the mean relative molecular weight $\bar{\mu}_{\mathrm{r}}$ reduces, i.e. α is directly proportional to the squared T and inversely proportional to the squared $\bar{\mu}_{\mathrm{r}}$. The finding $\bar{\mu}_{\mathrm{r}}$ is not simple task in the case of a highly ionized stellar substance [26], so that the formula (33) can be used for calculating $\bar{\mu}_{\mathrm{r}}$ in the process of evolution of a star:

$$\bar{\mu}_{\mathrm{r}2} = \bar{\mu}_{\mathrm{r}1} \cdot \frac{T_2}{T_1} \cdot \frac{\sqrt{\alpha_1}}{\sqrt{\alpha_2}}. \tag{34}$$

Now with aim to answer the main question on how a stable level of the gravitational potential φ_g is supported around a star in exoplanet system and how owing to it the stability of planetary orbits occurs let us consider jointly the USL (30) and Kepler 3rd law (3KL) [28], [29]:

$$\frac{a^3}{\mathrm{T}_K^2} = \frac{\gamma M}{4\pi^2}, \tag{35}$$

where a is a major semi-axis of a planetary orbit, T_K is a Keplerian period of motion of a planet around its star (belonging to the Solar or exoplanet system).

Taking into account the formula for the universal stellar constant $\kappa = 3\sqrt{\pi} \cdot \frac{k_{\mathrm{B}}}{\gamma}$ let us rewrite the USL (30) in the following way:

$$\sqrt{\frac{\pi}{\alpha}} \cdot \frac{3\bar{i}}{m_{\mathrm{p}}\bar{\mu}_{\mathrm{r}}} \cdot k_{\mathrm{B}}T = \gamma M \,. \tag{36}$$

Then substituting the left part of (36) to the right part of Eq. (35) we obtain *the combined Kepler's 3rd law with the universal stellar law* (3KL-USL):

$$\frac{a^3}{\mathrm{T}_K^2} = \frac{3}{4\pi^{3/2} m_{\mathrm{p}}} \cdot \frac{\bar{i}}{\bar{\mu}_{\mathrm{r}}} \cdot \frac{\theta}{\sqrt{\alpha}} \ , \tag{37a}$$

where $\theta = k_{\mathrm{B}}T$ is a statistical temperature [30]. The combined law (37a) shows that the stability of parameters of moving planet in orbit is determined by constancy of the value:

$$\frac{\theta}{\sqrt{\alpha}} \cdot \frac{\bar{i}}{\bar{\mu}_{\mathrm{r}}} = \mathrm{const}\,, \tag{37b}$$

that is confirmed by Eq. (32) completely.

Introducing a Keplerian angular velocity $\Omega_K = 2\pi/\mathrm{T}_K$ of rotation of a planet around star the combined law (37a) can be written in the following form:

$$a^3\Omega_K^2 = s \cdot \frac{\bar{i}}{\bar{\mu}_{\mathrm{r}}} \cdot \frac{T}{\sqrt{\alpha}}\,, \tag{38}$$

where

$$s = 3\sqrt{\pi} \cdot \frac{k_{\mathrm{B}}}{m_{\mathrm{p}}} \tag{39a}$$

is a constant with the value equal to

$$s \approx 3\sqrt{3.14159265} \cdot \frac{1.38049 \cdot 10^{-23}\,\mathrm{J/K}}{1.67248 \cdot 10^{-27}\,\mathrm{kg}} = 4.3890297 \cdot 10^4 (\mathrm{J/kg \cdot K})\,.$$

Let us note that this constant has physical measure of a specific heat capacity c_V or a specific entropy s [28], i.e. according to Eqs. (38), (39a) the stability of parameters of planetary orbits is determined by a *constancy of the specific entropy* s in conformity with the principles of self-organization in complex systems [24]. Really, the constant s can be represented as the following:

$$s = 3\sqrt{\pi} \cdot \frac{k_{\mathrm{B}}}{\mu_{\mathrm{p}}} \cdot N_A = 3\sqrt{\pi} \cdot \frac{\Re}{\mu_{\mathrm{p}}} = \frac{3\Re}{2} \cdot \frac{2\sqrt{\pi}}{\mu_{\mathrm{p}}} = c_{\mu V}^{(1)} \cdot \frac{2\sqrt{\pi}}{\mu_{\mathrm{p}}}\,, \tag{39b}$$

where $c_{\mu V}^{(1)}$ is a molar heat capacity of one-atomic gas under the condition of its constant volume V, N_A is Avogadro's constant, $\Re$ is the universal gaseous constant [28]. Then according to Eq. (35), (38) and (39b) we obtain:

$$\gamma M = a^3\Omega_K^2 = s\cdot\frac{\bar{i}}{\bar{\mu}_{\mathrm{r}}}\cdot\frac{T}{\sqrt{\alpha}} = c_{\mu V}^{(1)}\cdot\frac{2\sqrt{\pi}}{\mu_{\mathrm{p}}}\cdot\frac{\bar{i}}{\bar{\mu}_{\mathrm{r}}}\cdot\frac{T}{\sqrt{\alpha}} = \\ = 2\sqrt{\pi}\cdot\frac{c_{\mu V}^{(i)}}{\mu}\cdot\frac{T}{\sqrt{\alpha}} = \frac{2\sqrt{\pi}}{\sqrt{\alpha}}\cdot c_V T = \frac{2\sqrt{\pi}}{\sqrt{\alpha}}\cdot u \quad , \quad (40a)$$

where $u = c_V T$ is a specific value of the interior energy of ideal gas, $c_V = c_{\mu V}^{(i)} / \mu$ is a specific heat capacity, $c_{\mu V}^{(i)} = \bar{i}\, c_{\mu V}^{(1)} = 3\bar{i}\,\Re/2 = i\Re/2$ is a molar heat capacity of ideal gas, besides $\mu = \bar{\mu}_{\mathrm{r}}\mu_{\mathrm{p}}$. Since the specific value of the interior energy is $u = dU/dm$ (where U is an interior energy of a spheroidal configuration of ideal gas, dm is an elementary mass) we can suppose that $dU/dm \approx U/M$ at the initial stage of formation of a spheroidal body with the total mass M. Then taking into account the formulas (19), (40a) we obtain that

$$U \approx \sqrt{\frac{\alpha}{\pi}}\cdot\frac{\gamma M^2}{2} = \left|E_{\mathrm{g}}\right|, \quad (40b)$$

whence $\imath \approx 4/3$ in accord with Eq. (26), i.e. the gravitational potential energy is spent by the interior energy during the *initial stage of formation* of a spheroidal body.

The combined law connects the mechanical values a and Ω in the left part of Eq. (38) and the statistical (thermodynamic) values α, T, $\bar{i}$ and $\bar{\mu}_{\mathrm{r}}$ in the right part of this Eq. (38). It means that a stability of the mechanical values (including the angular velocity Ω_K and the major semi-axis a of a planetary orbit) depends on a statistical regularity of the right part of Eq.(38). Thus, we conclude about a possibility of presence of *statistical oscillations* of motion in planetary orbit, i.e. the oscillations of the major semi-axis a and the orbital angular velocity Ω_K of rotation of planets and bodies around stars. Really, this conclusion is completely confirmed by existing the radial and the axial orbital oscillations of bodies for the first time described by Alfvén and Arrhenius [1]-[3].

3 Conclusions

We investigate the stability of planetary orbits based on the statistical theory of gravitating spheroidal bodies [4]-[11]. Using the obtained universal stellar law (USL) and the modification of the USL connecting temperature, size and mass of star (see Table 1) we show that knowledge of some orbital characteristics of multi-planet extrasolar systems refines the knowledge of the parameters of the stars based on the combined Kepler 3rd law with universal stellar law (3KL-USL).

Really, the combined 3KL-USL law connects among themselves both the mechanical values (the Keplerian angular velocity Ω_K and the major semi-axis a of a planetary orbit) and the statistical (thermodynamic) values (the parameter of gravitational condensation α and the temperature T) in the Eq.(38). It means that a stability of the mechanical values (entering in the left part) directly depends on a statistical regularity of the right part of the 3KL–USL equation. This relation points to a possibility of presence of *statistical oscillations* of body motion in planetary orbit. This conclusion is confirmed by existing Alfvén–Arrhenius' radial and axial orbital oscillations of bodies [1]-[3]. This work also shows that the stability of parameters of planetary orbits is determined by a *constancy of the specific entropy* (39b) in conformity with the principles of self-organization in complex systems. Therefore, the proposed 3KL–USL explains the stability of planetary orbits in the extrasolar systems.

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